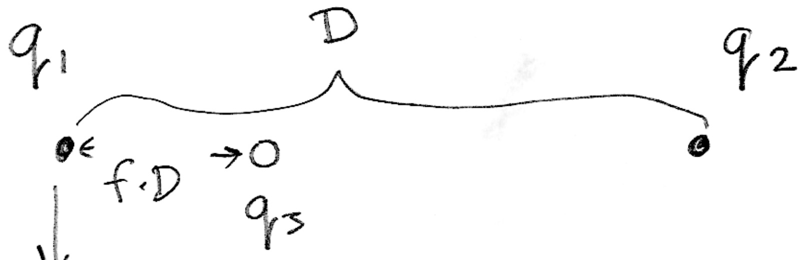
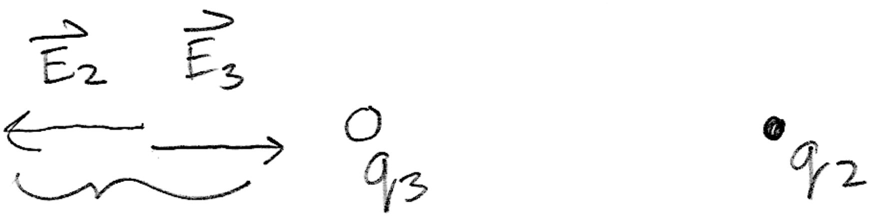
Electric Field Reminder



f set...

want 0 force   How? Ignore q_1 (remove it)



to cancel, q_3 opposite in sign to q_2

$$\frac{q_2}{D^2} = -\frac{q_3}{f^2 D^2}$$

$$\boxed{q_3 = -f^2 q_2}$$

(now imagine putting q_1 back)

$$\vec{F} = q_1 \cdot \vec{E}\left(\begin{matrix}\text{at 1, due}\\ \text{to 2 + 3}\end{matrix}\right)$$

$$\vec{F} = q_1 \cdot \vec{0} = 0$$

Potential Energy is to Electric Potential

as

Force is to Electric Field

A Difference in Electric Potential is called a... "Voltage"

$$U_{21} = -\int_{P_1}^{P_2} \vec{F} \cdot d\vec{s}$$

$-$: you need to push against existing force to increase U

$$\phi_{21} = -\int_{P_1}^{P_2} \vec{E} \cdot ds$$

$$\left(U_{21} = q\,\phi_{21} \right)$$

energy to move

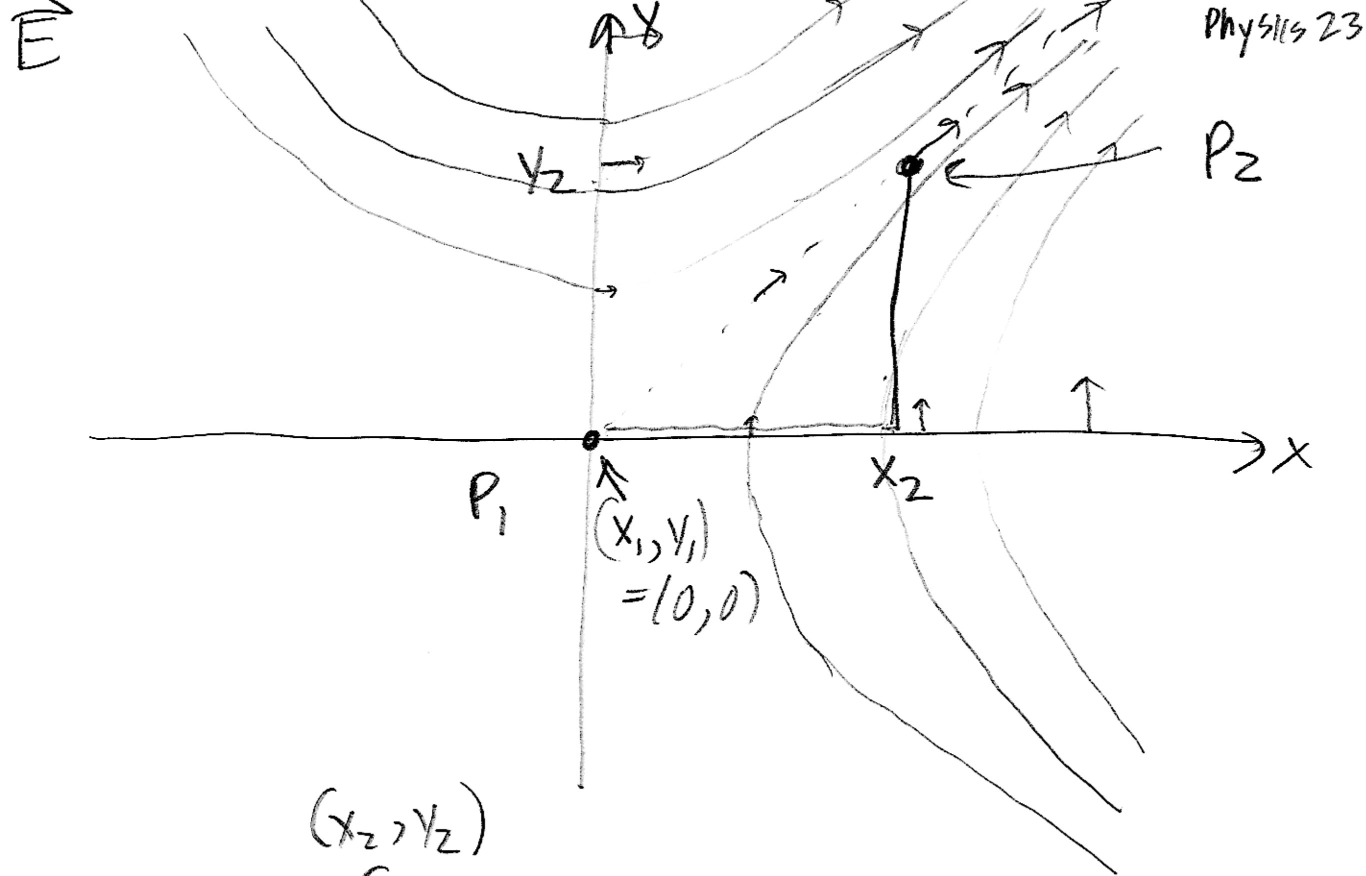
Electric Potential is Energy/Charge

concept works when integral is path independent, which is true for $\vec{E}$ fields only when $\frac{\partial \vec{E}}{\partial t} = 0$. Assume true.

Do example...

$$E_x = Ky \qquad E_y = Kx$$

$$K = \text{constant}$$



$$\phi(x_2, y_2) = -\int_{(0,0)}^{(x_2, y_2)} \vec{E} \cdot d\vec{s} \quad \leftarrow \text{path independent.}$$

path shown: $d\vec{s} \propto \hat{\imath}$ first leg

$d\vec{s} \propto \hat{\jmath}$ second leg.

along first leg, $\vec{E} = E_x \hat{\imath} + E_y \hat{\jmath}$

$$= \underbrace{\cancel{Ky}}_{0} \hat{\imath} + \underbrace{Kx \hat{\jmath}}$$

0 along first leg

first leg $d\vec{s} \propto \hat{\imath}$, dot gives 0 along <u>second</u> leg, $\vec{ds} \propto \hat{j}$, $= dy\,\hat{j}$

$$\vec{E} = Ky\,\hat{\imath} + Kx_2\,\hat{j}$$

$$\vec{E}\cdot\vec{ds} = (Ky\,\hat{\imath} + Kx_2\,\hat{j})\cdot dy\,\hat{j}$$

$$= Kx_2\,dy$$

$$-\int_{(x_2,0)}^{(x_2,y_2)} \vec{E}\cdot\vec{ds} = -Kx_2y_2 + \text{constant}.$$

$$\phi(x,y) = -Kxy \qquad \left(\begin{array}{c}\text{setting}\\ \phi(0,0)=0\end{array}\right)$$

Energy change when a charge q moves from $(0,0)$ to (x,y)?

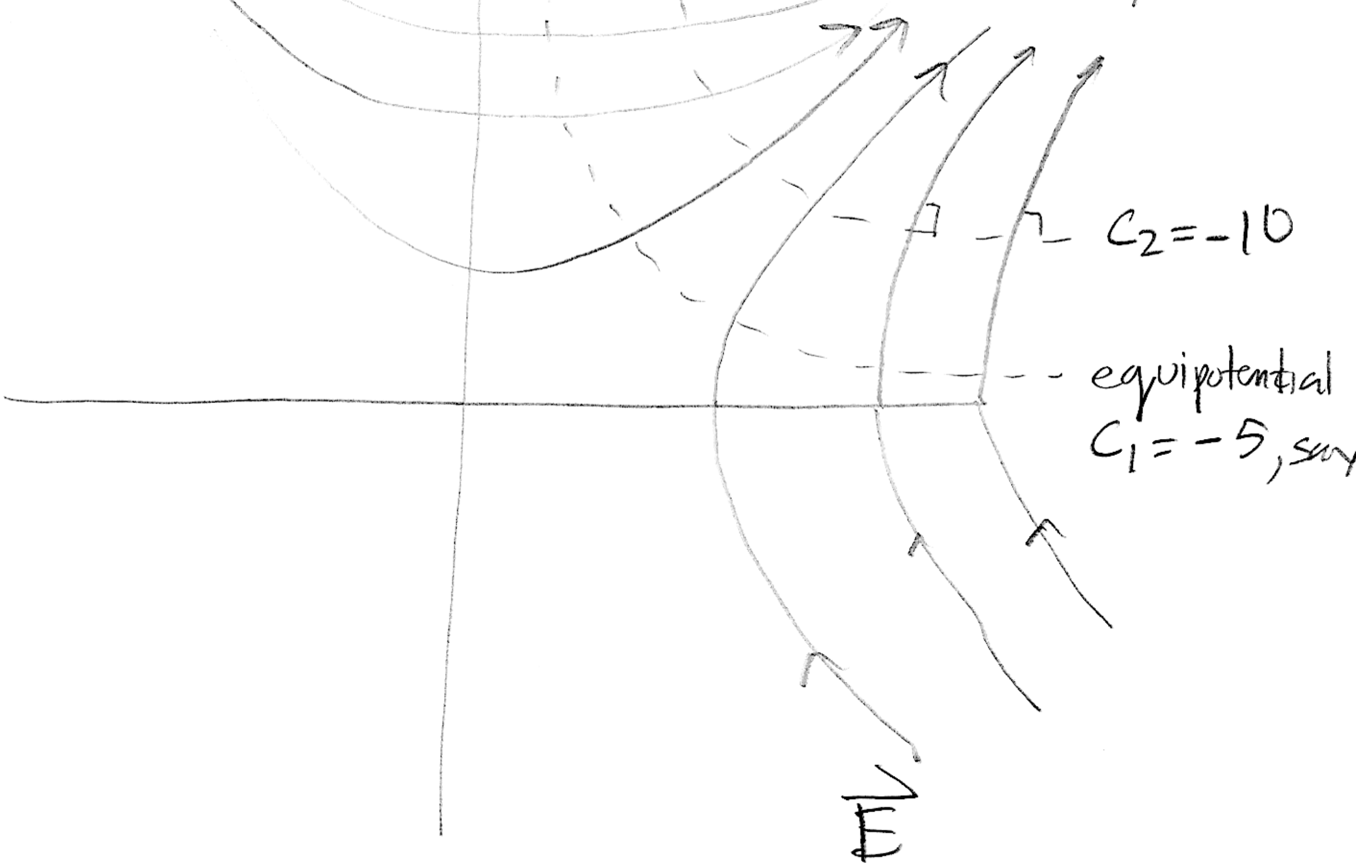
$$U(x,y) = q\phi(x,y) = -qKxy$$

"downhill"

$$\phi(x,y) = \text{constant} = -Kxy = C$$

$$y = \frac{-C}{Kx}$$

(draw on field plot)



Other direction... given $\phi(x, y)$, which is a scalar function, how do you get $\vec{E}$?

Consider $f(x, y, z)$ $\quad (= x^2 y z^3)$

$$\frac{\partial f}{\partial x} = 2xyz^3 \qquad \frac{\partial f}{\partial y} = x^2 z^3 \qquad \frac{\partial f}{\partial z} = 3x^2 y z^2$$

$$\vec{\nabla} f \equiv \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}$$

direction that gives maximum change in f)

purcell says $\hat{x}$ $\hat{y}$ $\hat{z}$