$\vec{\nabla} f$ is called the gradient of f.

Think about it....

when $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ all > 0, $\vec{\nabla} f$ gets the most out of each derivative.

"equal representation"... problem on set.

If $f(r)$, then $\vec{\nabla} f = \hat{r} \frac{\partial f}{\partial r}$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial r}\frac{\partial r}{\partial x} \qquad r = \sqrt{x^2+y^2+z^2}$$

$$\frac{\partial r}{\partial x} = \frac{1}{2}\frac{2x}{\sqrt{x^2+y^2+z^2}} = \frac{x}{r}$$

$$= \frac{\partial f}{\partial r}\frac{x}{r}$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial r}\frac{y}{r} \qquad \frac{\partial f}{\partial z} = \frac{\partial f}{\partial r}\frac{z}{r}$$

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k} = \frac{\partial f}{\partial r}\overbrace{\frac{(x\hat{\imath} + y\hat{\jmath} + z\hat{k})}{r}}^{\hat{r}}$$

$$\vec{\nabla} f = \hat{r}\frac{\partial f}{\partial r}$$

Relationship between φ & $\vec{E}$

$$\varphi_{21} = -\int_{P_1}^{P_2} \vec{E}\cdot d\vec{s}$$

$-$: push + charge against

$$\varphi_{21}^{d\vec{s}\,'} = -\int_{P_1}^{P_2 + d\vec{s}\,'} \vec{E}\cdot d\vec{s} \qquad d\vec{s}\,' = dx'\hat{\imath} + dy'\hat{\jmath} + dz'\hat{k}$$

$$\varphi_{21}^{d\vec{s}'} \equiv \varphi_{21} - d\vec{s}' \cdot \vec{E}(P_2)$$

$$\cong \varphi_{21} + dx' \frac{\partial \varphi_{21}}{\partial x} + dy' \frac{\partial \varphi_{21}}{\partial y} + dz' \frac{\partial \varphi_{2}}{\partial z}$$

$$= \varphi_{21} + \underbrace{(dx'\hat{\imath} + dy'\hat{\jmath} + dz'\hat{k})}_{d\vec{s}'} \cdot \left(\frac{\partial \varphi_{21}}{\partial x}\hat{\imath} + \frac{\partial \varphi_{21}}{\partial y}\hat{\jmath} + \frac{\partial \varphi_{21}}{\partial z}\hat{k} \right)$$

$$\frac{\partial \varphi_{21}}{\partial x}\hat{\imath} + \frac{\partial \varphi_{21}}{\partial y}\hat{\jmath} + \frac{\partial \varphi_{21}}{\partial z}\hat{k} = -E_x(P_2)\hat{\imath} - E_y(P_2)\hat{\jmath} - E_z(P_2)\hat{k}$$

$$= -\vec{E}(P_2$$

so,

$$\vec{E} = -\vec{\nabla}\varphi$$

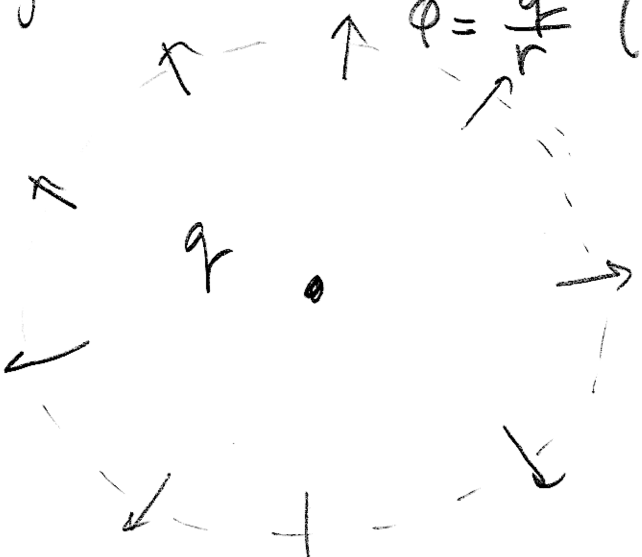
examples: $\varphi = -Kxy$

$$-\frac{\partial \varphi}{\partial x} = Ky = E_x \qquad -\frac{\partial \varphi}{\partial y} = +Kx = E_y$$

Calculating Potentials

One charge:

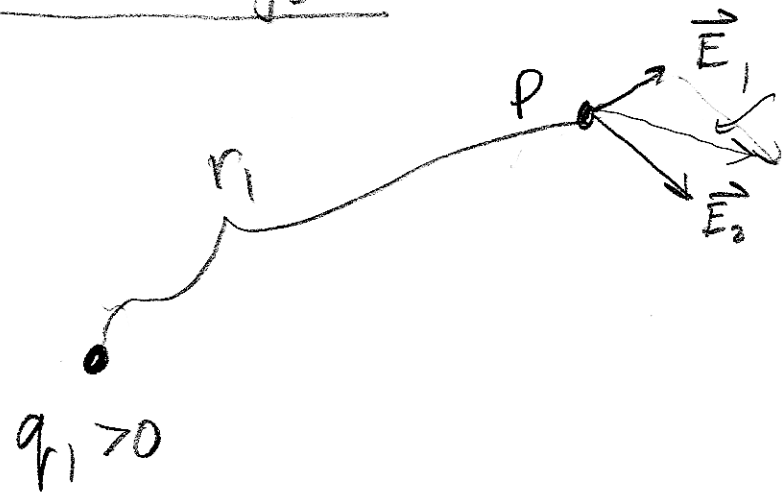
$\varphi = \frac{q}{r}$ (as $r \to \infty$, $\varphi \to 0$)



$E(\text{radial}) = \frac{q}{r^2}$

$$\vec{E} = \frac{q}{r^2}\hat{r}$$

Two Charges

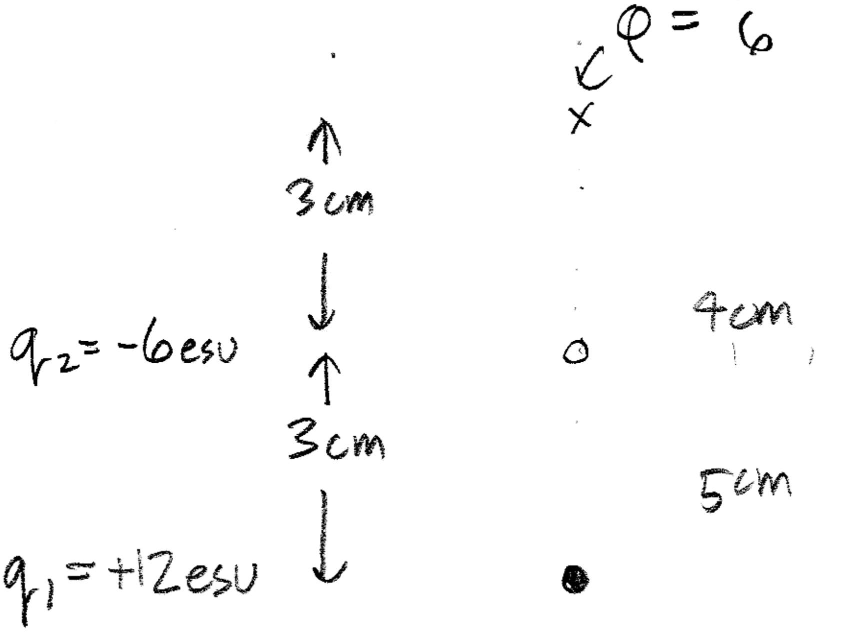


$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2$ vector calc!

$$\varphi(P) = \frac{q_1}{r_1} - \frac{|q_2|}{r_2}$$

no vector sum!

$$\varphi = \frac{12}{6} - \frac{6}{3} = 2 - 2 = 0 \ \frac{esu}{cm}$$

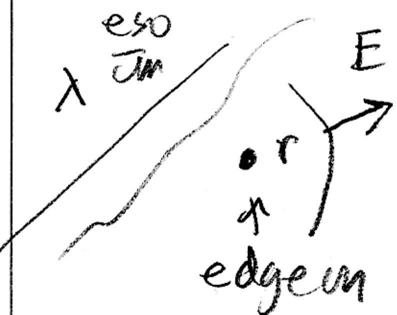


$$\varphi = \frac{-6}{4} + \frac{12}{5}$$

$$= -\frac{3}{2} + \frac{12}{5}$$

$$= \frac{-15}{10} + \frac{24}{10} = \frac{9}{10} \ \frac{esu}{cm}$$

Line Charge:



$E = \frac{2\lambda}{r}$

$$\varphi_{21} = -\int_{r_1}^{r_2} \frac{2\lambda}{r}\, dr$$

$$\varphi_{21} = -2\lambda \ln \frac{r_2}{r_1}$$

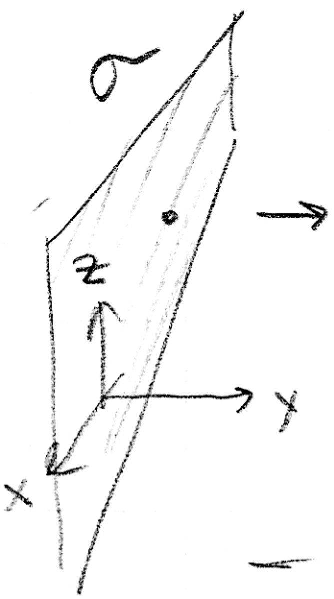
Cannot choose $\varphi(r=\infty)$ or $\varphi(r=0)$ to be zero!

$$\varphi(r) = -2\lambda \ln r + \text{constant}$$

$$\varphi(r) = -2\lambda \ln\left(\frac{r}{r_0}\right)$$ r_0 = radius where $\varphi(r_0) = 0$

$$E_r = -\frac{\partial \varphi}{\partial r}\hat{r} = \frac{2\lambda}{r}$$

∞ Sheet Charge:



$$E_y = 2\pi\sigma$$

$$\varphi_{21} = -2\pi\sigma \int_{y_1}^{y_2} dy = -2\pi\sigma(y_2 - y_1)$$

$$\boxed{\varphi = -2\pi\sigma y} \quad y > 0$$

$\varphi = 0,\ y = 0$

− sign: downhill going outward

$$\boxed{\varphi = 2\pi\sigma y} \quad y < 0$$