1. Let’s investigate whether the gradient of a function $f(x, y)$ is really the direction that maximizes the change in $f$. Imagine taking a step, starting at position $x_0 \hat{i} + y_0 \hat{j}$, of length $\Delta \xi$, and in direction $\theta$ with respect to the $x$ axis.

(a) Describe the step by a vector of the form $\Delta x \hat{i} + \Delta y \hat{j}$, and express $\Delta x$ and $\Delta y$ in terms of $\Delta \xi$ and $\theta$. Make a graph in the $x-y$ plane showing the starting position $x_0 \hat{i} + y_0 \hat{j}$ and the vector that describes the step (you must assume illustrative values for all the quantities to make the graph).

(b) What is the change in the value of $f$ as one steps from $x_0 \hat{i} + y_0 \hat{j}$ to the final position?

(c) Now find the extrema in the change in the value of $f$ as a function of $\theta$. In particular, what values of $\tan \theta$ correspond to the extrema? What do you conclude about the direction of the step that leads to the maximum and minimum changes in $f$?

(d) Just for fun, find the direction that corresponds to no change in $f$... this is the direction of an ‘iso-$f$’ line, like a line of constant altitude on a topo map. What is the direction of the ‘iso-$f$’ line relative to that of the gradient?

2. Purcell 2.2

3. Purcell 2.4

4. Purcell 2.7

5. Purcell 2.12