along <u>second</u> leg, $\vec{ds} \propto \hat{j}$, $= dy\,\hat{j}$

$$\vec{E} = Ky\,\hat{i} + Kx_2\,\hat{j}$$

$$\vec{E} \cdot \vec{ds} = (Ky\,\hat{i} + Kx_2\,\hat{j}) \cdot dy\,\hat{j}$$

$$= Kx_2\,dy$$

$$-\int_{(x_2,0)}^{(x_2,y_2)} \vec{E} \cdot \vec{ds} = -Kx_2 y_2 + \text{constant}$$

$$\phi(x,y) = -Kxy \qquad \left(\begin{array}{c}\text{setting} \\ \phi(0,0) = 0\end{array}\right)$$

Energy change when a charge q moves from $(0,0)$ to (x,y)?

$$U(x,y) = q\phi(x,y) = -qKxy$$

↑ "downhill"

$$\phi(x,y) = \text{constant} = -Kxy = C$$

$$y = \frac{-C}{Kx}$$

(draw on field plot)

Other direction... given $\phi(x,y)$, which is a scalar function, how do you get $\vec{E}$?

Consider $f(x,y,z)$ $\left(= x^2 y z^3 \right)$

$$\frac{\partial f}{\partial x} = 2xyz^3 \qquad \frac{\partial f}{\partial y} = x^2 z^3 \qquad \frac{\partial f}{\partial z} = 3x^2 y z^2$$

$$\vec{\nabla} f \equiv \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}$$

direction that gives maximum change in f.

purcell says $\hat{x}$ $\hat{y}$ $\hat{z}$

$\vec{\nabla} f$ is called the gradient of f.

Think about it....

when $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ all >0, $\vec{\nabla} f$

gets the most out of each derivative

"equal representation"... problem on set.

If $f(r)$, then $\vec{\nabla} f = \hat{r}\frac{\partial f}{\partial r}$

3d 2d

$r = \sqrt{x^2+y^2+z^2}$ or $\sqrt{x^2+y^2}$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial r}\frac{\partial r}{\partial x}$$

$$\frac{\partial r}{\partial x} = \frac{1}{2}\frac{2x}{\sqrt{x^2+y^2+z^2}} = \frac{x}{r}$$

$$= \frac{\partial f}{\partial r}\frac{x}{r}$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial r}\frac{y}{r}$$

$$\frac{\partial f}{\partial z} = \frac{\partial f}{\partial r}\frac{z}{r}$$

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k} = \frac{\partial f}{\partial r}\underbrace{\frac{(x\hat{\imath}+y\hat{\jmath}+z\hat{k})}{r}}_{\hat{r}}$$

$$\vec{\nabla} f = \hat{r}\frac{\partial f}{\partial r}$$

Relationship between φ + $\vec{E}$

$$\varphi_{21} = -\int_{P_1}^{P_2} \vec{E}\cdot d\vec{s}$$

$-$: push + charge against

$$\varphi_{21}^{d\vec{s}\,'} = -\int^{P_2 + d\vec{s}\,'} \vec{E}\cdot d\vec{s}$$

$d\vec{s}\,' = dx'\hat{\imath} + dy\hat{\jmath} + dz'\hat{k}$

$$\varphi_{21}^{d\vec{s}\,'} \cong \varphi_{21} - d\vec{s}\,' \cdot \vec{E}(P_2)$$

$$\cong \varphi_{21} + dx'\frac{\partial \varphi_{21}}{\partial x} + d\cancel{x}y'\frac{\partial \varphi_{21}}{\partial y} + dz'\frac{\partial \varphi_{2}}{\partial z}$$

$$= \varphi_{21} + \underbrace{(dx'\hat{\imath} + dy'\hat{\jmath} + dz'\hat{k})}_{d\vec{s}\,'} \cdot \left(\frac{\partial \varphi_{21}}{\partial x}\hat{\imath} + \frac{\partial \varphi_{21}}{\partial y}\hat{\jmath} + \frac{\partial \varphi_{21}}{\partial z}\hat{k}\right)$$

$$\frac{\partial \varphi_{21}}{\partial x}\hat{\imath} + \frac{\partial \varphi_{21}}{\partial y}\hat{\jmath} + \frac{\partial \varphi_{21}}{\partial z}\hat{k} = -E_x(P_2)\hat{\imath} - E_y(P_2)\hat{\jmath} - E_z(P_2)\hat{k}$$
$$= -\vec{E}(P_2$$

so,
$$\vec{E} = -\vec{\nabla}\varphi$$

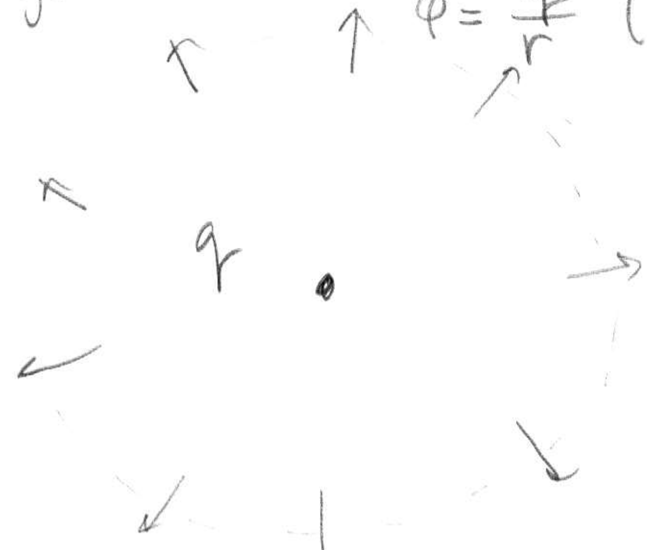
examples: $\varphi = -Kxy$

$$-\frac{\partial \varphi}{\partial x} = Ky = E_x \qquad -\frac{\partial \varphi}{\partial y} = +Kx = E_y$$

Calculating Potentials

One charge:

$\varphi = \frac{q}{r}$ (as $r \to \infty$, $\varphi \to 0$)



$E(\text{radial}) = \frac{q}{r^2}$

$$\vec{E} = \frac{q}{r^2}\hat{r}$$