Topics this week:

- Simple $\vec{B}$ field:
  - Magnetic Dipoles: Earth, Alignment with $\vec{B}$
  
  \[ \vec{F} = q(\vec{E} + \vec{B} \times \vec{B}) \]
  
  in cgs!

- How $\vec{E}$ fields transform between inertial reference frames.

- Field of a moving point charge.

- Radiation!!

- Origin of $\vec{B}$ from \ldots RELATIVITY!
Super-Quick Relativity (Appendix A)

Lengths are longest in the rest frame
time intervals are shortest (twin
younger)

\[ \beta = \frac{v}{c} < 1 \quad \Rightarrow \quad \beta = \frac{v}{c} \]

\[ \gamma = \frac{1}{\sqrt{1 - \beta^2}} > 1 \quad \Rightarrow \quad \beta = \sqrt{1 - \frac{1}{8^2}} \]

\[ \Delta t = \Delta t' \quad \Rightarrow \quad \Delta t' = \frac{\Delta t}{\gamma} \quad \Rightarrow \quad (L \text{ in } F') \]

\[ \Delta t = \gamma \Delta t' \quad \Rightarrow \quad \text{clock, say at rest in } F' \]

\[ \Delta x' = 0 \]

4 vectors: \( (\vec{x}, t) \) or \( (t, \vec{x}) \)

\[ (\vec{p}, E) \text{ or } (E, \vec{p}) \]

\[ u_x' = \frac{u_x - v}{1 - u_x v / c^2} \]
Forces: \[ dp_{ii} = \gamma dp_{ii} \]
\[ d+ = \sigma d+ \]

\[ F_{ii} = \frac{dp_{ii}}{d+} = \frac{dp_{ii}'}{d+}, \quad \text{like length} \]

but \[ F_{i} = \frac{dp_{i}}{d+} = \frac{1}{\sigma} \frac{dp_{i}'}{d+} \]
Magnetism

Currents attract

Like currents attract

Putting a conducting plate between the wires does not eliminate or reduce the forces. \[ \vec{F} = q \left( \vec{E} + \frac{\vec{v}}{c} \times \vec{B} \right) \]
Topics this week:

- Simple $\vec{B}$ field:
  - Magnetic Dipoles: Earth, alignment with $\vec{B}$
  - $\vec{F} = q(\vec{E} + \vec{B} \times \vec{v})$
  - in cgs!

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\[ \gamma = \frac{1}{\sqrt{1 - \beta^2}} > 1 \quad \beta = \sqrt{1 - \frac{1}{\gamma^2}} \]

\[ \Delta t = 0 \quad L / \gamma \quad (L \text{ in } \mathcal{F}') \quad \Delta t' = \gamma \Delta t \] at rest
clock, say at rest in \( \mathcal{F} \),

\[ \Delta x' = 0 \]

4 vectors: \((x, t)\) or \((t, x)\)
\((\vec{p}, E)\) or \((E, \vec{p})\)

\[ U_x' = \frac{U_x - \frac{v}{c}}{1 - \frac{U_x v}{c^2}} \]
\[ \frac{dF}{dp} = \frac{d^2p}{dx^2} = \frac{d}{dx} \left( \frac{dE}{dF} \right) \]

\[ E = \int_0^L F(x) \, dx \]

Forces:
\[ dp_i = \frac{\partial F}{\partial p_i} \]

\[ F = \int_{p_0}^{p_1} \left[ \frac{\partial F}{\partial p_1} \right] \, dp_1 \]
Magnetism

- Compass needle initially II to wire, turns to I to current
- Current in wire

Currents attract

- Like currents attract
- Opposite currents repel

Putting a conducting plate between the wires does not eliminate or reduce the forces!

\[ \mathbf{F} = q \left( \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right) \]
Charge Independent of $\vec{V}$ \( (\!\!) \)

mass was not: \( \frac{m_0}{\sqrt{1 - (\frac{V}{c})^2}} \)

use Gauss' Law:

\[
\int_{\text{Surface}} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}
\]

even if charge moving at speed of light \( c \).

Note: \((x,t)\) of surface is actually different as viewed in different frame: \((x',t')\). But

\[
\int_{S} \vec{E} \cdot d\vec{A} = \int_{S'} \vec{E}' \cdot d\vec{A}' \quad (\!\!)
\]

since charge is enclosed in both cases
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Field Viewed From a Moving Charge

\[ \vec{E} = 0 \]

\[ \vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \]

"primed frame"

\[ E' = 8E \]

more generally, \[ E'_i = 8E_i \]

\[ \vec{E}' \perp \text{to direction of motion} \]

How about \( d \) to direction of motion

\[ d \cdot (1-\beta^2) = d/8 \]

but, \[ E'_{ii} = \vec{E}_{ii} \]
Field of a Point Charge That Is Moving

At $t = t' = 0$
$x = x'_0, y = y'_0, z = z'_0$

$\mathbf{E} = \frac{Q}{r^2} \cos \theta = \frac{Q}{r^2} \cdot \frac{x}{r} = \frac{Qx}{r^3}$

$= \frac{Qx}{(x^2 + z^2)^{3/2}}$

$\mathbf{E}_z = \frac{Q}{r^2} \cdot \sin \theta = \frac{Q}{r^2} \cdot \frac{z}{r} = \frac{Qz}{r^3}$

$= \frac{Qz}{(x^2 + z^2)^{3/2}}$

unprimed frame
\[ x = \gamma (x' - \beta ct') = \gamma x' \quad a^+ + t' = 0 \]
\[ y = y' \]
\[ z = z' \]
\[ + = \gamma (t' - \beta \frac{x'}{c}) = -\gamma \beta \frac{x'}{c} \quad a^+ + t' = 0 \]

\[ E'_x = \frac{Qx'}{(x'^2 + z'^2)^{3/2}} = \frac{\gamma Qx'}{(\gamma x')^2 + z'^2)^{3/2}} \]

\[ \text{ill to direction of motion} \]

\[ E'_z = \gamma E_z = \frac{\gamma Qz}{(x'^2 + z'^2)^{3/2}} = \frac{\gamma Qz'}{(\gamma x')^2 + z'^2)^{3/2}} \]

\[ \text{perpendicular to direction of motion} \]

\[ \text{note: } \frac{E'_x}{E'_z} = \frac{x'}{z'} \quad \{ E'_1 \text{ still points along radii} \} \]

How strong is it?

\[ E_{x'}^{12} + E_{z'}^{12} = \frac{\gamma^2 Q^2 (x'^2 + z'^2)}{(\gamma^2 x'^2 + z'^2)^3} \]

\[ = \frac{\gamma^2}{\gamma^6} \frac{Q^2 (x'^2 + z'^2)}{(x'^2 + \frac{1}{\gamma^2} z'^2)^3} \]

\[ \frac{1}{\gamma^2} = 1 - \beta^2 \]

\[ = \frac{1}{\gamma^4} \frac{(x'^2 + z'^2)}{(x'^2 + z'^2)^3} \cdot \frac{Q^2}{(1 - \frac{\beta^2 z'^2}{x'^2 + z'^2})^3} \]
\[ E' = \frac{Q}{r'^2} \cdot \frac{1 - \beta^2}{(1 - \beta^2 \sin^2 \theta')^{3/2}} \]

1) \( \theta' = 0 \rightarrow E' = \frac{Q}{r'^2} \cdot (1 - \beta^2) \)

2) \( \theta' = \frac{\pi}{2} \rightarrow E' = \frac{Q}{r'^2} \cdot \frac{1 - \beta^2}{(1 - \beta^2)^{3/2}} = \frac{Q}{r'^2} \cdot \frac{1}{\sqrt{1 - \beta^2}} = \gamma \cdot \frac{Q}{r'^2} \)

Looks like: \( z' \) strong

\[ \vec{S} \cdot d\vec{I} \neq 0 \]

\[ \text{closed} \]

\[ \text{weak} \]

\[ \leftarrow \rightarrow \text{ symmetric} \]
Now imagine motion... 

Imagine "light emitted"

Can't be influenced by what happened at $t_1$!

Suppose at $t_1$, charge sped up.
What if charge stops or speeds up!!!

ACCELERATION OF CHARGES CAUSES RADIATION.

5.11 (Scan figure)

Stopped charge: \( \frac{1}{2\pi} \theta_0 \int_0^{2\pi} d\phi \int_0^\infty \frac{r^2 \sin^2 \phi \cdot q}{r^2 \sin^2 \phi} \cdot \frac{1}{(r^2 - \beta^2 \sin^2 \phi \beta^2)^{3/2}} \cdot q \left(1 - \cos^2 \phi \right) \) 

\[ = 2\pi q \int d\nu = 2\pi q \left(1 - \cos \theta_0 \right) \frac{1}{\theta_0} \]

\[ = 2\pi q \left(1 - \beta^2 \right) \int_0^\infty d\phi \frac{1}{\left((1 - \beta^2) + \beta^2 \mu^2 \right)^{3/2}} \]

\[ x = \mu \beta \]
\[ 2\pi \left[ \frac{1 - \beta^2}{\beta} \right] \int_0^\beta \frac{dx}{(1 - \beta^2 + x^2)^{3/2}} \]

\[ = 2\pi \frac{\beta}{\beta} \left[ \frac{1 - \beta^2}{(1 - \beta^2 + \beta^2)^{3/2}} - \frac{\beta \cos \phi_0}{(1 - \beta^2 (1 - \omega^2 \phi_0))^{1/2}} \right] \]

\[ = 2\pi \frac{\beta}{\beta} \left[ 1 - \frac{\cos \phi_0}{(1 - \beta^2 \sin^2 \phi_0)^{1/2}} \right] \]

Set equal

\[ 2\pi \frac{\beta}{\beta} (1 - \cos \Theta_0) = 2\pi \frac{\beta}{\beta} \left[ 1 - \frac{\omega \sin \phi_0}{(1 - \beta^2 \sin^2 \phi_0)^{1/2}} \right] \]

\[ \cos \Theta_0 = \frac{\cos \phi_0}{(1 - \beta^2 \sin^2 \phi_0)^{1/2}} \]

\[ \sin \Theta_0 = \left[ 1 - \cos^2 \Theta_0 \right]^{1/2} = \left[ 1 - \frac{\omega \sin \phi_0}{1 - \beta^2 \sin^2 \phi_0} \right]^{1/2} \]

\[ \sin \Theta_0 = \sqrt{1 - \beta^2 \sin^2 \phi_0 - \omega \sin \phi_0} \]

\[ \text{set equal} \]

\[ 2\pi \frac{\beta}{\beta} (1 - \cos \Theta_0) = 2\pi \frac{\beta}{\beta} \left[ 1 - \frac{\omega \sin \phi_0}{(1 - \beta^2 \sin^2 \phi_0)^{1/2}} \right] \]

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\[ \sin \Theta_0 = \sqrt{1 - \beta^2 \sin^2 \phi_0 - \omega \sin \phi_0} \]
\[
\sin \theta_0 = \left[ \frac{\sin^2 \varphi_0 (1-\beta^2)}{1-\beta^2 \sin^2 \varphi_0} \right]^{1/2} = \frac{\sin \varphi_0}{\gamma (1-\beta^2 \sin^2 \varphi_0)^{1/2}}
\]

So

\[
\frac{\sin \theta_0}{\cos \theta_0} = \frac{\sin \varphi_0}{\gamma (1-\beta^2 \sin^2 \varphi_0)^{1/2}} \frac{1}{\cos \varphi_0} \left(1-\beta^2 \sin^2 \varphi_0\right)^{1/2}
\]

or

\[
\tan \varphi_0 = \gamma \tan \theta_0
\]
Radiation + Acceleration (App B)

Easiest to reason from deceleration...

![Graph showing velocity over time with constant acceleration](image)

\[ v_0 \ll c \]

\[ +\infty \quad a = \frac{-v_0}{T} \]

\[ v_0 + aT = 0 \]

\[ t = 0 \quad a \text{ at rest} \quad a^2 = -v_0 \]

\[ x = v_0 t + \frac{1}{2} a^2 t^2 \]

\[ = v_0 t - \frac{1}{2} v_0 T^2 \]

\[ \frac{E_r}{E_0} = \frac{cT}{v_0 T \sin \theta} \]

Region I

Region II

Time T later
\[ E = \frac{V_0 T \sin \theta}{c^4 R} \]
\[ E = \frac{q |a| \sin \theta}{c^2 R} \]

Falls ONLY AS \( \frac{1}{R} \)
not \( \frac{1}{R^2} \) !!

\[ \text{Energy} = \frac{E^2}{\text{Volume}} = \frac{q^2 a^2 \sin^2 \theta}{8 \pi 1} = \frac{q^2 a^2 \sin^2 \theta}{8 \pi c^4 R^2} \]

Do volume integral...

\[ \text{Energy} = \frac{q^2 a^2 \cdot \frac{2}{3}}{8 \pi c^4 R^2} \cdot 4 \pi R^2 c^4 \]

\[ \text{Energy} = \text{Power} = \frac{1}{3} \frac{q^2 a^2}{c^3} \]

[\text{E field}]

[\text{B field} \iff \text{DOUBLES}]

\[ \text{Power} = \frac{2}{3} \frac{q^2 a^2}{c^3} \]

\( \text{Larmor Power Formula} \)