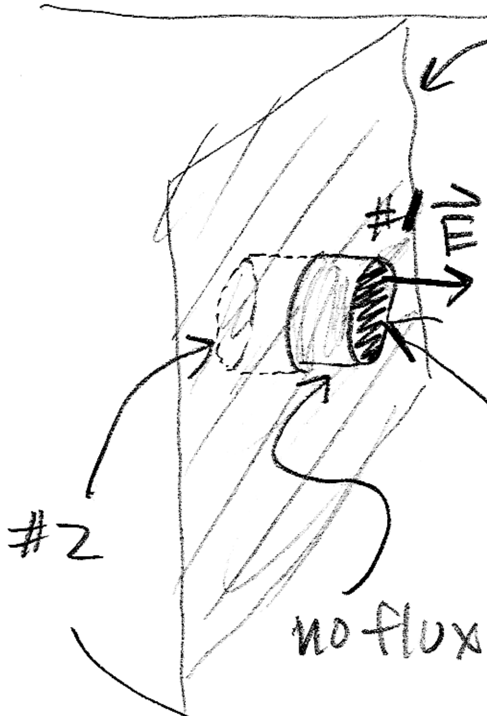
Uniform Flat Charge Density

σ, $\frac{esu}{cm^2}$, extending to ∞



E must be $\perp$ to surface (what direction would such a component point?)

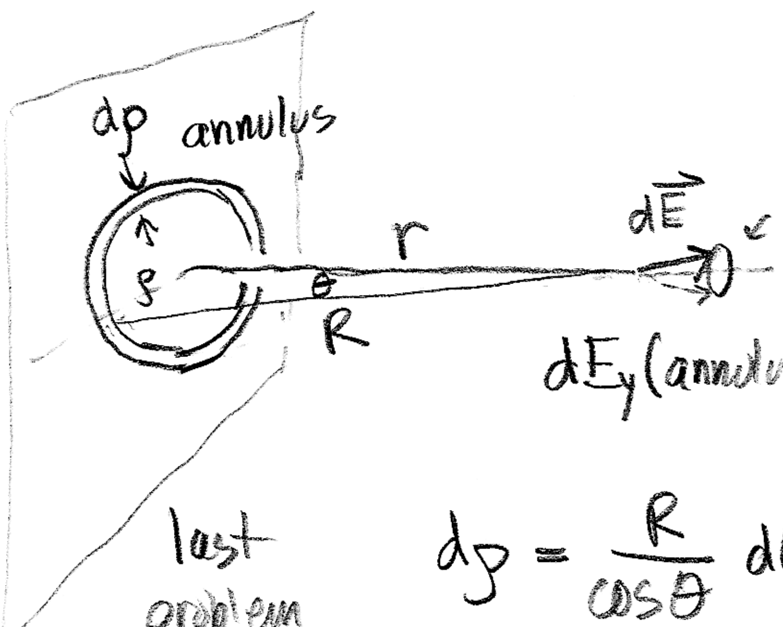
Area A

$$\underbrace{\int \vec{E} \cdot d\vec{a}} = 4\pi\, q(\text{enclosed})$$

2 "caps" $\rightarrow$ $2 \cdot E \cdot A = 4\pi\sigma \cdot A$

$$\boxed{E = 2\pi\sigma}$$

Old, non-gaussian way ...



all $d\vec{E}$'s from annulus

$$dE_y(\text{annulus}) = \frac{2\pi\rho\, d\rho \cdot \sigma}{R^2} \cos\theta$$

last problem

$$d\rho = \frac{R}{\cos\theta}\, d\theta = \frac{r}{\cos^2\theta}\, d\theta$$

$$R = \frac{r}{\cos\theta} \qquad \rho = r\tan\theta$$

$$dE_y(\text{annulus}) = \frac{2\pi \cdot \cancel{r} \cdot \frac{\sin\theta}{\cos\theta} \cdot \frac{r}{\cos^2\theta}\, \sigma \cdot \cancel{\cos\theta}\, d\theta}{\frac{\cancel{r^2}}{\cancel{\cos^2\theta}}}$$

$$= 2\pi\sigma \sin\theta \, d\theta$$

now: $\rho \to 0$, $\rho \to \infty$

$\theta \to 0$, $\theta \to \pi/2$

$$E_y = 2\pi\sigma \cdot \int_0^{\pi/2} \sin\theta \, d\theta$$

$$= 2\pi\sigma \cdot \left(-\cos\theta \Big|_0^{\pi/2}\right)$$

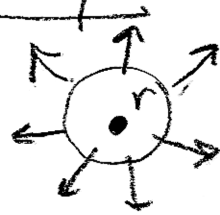
$$= 2\pi\sigma\left(-(\cancel{\cos\tfrac{\pi}{2}}) + \cos(0)\right)$$

0

$$\boxed{E_y = 2\pi\sigma}$$

Summary:

Point Charge



$E_r \propto \frac{1}{r^2}$, $= \frac{4\pi q}{4\pi r^2} = \frac{q}{r^2}$ gCGS

$= \frac{q}{4\pi\varepsilon_0 r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ SI

Line Charge

$E_r \propto \frac{1}{r}$, $= \frac{4\pi\lambda h}{2\pi r h} = \frac{2\lambda}{r}$ gCGS

$= \frac{\lambda h}{2\pi\varepsilon_0 r h} = \frac{\lambda}{2\pi\varepsilon_0 r}$ gCGS

Sheet Charge

$E \propto r^0$, $= 2\pi\sigma$ gCGS

$= \frac{\sigma}{2\varepsilon_0}$ SI

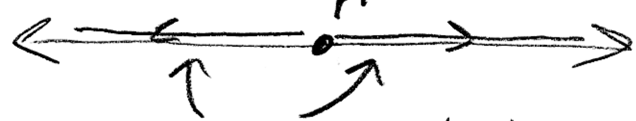
Like living in worlds of varying #'s of dimensions...

3d

independent of distance

$\sigma = \frac{\text{3d charge}}{cm^2}$

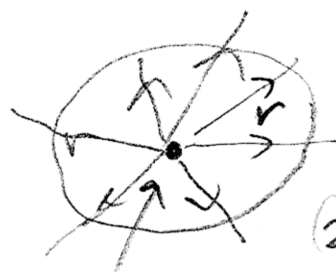
like: one-dimensional world... 1-d charge



field → constant, $\propto r^0$

(where does field go?)

$\lambda = \frac{\text{3d charge}}{cm}$



circum $2\pi r$

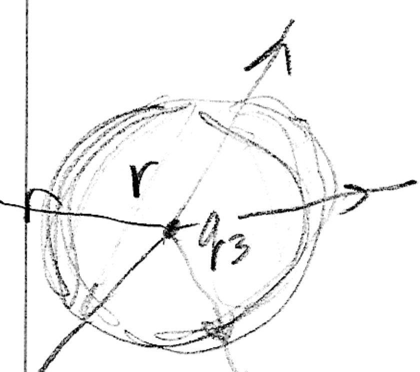
$(2\pi r) \cdot E = 2\pi q_2$

2-d

2-d charge

$E = \frac{q_2}{r}$

$E \propto r^{-1}$



$(4\pi r^2)E = 4\pi q_3$

$E = \frac{q_3}{r^2}$

$E \propto r^{-2}$

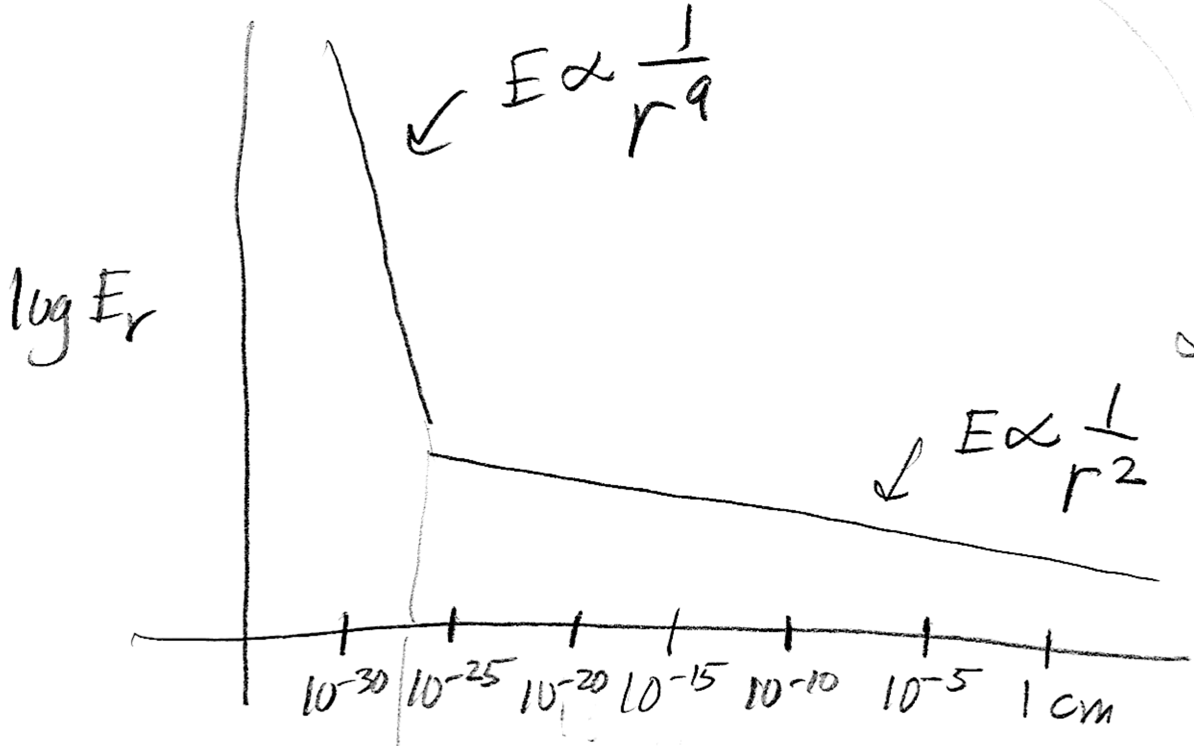
n-dimensions...

$E \propto r^{-(n-1)}$

Maybe 10 spatial dimensions!

7 "curled up"

3 not



dimensions curled up to 10^{-27} cm...