Three perfect conductors put charge $Q_1$, $Q_2$, $Q_3$ on them, none = 0

- Electric fields will eminate from the charges
- No $\vec{E}$ field inside $\Rightarrow$ inside at one electric potential (no work needed to move charge around inside)
- $\vec{E}$ must be $\perp$ to the surface at the surface... $\vec{E}$ always $\perp$ to equipotential

$\Phi$ = 0

$E \cdot A = 4\pi \sigma \cdot A$

$\sigma$: depends on where you are
\[ E = 4\pi \sigma \]

**Conductor**

- \( \vec{E} = 0 \)
- \[ E = 4\pi \sigma \]

**Sheet Charge**

- \[ E = -2\pi \sigma \]
- \[ \sigma \]
- \[ + \]
- \[ + \]
- \[ \Rightarrow \]

Change in \( \vec{E} \) due to \( \sigma \)

is \( 4\pi \sigma \) in horizontal direction.

\[ \oint_{\partial S} \vec{E} \cdot d\vec{A} = \sigma \]

General Electrostatic Problem

Outside a System of Conductors

\[ \nabla^2 \varphi = 0 = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} \]
"Boundary Condition"

1. Shapes of the conductors
2. a) Potentials of them (or some of them)
   b) Charge on them (or some of them)
3. in between them
   \[ \nabla^2 \phi = 0 \]
4. \( \phi \to 0 \) as \( r \to \infty \)

How many solutions?

A) none \( \nRightarrow \) physics
B) exactly one \( \nRightarrow \) that is it.
C) more than one

"Uniqueness": \( \phi(x, y, z) \)
\( \psi(x, y, z) \)

Suppose two solutions

Both satisfy boundary conditions...

at the conductors and

\( \nabla^2 \phi = 0, \ \nabla^2 \psi = 0 \) outside
Consider

\[ W(x, y, z) = \Phi(x, y, z) - \Psi(x, y, z) \]

1. At the conductors, \( W(x_c, y_c, z_c) = 0 \) because

\[ \Phi(x_c, y_c, z_c) = \Psi(x_c, y_c, z_c) \] (same boundary conditions)

2. \( \nabla^2 W = \)

\[ \frac{\partial^2}{\partial x^2} (\Phi(x, y, z) - \Psi(x, y, z)) \]

\[ + \frac{\partial^2}{\partial y^2} (\Phi(x, y, z) - \Psi(x, y, z)) \]

\[ + \frac{\partial^2}{\partial z^2} (\Phi(x, y, z) - \Psi(x, y, z)) \]

\[ = \frac{\partial^2}{\partial x^2} \Phi + \frac{\partial^2}{\partial y^2} \Phi + \frac{\partial^2}{\partial z^2} \Phi \]

\[ + \frac{\partial^2}{\partial x^2} \Psi + \frac{\partial^2}{\partial y^2} \Psi + \frac{\partial^2}{\partial z^2} \Psi \]

\[ = \nabla^2 \Phi + \nabla^2 \Psi = 0 \]
3. \( W = 0 \) everywhere...
   \( W = 0 \) on conductors
   \( W = 0 \) as \( r \to \infty \)

4. \( \psi(x, y, z) = \psi(x, y, z) \)

Consequence: Cavity

- \( \psi = \) same constant in here
- \( \psi = \) constant
- Conductor

\[ \vec{E} = 0 \] inside then, since
\[ -\nabla \psi = 0 \]

matches boundary conditions

\[ \nabla^2 \psi = 0 \]
Better yet:

\[ \vec{E} = 0 \text{ outside} \]
\[ \vec{E} = 0 \text{ inside} \]

→ perfect conductor

→ electrostatic limit (moving \( \vec{E} \) field can "penetrate"

Conductors

\[
\begin{align*}
Q_1 & \quad \text{if } R_1 < r \\
Q_1 + Q_2 & \quad \text{if } r = R_1 \\
Q_1 + \frac{Q_2}{R_1} & \quad \text{if } R_2 \leq r < R_1 \\
\frac{Q_1}{R_1} + \frac{Q_2}{R_2} & \quad \text{if } r < R_2
\end{align*}
\]