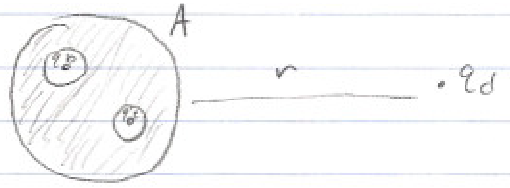
Physics 24 - Homework Set 3 : 3.1, 3.2, 3.3, 3.6, 3.8

3.1 Physics: q_b and q_c will attract charges to the surfaces of the interior cavities such that the interior of A has $E=0$.



This results in a surface charge on the exterior surface of A, $q_A = q_b + q_c$

5 points

The presence of q_d, at $r \gg 1$, ensures a uniform electric field in the vicinity of A, which will rearrange the external surface charge density σ such that the field in A is again zero. Note that this does not affect the (uniform) distribution over both cavities.

Thus since q_b and q_c are sitting inside uniform shells of charge, the force on them is zero. (N.B.: because $E=0$ inside A, no information about the external charge configurations can be carried to q_b and q_c)

Now since A is far from q_d, A can be approximated by a point charge $q_A = q_b + q_c$ from q_d's point of view. By Newton's third law the force on q_d is the opposite.

In a usual coordinate system:

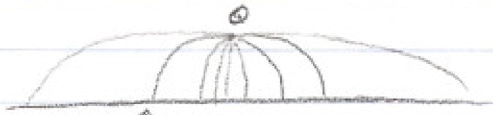
$$\boxed{\begin{array}{ll} \vec{F}_d = \dfrac{(q_b+q_c)\,q_d}{r^2}\,(\hat{x}) & \vec{F}_c = 0 \\[2ex] \vec{F}_A = \dfrac{(q_b+q_c)\,q_d}{r^2}\,(-\hat{x}) & \vec{F}_b = 0 \end{array}}$$

Now $F_c = F_b = 0$ regardless of r. But our assumption that we may take q_A as a point charge depends on having r large. Thus F_A and F_d are approximate, and will change if r is comparable to r_A.

3.2 A gravitational screen would require having both positive and negative sources of gravitational field, i.e. positive and negative mass, and the ability to move them freely. However to the best of our knowledge negative mass is impossible.

2 points

3.3 Our plate is grounded at infinity, so the field below it is zero.



Field line starting $\perp$ to plane.

10 points

Therefore let us construct a Gaussian surface that follows the field lines leaving parallel to the plane, and running through the plane below. This will clearly be hemispherical in shape. The electric field at the plane is zero, and on the hemisphere $\vec{E} \perp d\vec{A}$ so Gauss' law applied to this surface yields zero flux. This can be only true if $q_{encl} = 0$. We include half of Q in our surface, so thus the line that the parallel field lines intersect the plane on must encircle half the charge on the plane below. ∴ The integral of σ must yield $-\frac{Q}{2}$.

from (3.8): $\sigma = \frac{-Qh}{2\pi(r^2+h^2)^{3/2}}$

$$\text{so} \int_0^R \sigma \cdot 2\pi r\, dr = -\frac{Q}{2} = \int_0^R dr \frac{-Qhr}{(r^2+h^2)^{3/2}} = -Qh \int_0^R \frac{r\,dr}{(r^2+h^2)^{3/2}}$$

$$= -Qh \left[\frac{-1}{(r^2+h^2)^{1/2}}\right]_0^R$$

$$= \frac{Qh}{(R^2+h^2)^{1/2}} - \frac{Qh}{(0+h^2)^{1/2}} = \frac{Qh}{(R^2+h^2)^{1/2}} - Q = -\frac{Q}{2}$$

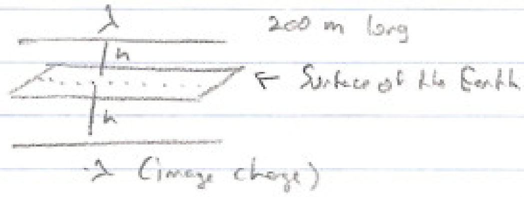
$$\rightarrow \frac{Q}{2} = \frac{Qh}{(R^2+h^2)^{1/2}} \qquad \text{so} \quad (R^2+h^2)^{1/2} = 2h$$

$$R^2 + h^2 = 4h^2$$
$$R^2 = 3h^2$$

$$\therefore \boxed{R = \sqrt{3}\,h}$$

So a parallel field line meets the plane at a distance $\sqrt{3}\,h$ from the point directly under the charge Q.

3.6 By applying image charges, we see that we get an image charge a height h below the surface of the earth.



5 points Thus the field at the surface should be $\vec{E}(\lambda, -h) + \vec{E}(-\lambda, h)$

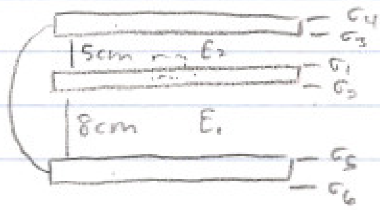
E of a line charge: $\vec{E}_r = \frac{2\lambda}{r}\hat{r}$ (we make the assumption that this is an infinite line)

so $\vec{E} = \frac{2\lambda}{h}(-\hat{y}) + \frac{2(-\lambda)}{h}(\hat{y}) = -\frac{4\lambda}{h}\hat{y}$ so $E = \frac{4\lambda}{h}$

$\lambda = 10^3$ esu/cm $\quad h = 500$cm $\quad$ so $E = \frac{4\times10^3}{500}\frac{\text{esu}}{\text{cm}^2} = \boxed{8\ \text{esu/cm}^2}$

Force is $\approx$ constant along length: $F = L\lambda E = \frac{2\lambda^2 L}{2h} = \frac{2\cdot10^6\,\text{esu/cm}\cdot 20{,}000\,\text{cm}}{1000\,\text{cm}} = \boxed{4\times10^7\ \text{dyne}}$

3.8 We can find σ_1 and σ_2 by applying Gauss' law to pillboxes; let each end lie inside a conductor; $\vec{E} \perp d\vec{A}$ in vacuum, so $\int \vec{E}\cdot d\vec{A}$ over the pillbox $= 0$.



10 points

$E_{\text{surface charge}} = 4\pi\sigma$ $\quad$ so $\quad E_1 = 4\pi(\sigma_5 - \sigma_2)$

$E_2 = 4\pi(\sigma_1 - \sigma_3)$

Continuity: $\sigma_1 + \sigma_2 = 10\ \text{esu/cm}^2$ $\quad \sigma_3 + \sigma_4 + \sigma_5 + \sigma_6 = 0$

Using Gauss' law: upper plate $\Rightarrow \sigma_3 = -\sigma_1$

Lower plate $\Rightarrow \sigma_2 = -\sigma_5$

All plates $\Rightarrow \sigma_3 + \sigma_1 + \sigma_2 + \sigma_5 = -\sigma_1 + \sigma_1 + \sigma_2 - \sigma_2 = 0$ ✓

Top + bottom plates are equipotentials: $\int_{\text{bottom}}^{\text{top}} \vec{E}\cdot d\vec{s} = 0 = 4\pi\left[8\sigma_5 - 8\sigma_2 + 5\sigma_1 - 5\sigma_3\right]$

but $\sigma_5 = -\sigma_2$ and $-\sigma_3 = \sigma_1$ so $0 = 8(-2\sigma_2) + 5(2\sigma_1) \Rightarrow 8\sigma_2 = 5\sigma_1$

with $\sigma_1 + \sigma_2 = 10$: $\quad \sigma_1 + \frac{5}{8}\sigma_1 = 10$ $\quad$ so $\boxed{\sigma_1 = \frac{80}{13}\ \text{esu/cm}^2}$ $\quad \sigma_1 = 6.15\ \text{esu/cm}^2$

$\sigma_2 + \frac{8}{5}\sigma_2 = 10$ $\quad \boxed{\sigma_2 = \frac{50}{13}\ \text{esu/cm}^2}$ $\quad \sigma_2 = 3.85\ \text{esu/cm}^2$