Capacitance

Consider an isolated conductor; its electric potential, relative to infinity, is \( \Phi_0 \). The charge on the conductor is \( Q \).

Should not be too surprising that:

\[
\Phi_0 \propto Q \quad \text{surface argument: } \Phi_0 \propto \int \vec{E} \cdot d\vec{s}, \quad \vec{E} \propto Q
\]

Example: sphere:

\[
\Phi_0 = \frac{Q}{a}
\]

Constant of proportionality is called \( \frac{1}{V \text{ capacitance}} \).

\[
\Phi_0 = \left( \frac{1}{a} \right) \cdot Q
\]

or \( Q = a \cdot \Phi_0 = CV \Phi_0 \).
CGS: units are centimeters.

SI/MKS: spherical shell:

\[ \Phi_0 = \frac{Q}{4\pi \varepsilon_0 a} \]

\[ Q = \left(4\pi \varepsilon_0 a\right) \cdot \Phi_0 \]

has units of Farads.

\[ \varepsilon_0 = 8.85 \cdot 10^{-12} \text{ Farads/meter} \]

\[ Q = C\Phi_0 \leftarrow \text{set this, like, with a battery.} \]

more capacitance, more charge!

more charge, more energy.

Parallel Plate Conductor

\[ \Phi_1 \quad Q_1 \]

\[ \Phi_2 \quad Q_2 \]

\[ \leq \text{big compared to } s \rightarrow \]

want smallest field as possible

as one goes far away...
\[ Q_1 = -Q_2 = Q \quad \text{"balanced"} \quad \text{usually true.} \]

\[ E \cdot \Delta = 4\pi \sigma \quad \text{usually true when width/depth much bigger than } s. \]

\[ \frac{Q}{A} = \frac{4\pi}{s} \cdot (\phi_1 - \phi_2) \quad \text{usually called "the" capacitance.} \]
Symbol: \[ \begin{array}{c} \frac{1}{-Q} \\ Q \end{array} \]

"conducting wire."

**Parallel Connection:**

![Parallel Connection Diagram]

\[ C_1 \begin{array}{c} \frac{Q_1}{-Q_1} \\ -Q_2 \end{array} \begin{array}{c} Q_2 \end{array} \begin{array}{c} \frac{C_2}{V} \end{array} \]

\[ Q_1 = C_1 V \quad Q_2 = C_2 V \]

\[ Q_1 + Q_2 = Q = (C_1 + C_2) \cdot V = CV \quad \text{since} \quad C = C_1 + C_2 \]

\[ \frac{Q_1}{Q} = \frac{C_1}{C_1 + C_2} \]

\( \text{more capacity, more charge.} \)

\[ = \frac{A_1}{S_1} + \frac{A_2}{S_2} \]

\( \text{bigger } A, \text{ more cap.} \)

\( \text{smaller } S, \text{ more cap.} \)

**Series Connection:**

![Series Connection Diagram]

\[ C_1 \begin{array}{c} \frac{Q_1}{-Q_1} \end{array} \begin{array}{c} Q = C_1 V_1 \end{array} \begin{array}{c} Q = C_2 V_2 \end{array} \begin{array}{c} \frac{C_2}{V} \end{array} \begin{array}{c} -Q \end{array} \begin{array}{c} V = V_1 + V_2 = \left( \frac{1}{C_1} + \frac{1}{C_2} \right) Q \end{array} \begin{array}{c} \frac{C_1}{S_1} \end{array} \begin{array}{c} \frac{C_2}{S_2} \end{array} \]

\( \text{Series Connection} \)