

$\vec{\nabla}f$ is called the gradient of f .

Think about it....

when $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ all > 0 , $\vec{\nabla}f$ gets the most out of each derivative "equal representation".... problem on set.

If $f(r)$, then $\vec{\nabla}f = \hat{r} \frac{\partial f}{\partial r}$

$$\begin{aligned} \frac{\partial f}{\partial x} &= \frac{\partial f}{\partial r} \frac{\partial r}{\partial x} \\ &= \frac{\partial f}{\partial r} \frac{x}{r} \end{aligned}$$

$$\begin{aligned} r &= \sqrt{x^2 + y^2 + z^2} \\ \frac{\partial r}{\partial x} &= \frac{1}{2} \frac{2x}{\sqrt{x^2 + y^2 + z^2}} = \frac{x}{r} \end{aligned}$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial r} \frac{y}{r}$$

$$\frac{\partial f}{\partial z} = \frac{\partial f}{\partial r} \frac{z}{r}$$

$$\vec{\nabla}f = \frac{\partial f}{\partial x} \hat{i} + \frac{\partial f}{\partial y} \hat{j} + \frac{\partial f}{\partial z} \hat{k} = \frac{\partial f}{\partial r} \frac{(x\hat{i} + y\hat{j} + z\hat{k})}{r}$$

$$\vec{\nabla}f = \hat{r} \frac{\partial f}{\partial r}$$

Relationship between ϕ + \vec{E}

$$\phi_{21} = - \int_{P_1}^{P_2} \vec{E} \cdot d\vec{s}$$

- : push + charge against

$$\phi_{21} \equiv - \int_{P_1}^{P_2 + d\vec{s}} \vec{E} \cdot d\vec{s}$$

$$d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\varphi_{21}^{d\vec{s}'} \equiv \varphi_{21} - d\vec{s}' \cdot \vec{E}(P_2)$$

$$\cong \varphi_{21} + dx' \frac{\partial \varphi_{21}}{\partial x} + dy' \frac{\partial \varphi_{21}}{\partial y} + dz' \frac{\partial \varphi_{21}}{\partial z}$$

$$= \varphi_{21} + \underbrace{(dx' \hat{i} + dy' \hat{j} + dz' \hat{k})}_{d\vec{s}'} \cdot \left(\frac{\partial \varphi_{21}}{\partial x} \hat{i} + \frac{\partial \varphi_{21}}{\partial y} \hat{j} + \frac{\partial \varphi_{21}}{\partial z} \hat{k} \right)$$

$$\frac{\partial \varphi_{21}}{\partial x} \hat{i} + \frac{\partial \varphi_{21}}{\partial y} \hat{j} + \frac{\partial \varphi_{21}}{\partial z} \hat{k} = -E_x(P_2) \hat{i} - E_y(P_2) \hat{j} - E_z(P_2) \hat{k} \\ = -\vec{E}(P_2)$$

$$\text{so, } \vec{E} = -\vec{\nabla} \varphi$$

examples: $\varphi = -kxy$

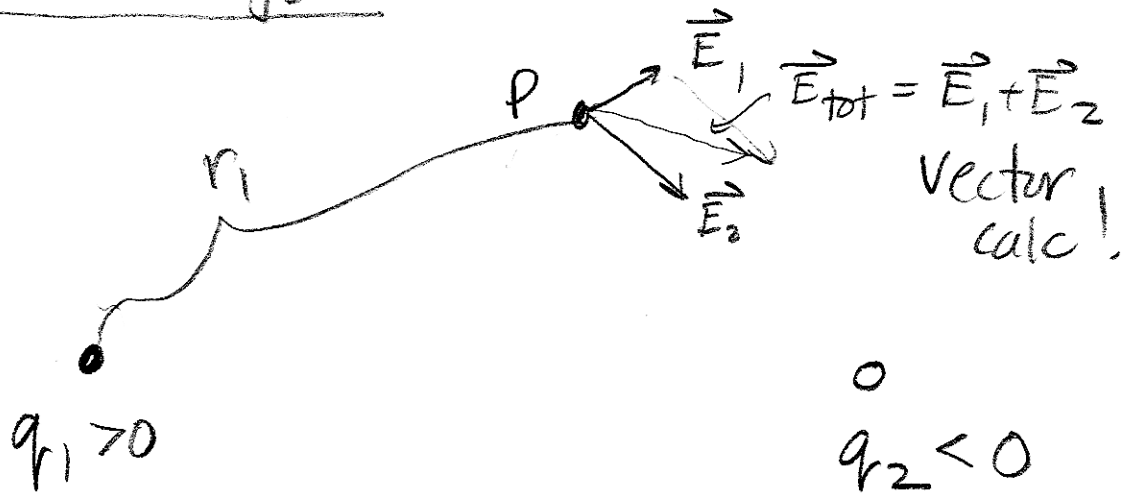
$$-\frac{\partial \varphi}{\partial x} = ky = E_x \quad -\frac{\partial \varphi}{\partial y} = +kx = E_y$$

Calculating Potentials.

One charge:

Diagram showing a point charge q at the center. Radial lines with arrows pointing outwards represent the electric field. A dashed line indicates a radial distance r from the charge. The potential is given by $\varphi = \frac{q}{r}$ (as $r \rightarrow \infty$, $\varphi \rightarrow 0$). The electric field is given by $E(\text{radial}) = \frac{q}{r^2}$ and $\vec{E} = \frac{q}{r^2} \hat{r}$.

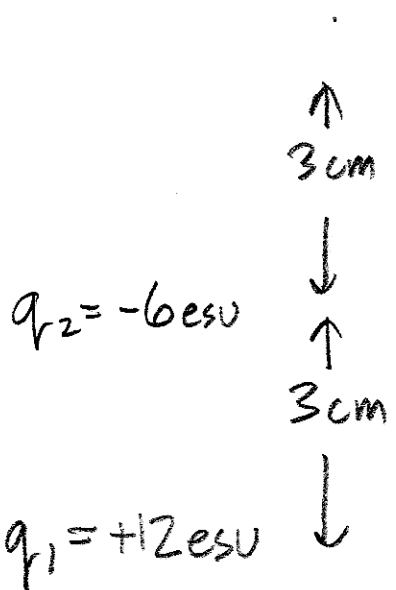
Two Charges



$$\phi(P) = \frac{q_1}{r_1} - \frac{|q_2|}{r_2}$$

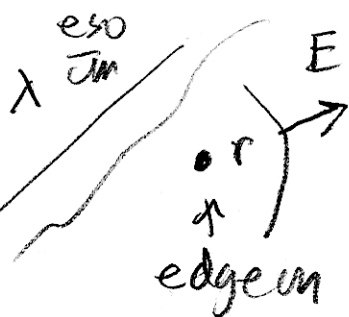
no vector sum!

$$\times \phi = \frac{12}{6} - \frac{6}{3} = 2 - 2 = 0 \frac{esu}{cm}$$



$$\begin{aligned} \times \phi &= \frac{-6}{4} + \frac{12}{5} \\ &= -\frac{3}{2} + \frac{12}{5} \\ &= \frac{-15}{10} + \frac{24}{10} = \frac{9}{10} \frac{esu}{cm} \end{aligned}$$

Line Charge:



$$E = \frac{2\lambda}{r}$$

$$\phi_{z1} = - \int_{r_1}^{r_2} \frac{2\lambda}{r} dr$$

$$\phi_{z1} = -2\lambda \ln \frac{r_2}{r_1}$$

or $\phi(r=0)$ Cannot choose $\phi(r=\infty)$ to be zero!

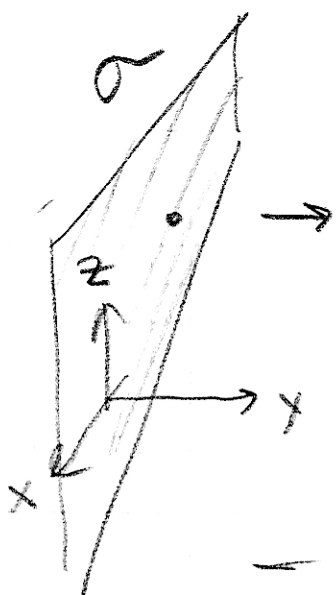
$$\phi(r) = -2\lambda \ln r + \text{constant}$$

$$\phi(r) = -2\lambda \ln \left(\frac{r}{r_0} \right)$$

$r_0 = \text{radius where } \phi(r_0) = 0$

$$E_r = -\frac{\partial \phi}{\partial r} \hat{r} = \frac{2\lambda}{r}$$

∞ Sheet Charge:



$$E_y = 2\pi\sigma$$

$$\phi_{z1} = -2\pi\sigma \int_{y_1}^{y_2} dy = -2\pi\sigma(y_2 - y_1)$$

$$\boxed{\phi = -2\pi\sigma y}$$

$y > 0$
 $\phi = 0, y = 0$

- sign : downhill going outward

$$\boxed{\phi = 2\pi\sigma y}$$

$y < 0$