- \( \vec{V} \) (at rest) \( \Rightarrow \) no radiation

- \( \vec{V} \) constant \( \vec{V} \) \( \Rightarrow \) no radiation

- \( \vec{V}(t_2) \neq \vec{V}(t_1) \)

Larmor: \( \text{Prad} = \frac{2}{3} \frac{q^2 |\vec{a}|^2 \vec{\epsilon}}{c^3} \)

- non-relativistic
- \( \frac{2}{3} \Rightarrow \text{peaks} \perp \text{to direction} \)
- Let's the Universe know... question...
  - electrons orbiting nuclei

\( \vec{B} \) fields: caused by charges in motion
5) Ampere's Law

\[ \int \mathbf{B} \cdot d\mathbf{s} = \frac{4\pi}{c} I \text{ (that penetrates surface)} \]

Check: \( \frac{2I}{rc} \cdot 2\pi r = \frac{4\pi}{c} I \)

Point: path need not be perfect

Analogous to Gaussian Surface "Amperean Loop"

\[ \mathbf{D} \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} = \text{current density} \]

6) Biot-Savart

\[ d\mathbf{B} = \frac{I d\mathbf{A} \times \hat{r}}{cr^2} \]
QUALITATIVE paradigm

Long straight wire. \( \vec{B} \) around \( \text{in circles} \)

1) Right Hand Rule:
   - Thumb along current
   - Fingers along \( \vec{B} \)

2) \( \vec{B} \): no start, no end!
   - \( \vec{E} \): starts on charge.
   \[
   \nabla \cdot E = \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right) (E_x + E_y + E_z) = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 4\pi \rho
   \]

   \( \nabla \cdot \vec{B} = 0 \) (no magnetic charge)

3) In this case,
   \[
   |B| = \frac{2I}{rc}
   \]

4) \( \vec{F} = q \frac{\vec{V}}{c} \times \vec{B} \) - Cross product important. \( \vec{V} \) (wires) rotate
Origin of Magnetic Force

\[ \text{wire with current } I \text{ no net charge!} \]

\[ q > 0 \quad \overrightarrow{V} \quad \text{(like a current II to I)} \]

\( \text{feels repulsive force. (opposite currents repel)} \)

\[ \text{Lab Frame: } \text{ions, } + \text{ charge, immobile, } \lambda_0 \text{ esu/cm} \]

\[ \text{Blow up wire: } \text{electrons moving left to right, Velocity } \overrightarrow{V_0} \]

\[ \beta_0 = \frac{V_0}{c} \]

\[ \text{negative charge current from right to left.} \]

\[ \text{NO } \overrightarrow{E} \text{ field!! } \lambda_0 - \lambda_0 = 0 \]

Boost to charge particle rest frame.

\[ \overrightarrow{V} \quad \text{+ charge density: } \lambda_0 \cdot \beta > \lambda_0 \text{ due to length contraction} \]

\[ \beta = \frac{V}{c} \]

\[ \beta_0' = \frac{\beta_0 - \beta}{1 - \beta \beta_0} \]

\[ 1 - \beta_0'^2 = 1 - \frac{(\beta_0 - \beta)^2}{(1 - \beta \beta_0)^2} = \frac{(1 - \beta \beta_0)^2 - (\beta_0 - \beta)^2}{(1 - \beta \beta_0)^2} \]

\[ q \text{ at rest } \rightarrow \overrightarrow{V_0} \rightarrow \text{what is } \Delta_0' ? \]
\[
\frac{1}{\gamma_0^1} = 1 - \beta_0^1 = \frac{1 - 2 \beta \beta_0 + \beta^2 \beta_0^2 - \beta_0^2 + 2 \beta \beta_0 - \beta^2}{(1 - \beta \beta_0)^2}
\]

\[
\frac{1}{\gamma_0^1} = \frac{1 - \beta_0^2 - \beta^2 + \beta^2 \beta_0^2}{(1 - \beta \beta_0)^2} = \frac{(1 - \beta^2)(1 - \beta \beta_0)}{(1 - \beta \beta_0)^2}
\]

\[
\gamma_0^1 = \frac{1 - \beta \beta_0}{\sqrt{1 - \beta^2}} = \gamma \gamma_0 (1 - \beta \beta_0)
\]

want the charge density of the negative charge in \(q\)’s rest frame.
Route: (2 steps)

\[
-\frac{\lambda_0 \text{ esu}}{\text{cm}} \Rightarrow -\frac{\lambda_0 \text{ esu}}{\gamma_0 \text{ cm}} \Rightarrow -\frac{\lambda_0 \gamma_0^1 \text{ esu}}{\gamma_0 \text{ cm}}
\]

wire rest frame already length contracted

charge density \(q\) rest in frame \(\frac{-\lambda_0}{\gamma_0} \text{ esu} (1 - \beta \beta_0)\)

\[
= -\lambda_0 \gamma_0 (1 - \beta \beta_0)
\]
Net charge density:

\[ \lambda' = -\lambda_0 - \lambda_0 (1 - \beta \beta_0) \]

for \( q \) at rest frame

\[ \lambda' = + \gamma \beta \beta_0 \lambda_0 \]

Note, linear in velocity \( \dot{q} \)

In \( q \)'s rest frame, there is a net charge density!

Radial:

\[ E_r' = \frac{2 \lambda'}{r'} = \frac{2 \gamma \beta \beta_0 \lambda_0}{r} \]

Boost back to the lab frame:

\[ F_r = \frac{d p_r}{d \tau} = \frac{1}{\gamma} \frac{d p_r'}{d \tau'} = \frac{1}{\gamma} q \cdot E_r' \quad r = r' \]

(C to base)

\[ F_r = \frac{2 \beta \beta_0 \lambda_0 q}{r} \]

\[ F_y = -F_r = -\frac{2 \beta \beta_0 \lambda_0 q}{r} \]

\[ I = -\lambda_0 \quad \nu_0 = -\lambda_0 \beta_0 c \quad \beta = v_x c \]

\[ F_y = \frac{2 I}{r \cdot c^2} q \cdot v_x \]
Pair of wires: reason through contractions, note:
now in + rest frame
in that frame, no force on + charge!

"boost" or "jump" to - charge rest frame, to analyze that
\[ \vec{V} \text{ bunches up} \]

\[ \sum \vec{F} (\text{sees net}) \]

\[ \text{A positive charge will feel ATTRACTION.} \]

net attraction, when we boost back to original frame
What about charge heading toward wire?

\[ \ell \quad \circ \quad \circ \quad \circ \quad \circ \quad \circ \quad \circ \quad \circ \quad \rightarrow \]

\[ \begin{array}{c}
\uparrow \quad \vec{V} \\
\rightarrow \quad \vec{E} \quad \text{(tends to)}
\end{array} \]

weak

\[ \vec{E} \quad \text{(not!)} \]

strong
Definition of $\vec{E}$: put charge $q$ at rest at some point in space. Measure $\vec{F}$.

$$\vec{E} = \frac{\vec{F}}{q}$$

Definition of $\vec{B}$: (magnetic field) go to that same spot in space, give $q$ a velocity $\vec{v}$, measure $\vec{F}'$.

"magnetic force" $= \vec{F}' - \vec{E}q$

$$\equiv q \frac{\vec{v}}{c} \times \vec{B} \quad \text{(by definition)}$$

Presence of cross product makes solving for $\vec{B}$ a challenge!

$\ I < 0$

$$\vec{E} = \frac{2I}{rc^2} q \nu_x \hat{\uparrow} \quad \text{(note)}$$

Deduce $\vec{B}$ in $\hat{z}$ direction.
Physics 24 Nelson

\[ F = qE + qv \times B \]

MKS: \[ F = \frac{qE}{c} + qv \times B \]

Newton's Second Law

- Coulomb's Law
- Ex. 11

- electric field
- not a conservative field

- \( E = \frac{\sigma B}{c} \) 
- \( B = \frac{2I}{nc} \) 
- \( \frac{9 \times 10^4}{B} = \frac{2I}{nc} \)

- Gauss's Law
- \( \nabla \cdot E = \frac{\rho}{\varepsilon_0} \)

- electric flux
- \( \Phi_E = B \cdot A \)

-remember

- right angles
- \( R \cdot \text{m}^2 \cdot \text{s}^{-1} \)

- \( \frac{9 \times 10^4}{B} = \frac{2I}{nc} \)

- \( B = \frac{2I}{nc} \)

- \( \frac{9 \times 10^4}{B} = \frac{2I}{nc} \)
The weird thing here is computing $B$ from $I$.

$$B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r} \quad \mu_0 = 4\pi \cdot 10^{-7}$$

1 Tesla = 10⁴ Gauss

**Force/Length on Wires**

![Diagram showing forces on wires](image)

$$\vec{B}_{21} = \frac{2I_1}{dc}$$

$$I_2 = n_2 q_2 v_2$$

$n_2 = \# \text{ of charges/length}$

$q_2 = \text{charge of carriers}$

$v_2 = \text{velocity of carriers}$

**Force on one carrier**

$$= \vec{B}_{21} \frac{q_2 v_2}{c}$$

**Force on $n_2$ carriers/length**

$$= n_2 \vec{B}_{21} \frac{q_2 v_2}{c} = \frac{2I_1 I_2}{dc^2}$$
\[
\frac{dF}{dl} = \frac{\mu_0 I^2}{2\pi} \frac{dA}{dl}
\]

This equation is used to define the magnetic field due to a current-carrying wire.
Ampere's Law

Current Spread Out... "Uniformly"

Current / (Area \( \rightarrow \) \( \uparrow \) to direction of flow.)

\[
J = \frac{I}{\left(\frac{\pi D^2}{4}\right)} = \frac{4I}{\pi D^2}
\]
everywhere!

Find \( \mathbf{B} \) everywhere! Look End on, use Ampere's Law!

\[
\oint \mathbf{B} \cdot d\mathbf{l} = \frac{4I}{\pi D^2}
\]

axis of symmetry
Imagined loop, radius \( r \)
Cylindrical symmetry
Imagine tiny tiny... I don't want to cut them.

\[ B_\theta = -\frac{8I}{D^2} \]

\[ \Theta = 2\pi r B_\theta = \frac{4\pi}{c} \times (-4\pi (\bar{5})^2) \]

\[ = \oint_{\text{circumference}} \frac{4I}{(D)^2} \]

\[ \int_{0}^{2\pi} \frac{4I}{(D)^2} \]

Current 'enclosed' (t. Right Final)

opposite to 'Radial'

I don't know...