No field inside a spherically symmetric shell of charge.

\[ \vec{E} \]

- No enclosed charge.
  - By symmetry, only radial component of \( \vec{E} \) possible.
  - (Imagine rotating the ball)

\[ E_r = \frac{0 \cdot e}{r^2} \]

\[ \vec{E} = 0 \text{ inside... recall gravity!} \]

Newton proved this in a very different (and more tedious) manner.
Application of Gauss's Law

Uniform Line Charge, $\infty$ long

dq = $\lambda$ dx

dE from this bit of charge cancels that from charge shown

Symmetry: $\int dE_x = 0$

dE_y = dE \cos \theta = \frac{\lambda dx}{R^2} \cos \theta

$\lambda \frac{\text{esu}}{\text{cm}}$

$\cos \theta = \frac{R d\theta}{dx}$

dx = $\frac{R}{\cos \theta} d\theta$

$\cos \theta = \frac{Y}{R} = \frac{r}{R}$ (defines $r$)

$R = \frac{r}{\cos \theta}$

dx = $\left( \frac{r}{\cos \theta} \right) \cdot \frac{1}{\cos \theta} d\theta = \frac{r}{\cos^2 \theta} d\theta$

so $dE_y = \frac{\lambda}{(\frac{r}{\cos \theta})^2} \cos \theta = \frac{\lambda}{r} \cos \theta d\theta$
when \( x = -\infty \), \( \theta = -\pi/2 \)
\( x = +\infty \), \( \theta = \pi/2 \)

\[
E_y = \frac{\lambda}{r} \int_{-\pi/2}^{\pi/2} \cos \theta \, d\theta = \frac{\lambda}{r} \sin \theta \bigg|_{-\pi/2}^{\pi/2} = \frac{\lambda}{r} (1 - (-1))
\]

\[
E_y = \frac{2\lambda}{r}
\]

**Gauss's Law Version**

Gaussian surface by symmetry (imagine rotating about line charge) \( \vec{E} \) only has component along direction to wire. (when wire \( \infty \) long)

Gauss: \( \oint \vec{E} \cdot d\vec{a} = 4\pi q \) (enclosed)

Endcaps: no flux, \( \vec{E} \parallel \) to them

Sides: \( E \cdot (2\pi r \cdot L) = 4\pi q \)

\[
E = \frac{2 \cdot \frac{q}{L}}{r} = \frac{2\lambda}{r}
\]