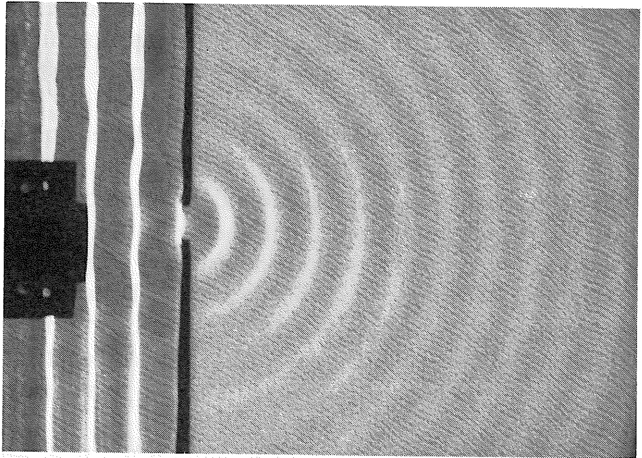

Figure 2 Diffraction of water waves at a slit in a ripple tank. Note that the slit width is about the same size as the wavelength. Compare with Fig. 1*c*.

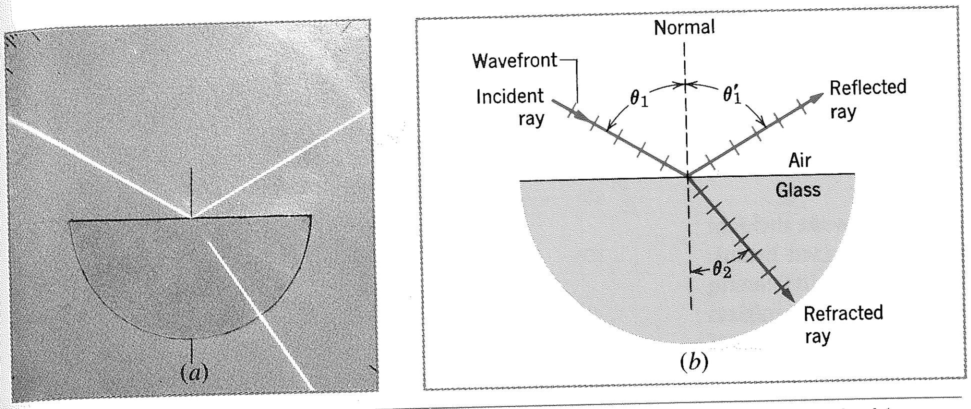

Figure 3 (*a*) A photograph showing the reflection and refraction of a light beam incident on a plane glass surface. (*b*) A representation using rays. The angles of incidence θ_1, reflection θ_1', and refraction θ_2 are marked. Note that the angles are measured between the normal to the surface and the appropriate ray.

Equation 2 is called Snell's law. Here n_1 and n_2 are dimensionless constants called the *index of refraction* of medium 1 and medium 2. The index of refraction n of a medium is the ratio between the speed of light c in vacuum and the speed of light v in that medium:

$$n = \frac{c}{v}. \tag{3}$$

We discussed the speed of light in various materials in Section 42-2. It is fair to say that refraction occurs because the speed of light changes from one medium to another. We develop this idea further in Section 43-5.

Table 1 shows some examples of the index of refraction of various materials. Note that, for most purposes, air can be regarded as equivalent to a vacuum in its refraction of light. The index of refraction of a material generally varies with the wavelength of the light (see Fig. 4). Refraction can thus be used to analyze a beam of light into its constituent wavelengths, such as occurs in a rainbow.

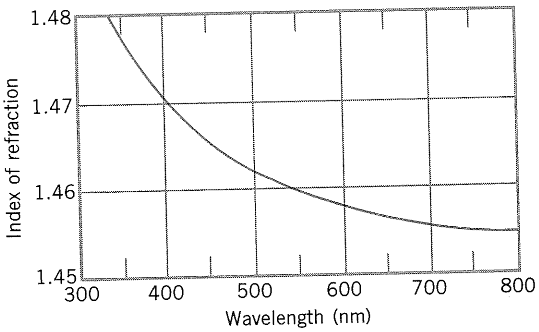

Figure 4 The index of refraction of fused quartz as a function of wavelength.

Reflection and Refraction of Electromagnetic Waves *(Optional)*

The laws of reflection and refraction hold for all regions of the electromagnetic spectrum, not just for light. In fact, Eqs. 1 and 2 can be derived from Maxwell's equations, which makes them generally applicable to electromagnetic waves. Experimental evidence for this general applicability includes the reflection of microwaves or radio waves from the ionosphere and the refraction of x rays by crystals.

We normally think of highly polished or smooth surfaces as "good" reflectors, but other surfaces may reflect as well, for example, a sheet of paper. The reflection by the paper (which is called a *diffuse reflection*) scatters the light more or less in all directions. It is largely by diffuse reflections that we see nonluminous objects around us. The difference between diffuse and specular (mirrorlike) reflection depends on the roughness of the surface: a reflected beam is formed only if the typical dimensions

TABLE 1 SOME INDICES OF REFRACTION[a]

Medium	*Index*	*Medium*	*Index*
Vacuum (exactly)	1.00000	Typical crown glass	1.52
Air (STP)	1.00029	Sodium chloride	1.54
Water (20°C)	1.33	Polystyrene	1.55
Acetone	1.36	Carbon disulfide	1.63
Ethyl alcohol	1.36	Heavy flint glass	1.65
Sugar solution (30%)	1.38	Sapphire	1.77
Fused quartz	1.46	Heaviest flint glass	1.89
Sugar solution (80%)	1.49	Diamond	2.42

[a] For a wavelength of 589 nm (yellow sodium light).

of the surface irregularities of the reflector are substantially less than the wavelength of the incident light. Thus the classification of the reflective properties of a surface depends on the wavelength of the radiation that strikes the surface. The bottom of a cast-iron skillet, for example, may be a good reflector for microwaves of wavelength 0.5 cm but is not a good reflector for visible light.

Maxwell's equations permit us to calculate how the incident energy is divided between the reflected and refracted beams. Figure 5 shows the theoretical prediction for (*a*) a light beam in air falling on a glass–air interface, and (*b*) a light beam in glass falling on a glass–air interface. Figure 5*a* shows that for angles of incidence up to about 60°, less than 10% of the light energy is reflected. At grazing incidence (that is, at angles of incidence near 90°), the surface becomes an excellent reflector. Another example of this effect is the high reflecting power of a wet road when light from automobile headlights strikes the road near grazing incidence.

Figure 5*b* shows clearly that at a certain critical angle (41.8° in this case), *all* the light is reflected. We consider this phenomenon, called *total internal reflection,* in Section 43-6. ■

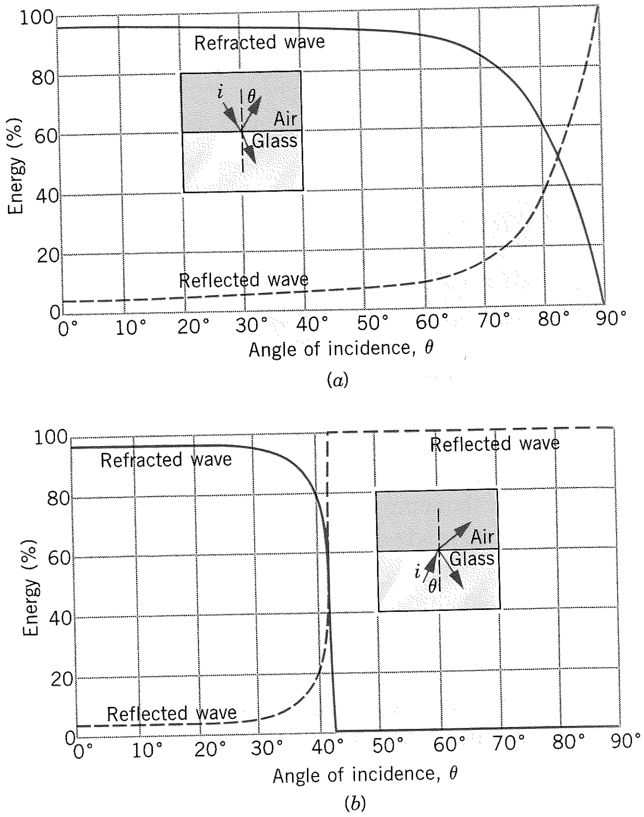

Figure 5 (*a*) The percentage of energy reflected and refracted when a wave in air is incident on glass ($n = 1.50$). (*b*) The same for a wave in glass incident on air, showing total internal reflection.

Sample Problem 1 Figure 6 shows an incident ray i striking a plane mirror MM' at angle of incidence θ. Mirror $M'M''$ is perpendicular to MM'. Trace this ray through its subsequent reflections.

Solution The reflected ray r makes an angle θ with the normal at b and falls as an incident ray on mirror $M'M''$. Its angle of incidence θ' on this mirror is $\pi/2 - \theta$. A second reflected ray r' makes an angle θ' with the normal erected at b'. Rays i and r' are antiparallel for any value of θ. To see this, note that

$$\phi = \pi - 2\theta' = \pi - 2\left(\frac{\pi}{2} - \theta\right) = 2\theta.$$

Two lines are parallel if their opposite interior angles for an intersecting line (ϕ and 2θ) are equal.

Repeat the problem if the angle between the mirrors is 120° rather than 90°.

The three-dimensional analogue of Fig. 6 is the *corner reflector,* which consists of three perpendicular plane mirrors joined like the positive sections of the coordinate planes of an xyz system. A corner reflector has the property that, for *any* direction of incidence, an incident ray is reflected back in the opposite direction. Highway reflectors use this principle, so that light from the headlights of an oncoming car is reflected back toward the car, no matter when the direction of approach of the car or the angle of the headlights above the road. Corner reflectors were placed on the Moon by the Apollo astronauts; timing a reflected laser beam from Earth permits precise determination of the Earth–Moon separation.

Sample Problem 2 A light beam in air is incident on the plane surface of a block of quartz and makes an angle of 30° with the normal. The beam contains two wavelengths, 400 and 500 nm. The indices of refraction for quartz at these wavelengths are 1.4702 and 1.4624, respectively. What is the angle between the two refracted beams in the quartz?

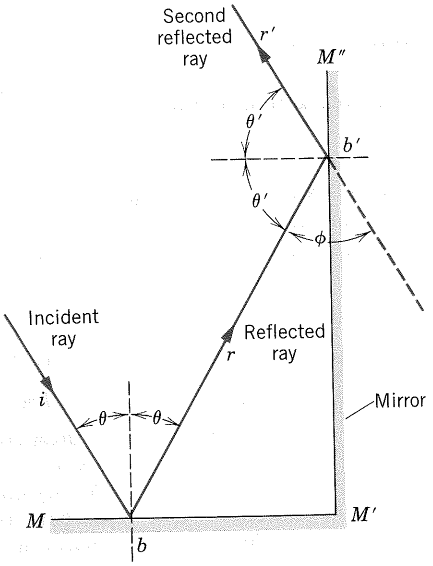

Figure 6 Sample Problem 1. A two-dimensional corner reflector.

All points on a wavefront can be considered as point sources for the production of spherical secondary wavelets. After a time t the new position of a wavefront is the surface tangent to these secondary wavelets.

Consider a trivial example. Given a wavefront (*ab* in Fig. 8) in a plane wave in free space, where will the wavefront be a time *t* later? Following Huygens' principle, we let several points on this plane (the dots in Fig. 8) serve as centers for secondary spherical wavelets. In a time *t* the radius of these spherical waves is *ct*, where *c* is the speed of light in free space. We represent the plane tangent to these spheres at time *t* by *de*. As we expect, it is parallel to plane *ab* and a perpendicular distance *ct* from it. Thus plane wavefronts are propagated as planes and with speed *c*. Note that the Huygens method involves a three-dimensional construction and that Fig. 8 is the intersection of this construction with the plane of the page.

We might expect that, contrary to observation, a wave should be radiated backward as well as forward from the dots in Fig. 8. This result is avoided by assuming that the

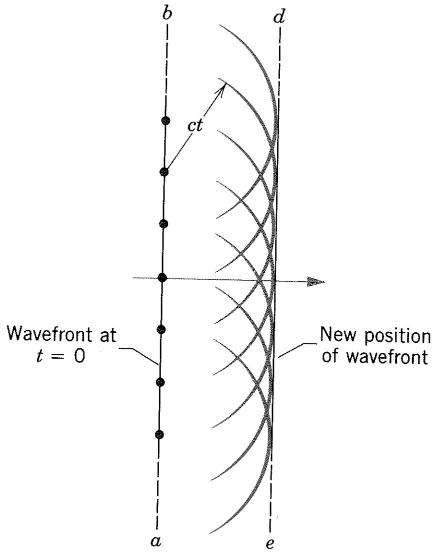

Figure 8 The propagation of a plane wave in free space is described by the Huygens construction. Note that the ray (horizontal arrow) representing the wave is perpendicular to the wavefronts.

Now we show how th
Huygens' principle. Figur
plane wave falling on a
nience the wavefronts ar
apart. Note that θ_1, the an
the mirror, is the same as
ray and the normal to the
angle of incidence. The
each other by the Huygen
Let us regard point *a* i
source of a Huygens wave
λ/c to include point *b* on
from point *p* in this same
the mirror but must expa

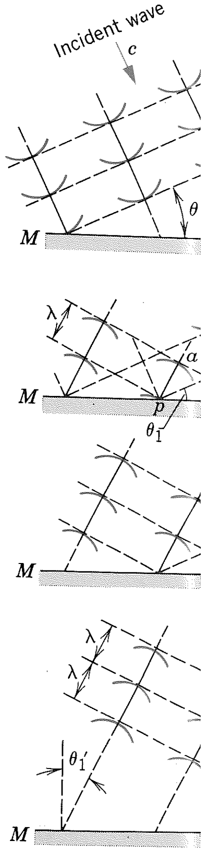

Figure 9 The reflection of a
as analyzed by the Huygens co

varies with its density? See "Mirages," by Alistair B. Fraser and William B. Mach, *Scientific American,* January 1976, p. 102.

28. Can a virtual image be photographed by exposing a film at the location of the image? Explain.

29. At night, in a lighted room, you blow a smoke ring toward a window pane. If you focus your eyes on the ring as it approaches the pane it will seem to go right through the glass into the darkness beyond. What is the explanation of this illusion?

30. In driving a car you sometimes see vehicles such as ambulances with letters printed on them in such a way that they read in the normal fashion when you look through the rear-view mirror. Print your name so that it may be so read.

31. We have seen that a single reflection in a plane mirror reverses right and left. When we drive down a highway, for example, the letters on the highway signs are reversed as seen through the rear-view mirror. And yet, as seen through this same mirror, you still seem to be driving down the right lane. Why does the mirror reverse the signs and not the lanes? Or does it? Discuss.

32. We all know that when we look into a mirror right and left are reversed. Our right hand will seem to be a left hand; if we part our hair on the left it will seem to be parted on the right, and so on. Can you think of a system of mirrors that would let us see ourselves as others see us? If so, draw it and prove your point by drawing some typical rays.

33. Devise a system of plane mirrors that will let you see the back of your head. Trace the rays to prove your point.

34. Design a periscope, taking advantage of total internal reflection. What are the advantages compared with silvered mirrors?

35. What characteristics must a material have in order to serve as an efficient "light pipe"?

36. A certain toothbrush has a red plastic handle into which rows of nylon bristles are set. The tops of the bristles (but not their sides) appear red. Explain.

37. Why are optical fibers more effective carriers of information than, say, microwaves or cables? Think of the frequencies involved.

38. What does "optical path length" mean? Can the optical path length ever be less than the geometrical path length? Ever greater?

39. A solution of copper sulfate appears blue when we view it through transmitted light. Does this mean that a copper sulfate solution absorbs blue light selectively? Discuss.

PROBLEMS

Section 43-2 Reflection and Refraction

1. In Fig. 25 find the angles (*a*) θ_1 and (*b*) θ_2.

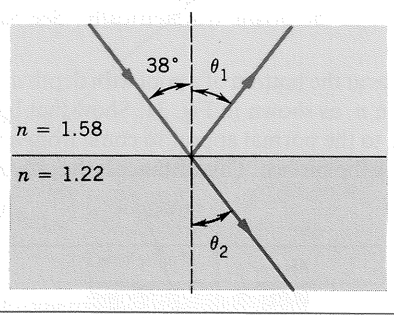

Figure 25 Problem 1.

2. Light in vacuum is incident on the surface of a glass slab. In the vacuum the beam makes an angle of 32.5° with the normal to the surface, while in the glass it makes an angle of 21.0° with the normal. Find the index of refraction of the glass.

3. The speed of yellow sodium light in a certain liquid is measured to be 1.92×10^8 m/s. Find the index of refraction of this liquid with respect to air, for sodium light.

4. Find the speed in fused quartz of light of wavelength 550 nm. (See Fig. 4.)

5. When an electron moves through a medium at a speed exceeding the speed of light in that medium, it radiates electromagnetic waves (the Cerenkov effect). What minimum speed must an electron have in a liquid of index of refraction 1.54 in order to radiate?

6. A laser beam travels along the axis of a straight section of pipeline 1.61 km long. The pipe normally contains air at standard temperature and pressure, but it may also be evacuated. In which case would the travel time for the beam be greater and by how much?

7. When the rectangular metal tank in Fig. 26 is filled to the top with an unknown liquid, an observer with eyes level with the top of the tank can just see the corner *E*. Find the index of refraction of the liquid.

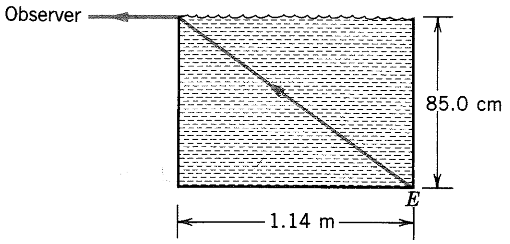

Figure 26 Problem 7.

8. Ocean waves moving at a speed of 4.0 m/s are approaching a beach at an angle of 30° to the normal, as shown in Fig. 27. Suppose the water depth changes abruptly and the wave speed drops to 3.0 m/s. Close to the beach, what is the angle θ between the direction of wave motion and the normal? (Assume the same law of refraction as for light.) Explain why

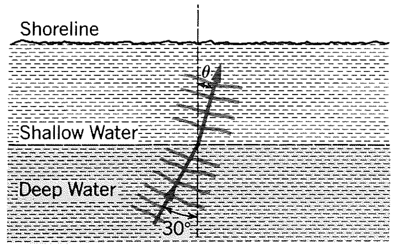

Figure 27 Problem 8.

most waves come in normal to a shore even though at large distances they approach at a variety of angles.

9. A ray of light goes through an equilateral prism in the position of minimum deviation. The total deviation is 37°. What is the index of refraction of the prism? See Sample Problem 3.

10. Two perpendicular mirrors form the sides of a vessel filled with water, as shown in Fig. 28. A light ray is incident from above, normal to the water surface. (*a*) Show that the emerging ray is parallel to the incident ray. Assume that there are two reflections at the mirror surfaces. (*b*) Repeat the analysis for the case of oblique incidence, the ray lying in the plane of the figure.

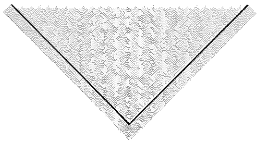

Figure 28 Problem 10.

11. In Fig. 7 (Sample Problem 3) show by graphical ray tracing, using a protractor, that if θ for the incident ray is either increased or decreased, the deviation angle ψ is increased.

12. Light from a laser enters a glass block at A and emerges at B; see Fig. 29. The glass block has a length $L = 54.7$ cm and an index of refraction $n = 1.63$. The angle of incidence is $\theta = 24.0°$. Find the time needed for light to pass through the block.

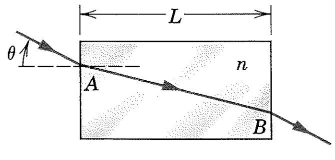

Figure 29 Problem 12.

13. A diver beneath the surface of water in a lake looks up at 27° from the vertical to see a life ring floating on the surface. Through the center of the ring can be seen the top of a smokestack known to be 98 m high. How far is the base of the smokestack from the life ring?

14. A bottom-weighted 200-cm-long vertical pole extends from the bottom of a swimming pool to a point 64 cm above the water. Sunlight is incident at 55° above the horizon. Find the length of the shadow of the pole on the level bottom of the pool.

15. Prove that a ray of light incident on the surface of a sheet of plate glass of thickness t emerges from the opposite face parallel to its initial direction but displaced sideways, as in Fig. 30. (*a*) Show that, for small angles of incidence θ, this displacement is given by

$$x = t\theta \frac{n-1}{n},$$

where n is the index of refraction and θ is measured in radians. (*b*) Calculate the displacement at a 10° angle of incidence through a 1.0-cm-thick sheet of crown glass.

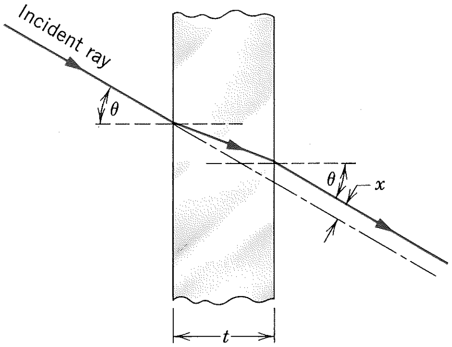

Figure 30 Problem 15.

16. A glass prism with an apex angle of 60° has $n = 1.60$. (*a*) What is the smallest angle of incidence for which a ray can enter one face of the prism and emerge from the other? (*b*) What angle of incidence would be required for the ray to pass through the prism symmetrically? See Sample Problem 3.

17. A coin lies at the bottom of a pool with depth d and index of refraction n, as shown in Fig. 31. Show that light rays that are close to the normal appear to come from a point $d_{\text{app}} = d/n$ below the surface. This distance is the *apparent depth* of the pool.

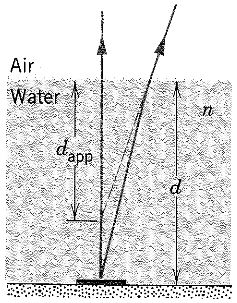

Figure 31 Problem 17.

18. The apparent depth of a pool depends on the angle of viewing. Suppose that you place a coin at the bottom of a swimming pool filled with water ($n = 1.33$) to a depth of 2.16 m. Find the apparent depth of the coin below the surface when viewed (*a*) at near normal incidence and (*b*) by rays that leave the coin making an angle of 35.0° with the normal to the bottom of the pool. See Problem 17.

19. A layer of water ($n = 1.33$) 20 mm thick floats on a layer of carbon tetrachloride ($n = 1.46$) 41 mm thick. How far below the water surface, viewed at near normal incidence, does the bottom of the tank seem to be?

20. The index of refraction of the Earth's atmosphere decreases monotonically with height from its surface value (about 1.00029) to the value in space (about 1.00000) at the top of the atmosphere. This continuous (or graded) variation can be approximated by considering the atmosphere to be composed of three (or more) plane parallel layers in each of which the index of refraction is constant. Thus, in Fig. 32, $n_3 > n_2 > n_1 > 1.00000$. Consider a ray of light from a star S that strikes the top of the atmosphere at an angle θ with the vertical. (*a*) Show that the apparent direction θ_3 of the star with the vertical as seen by an observer at the Earth's surface is obtained from

$$\sin\theta_3 = \frac{1}{n_3}\sin\theta.$$

(*Hint*: Apply the law of refraction to successive pairs of layers of the atmosphere; ignore the curvature of the Earth.) (*b*) Calculate the shift in position of a star observed to be 50° from the vertical. (The very small effects due to atmospheric refraction can be most important; for example, they must be taken into account in using navigation satellites to obtain accurate fixes of position on the Earth.)

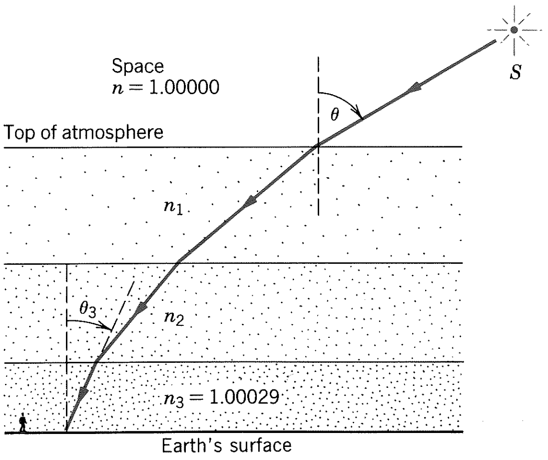

Figure 32 Problem 20.

21. You stand at one end of a long airport runway. A vertical temperature gradient in the air has resulted in the index of refraction of the air above the runway to vary with height y according to $n = n_0(1 + ay)$, where n_0 is the index of refraction at the runway surface and $a = 1.5 \times 10^{-6}\ \text{m}^{-1}$. Your eyes are at a height $h = 1.7$ m above the runway. Beyond what horizontal distance d can you not see the runway? See Fig. 33 and Problem 20.

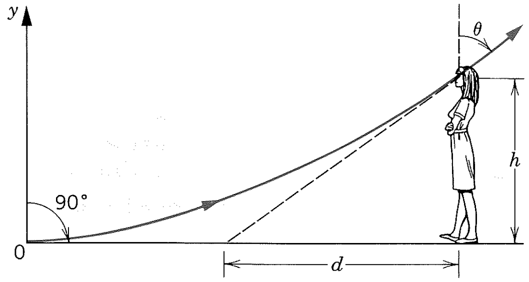

Figure 33 Problem 21.

22. A *corner reflector*, much used in optical, microwave, and other applications, consists of three plane mirrors fastened together as the corner of a cube. It has the property that an incident ray is returned, after three reflections, with its direction exactly reversed. Prove this result.

23. Muons (mass = 106 MeV/c^2) and neutral pions (mass = 135 MeV/c^2), each with momentum 145 MeV/c, pass through a transparent material. Find the range of index of refraction of the material so that only the muons emit Cerenkov radiation. (See Problem 5.)

Section 43-3 Deriving the Law of Reflection

24. One end of a stick is dragged through water at a speed v that is greater than the speed u of water waves. Applying Huygens' construction to the water waves, show that a conical wavefront is set up and that its half-angle α is given by

$$\sin\alpha = u/v.$$

This is familiar as the bow wave of a ship or the shock wave caused by an object moving through air with a speed exceeding that of sound, as in Fig. 14 of Chapter 20.

25. Using Fermat's principle, prove that the reflected ray, the incident ray, and the normal lie in one plane.

Section 43-4 Image Formation by Plane Mirrors

26. A small object is 10 cm in front of a plane mirror. If you stand behind the object, 30 cm from the mirror, and look at its image, for what distance must you focus your eyes?

27. You are standing in front of a large plane mirror, contemplating your image. If you move toward the mirror at speed v, at what speed does your image move toward you? Report this speed both (*a*) in your own reference frame and (*b*) in the reference frame of the room in which the mirror is at rest.

28. Figure 34 shows (top view) that Bernie B is walking directly toward the center of a vertical mirror M. How close to the mirror will he be when Sarah S is just able to see him? Take $d = 3.0$ m.

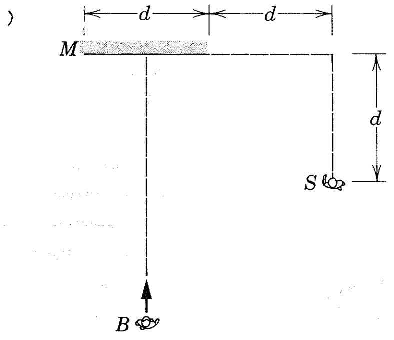

Figure 34 Problem 28.