$\nabla f$ is called the gradient of $f$.

Think about it...

when $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ all $> 0$, $\nabla f$ gets the most out of each derivative.

Problem on set...

If $f(r)$, then $\nabla f = \hat{r} \frac{\partial f}{\partial r}$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial r} \frac{\partial r}{\partial x} = \frac{\partial f}{\partial r} \frac{x}{r}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\frac{\partial r}{\partial x} = \frac{1}{2} \frac{2x}{\sqrt{x^2 + y^2 + z^2}} = \frac{x}{r}$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial r} \frac{y}{r}$$

$$\frac{\partial f}{\partial z} = \frac{\partial f}{\partial r} \frac{z}{r}$$

$$\nabla f = \frac{\partial f}{\partial x} \hat{i} + \frac{\partial f}{\partial y} \hat{j} + \frac{\partial f}{\partial z} \hat{k} = \frac{\partial f}{\partial r} \left( \hat{i} \frac{x}{r} + \hat{j} \frac{y}{r} + \hat{k} \frac{z}{r} \right)$$

$$\nabla f = \hat{r} \frac{\partial f}{\partial r}$$

Relationship between $\Phi + \vec{E}$

$$\Phi_{21} = - \int_{\vec{P}_{1}}^{\vec{P}_{2}} \vec{E} \cdot d\vec{s}$$

$- :$ push + charge against

$$ds^2 = dx^1 \hat{i} + dy^j \hat{j} + dz^k \hat{k}$$

$\Phi_{21} = - \int_{\vec{P}_{1}}^{\vec{P}_{2}} \vec{E} \cdot d\vec{s}$
\[ \phi_{21} = \phi_2 \cdot E(p_2) \]

\[ \approx \phi_{21} + dx \frac{\partial \phi_{21}}{\partial x} + dy \frac{\partial \phi_{21}}{\partial y} + dz \frac{\partial \phi_{21}}{\partial z} \]

\[ = \phi_{21} + (dx \hat{\imath} + dy \hat{j} + dz \hat{k}) \cdot (\frac{\partial \phi_{21}}{\partial x} \hat{\imath} + \frac{\partial \phi_{21}}{\partial y} \hat{j} + \frac{\partial \phi_{21}}{\partial z} \hat{k}) \]

\[ \frac{\partial \phi_{21}}{\partial x} \hat{\imath} + \frac{\partial \phi_{21}}{\partial y} \hat{j} + \frac{\partial \phi_{21}}{\partial z} \hat{k} = -E_x(p_2) \hat{\imath} - E_y(p_2) \hat{j} - E_z(p_2) \hat{k} \]

\[ \vec{E} = -\nabla \phi \]

Examples: \[ \phi = -kxy \]

\[ -\frac{\partial \phi}{\partial x} = ky = E_x \quad -\frac{\partial \phi}{\partial y} = -kx = E_y \]

Calculating Potentials.

One charge: \[ \phi = \frac{q}{4\pi r} \quad (\text{as } r \to \infty, \phi \to 0) \]

\[ \vec{E} = \frac{q}{r^2} \frac{\hat{r}}{r} \]
Two Charges

\[ \mathbf{E}_1 + \mathbf{E}_2 = \mathbf{E}_{\text{tot}} \]

\[ \varphi(P) = \frac{q_1}{r_1} - \frac{|q_2|}{r_2} \]

\[ q_1 > 0 \]

\[ q_2 < 0 \]

\[ \varphi = \frac{12}{6} - \frac{6}{3} = 2 - 2 = 0 \text{ esu/cm} \]

\[ q_1 = +12 \text{ esu} \]

\[ q_2 = -6 \text{ esu} \]

\[ 3 \text{ cm} \]

\[ 4 \text{ cm} \]

\[ 5 \text{ cm} \]
Line Charge:

\[ E = \frac{2\lambda}{r} \]

\[ \Phi_{21} = -\int_{r_1}^{r_2} \frac{2\lambda}{r} \, dr \]

\[ \Phi_{21} = -2\lambda \ln \frac{r_2}{r_1} \]

\[ \text{Cannot choose } \Phi(r=\infty) \]

or \( \Phi(r=0) \) to be zero.

\[ \Phi(r) = -2\lambda \ln r + \text{constant} \]

\[ \Phi(r) = -2\lambda \ln \left( \frac{r}{r_0} \right) \quad r_0 = \text{radius} \]

where \( \Phi(r) = 0 \)

\[ E_r = -\frac{\partial \Phi}{\partial r} \hat{r} = \frac{2\lambda}{r} \]

Sheet Charge:

\[ E_y = 2\pi \sigma \]

\[ \Phi_{21} = -2\pi \sigma \int_{y_1}^{y_2} dy = -2\pi \sigma (y_2 - y_1) \]

\[ \Phi = -2\pi \sigma \hat{y} \] \( y > 0 \)

\[ \phi = 0, y = 0 \]

- Sign: downhill going outward

\[ \Phi = 2\pi \sigma \hat{y} \] \( y < 0 \)