4.9

VRH

Free electrons

density $n = 10^6 / cm^3$

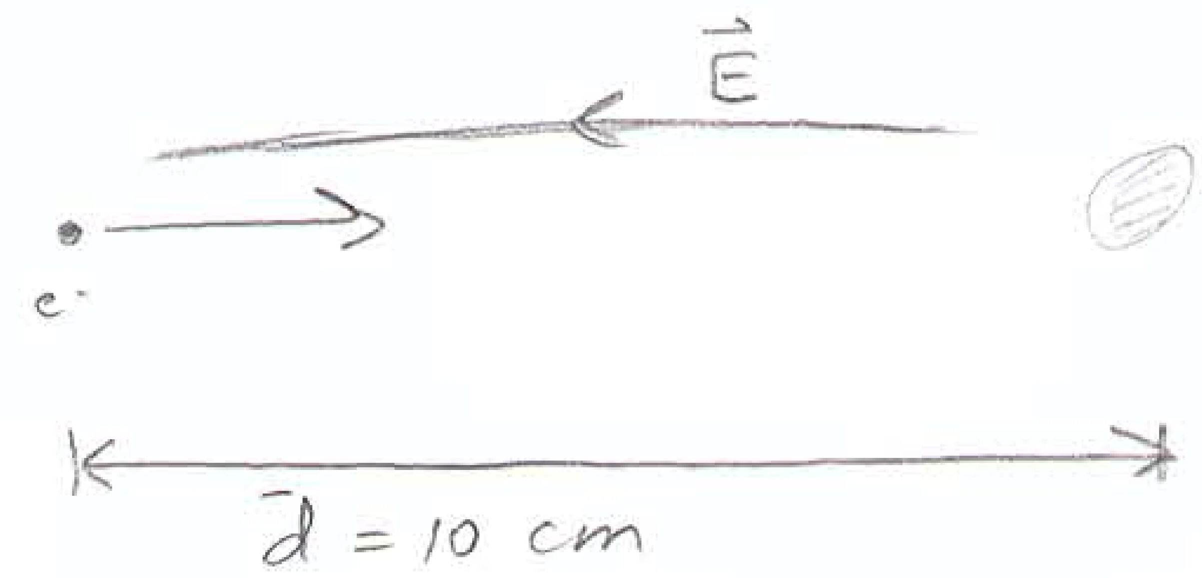
Mean free path $\bar{d} = 10$ cm

Mean speed $\bar{v} = 10^7$ cm/s

Mean free time $\bar{t} = \frac{\bar{d}}{\bar{v}} = \frac{10 \text{ cm}}{10^7 \text{ cm/s}} =$

$m = 9.110 \times 10^{-28}$ g

$q = -4.803 \times 10^{-10}$ esu.



conductivity $\sigma = ?$

We want to use Eq. (20), but τ doesn't have a precise definition in that equation, so we'll have to work it out.

The conductivity is defined by

$$\vec{J} = \sigma \vec{E} \qquad (p. 128)$$

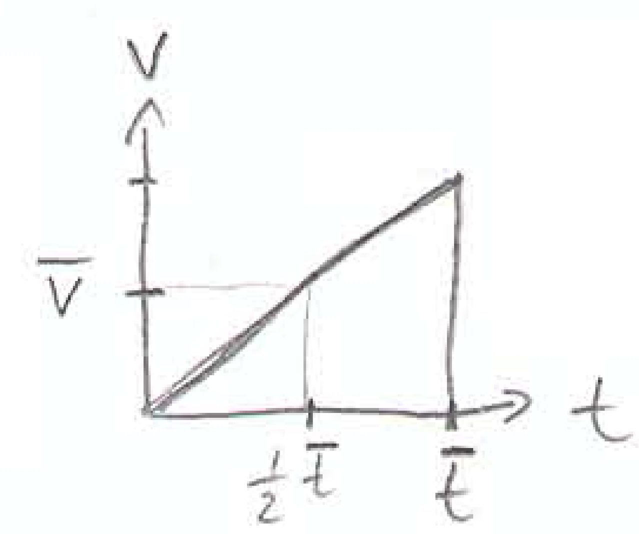
so let's assume a uniform electric field $\vec{E}$ for now. Pick coordinates so $\vec{E} = E_x \hat{x}$.
($\vec{E}$ won't appear in our answer.)

$\vec{F} = q\vec{E} = m\vec{a}$ force on electron

$a_x = \frac{qE_x}{m}$. $\quad V_x = a_x t$ if we assume the <u>average</u> velocity after a collision is zero (see bottom of page 136).

So $V_x(t) = \frac{qE_x}{m} t$

t = time since last collision.



$\bar{V} = V_x(t = \frac{\bar{t}}{2})$

$\bar{V} = \frac{qE_x}{m} \cdot (\frac{\bar{t}}{2}) \Rightarrow \bar{t} = \frac{2m\bar{v}}{qE_x}$.

With this distribution of electron speeds, the mean speed is half the expected speed of the electron when it collides.

Also,

$x = \frac{1}{2} a_x t^2$

so $\bar{d} = \frac{1}{2}\left(\frac{qE_x}{m}\right) \bar{t}^2$ but we already found that $\bar{t} = \frac{2m\bar{v}}{qE_x}$, so

$$\bar{d} = \frac{1}{2}\left(\frac{qE_x}{m}\right)\left(\frac{2m\bar{v}}{qE_x}\right)^2 = \frac{2m\bar{v}^2}{qE_x}$$

$$|\vec{E}| = |E_x| = \frac{2m\bar{v}^2}{q\bar{d}}$$

4.9 cont'd

VRH

current density $\quad J = qn\bar{v} = \sigma E_x$

$$qn\bar{v} = \sigma \frac{2m\bar{v}^2}{q\bar{d}}$$

conductivity $\quad \boxed{\sigma = \frac{q^2 n \bar{d}}{2m\bar{v}}}$

cgs:

$$\sigma = \frac{(-4.803\times10^{-10}\,\mathrm{esu})^2\,(10^6/\mathrm{cm}^3)\,(10\,\mathrm{cm})}{2\,(9.110\times10^{-28}\,\mathrm{g})\,(10^7\,\mathrm{cm/s})}$$

Recall:

$$\frac{\mathrm{esu}^2}{\mathrm{cm}^2} = \mathrm{dyn} = \frac{\mathrm{g\,cm}}{\mathrm{s}^2}$$

so $\quad \dfrac{\mathrm{esu}^2\,\mathrm{s}}{\mathrm{g\,cm}^3} = \dfrac{1}{\mathrm{s}}$

$$\boxed{\sigma = 1.3\times10^{8}\ \mathrm{s}^{-1}}$$

$$\sigma = (1.3\times10^8\ \mathrm{s}^{-1})\left(\frac{1.11\times10^{-10}\ \mathrm{s}}{100\ \mathrm{ohm\ cm}}\right) = \boxed{1.4\times10^{-4}\ \frac{1}{\mathrm{ohm\ cm}}}$$

Notation: The hyphen in "ohm-cm" is not a minus sign. It's the same as "ohm cm." I usually think of it as multiplying the two units.