Gauss's Law

Put a spherical surface around a charge, centered on the charge. Compute the flux of $\mathbf{E}$ through that surface:

$$\mathbf{E}_i \text{ is parallel to } \mathbf{n}_i \quad \mathbf{E}_i \text{ is constant}$$

$$= \frac{q}{r^2}$$

everywhere on the surface

$$\int \mathbf{E} \cdot d\mathbf{a} = \frac{q}{r^2} \int d\mathbf{a}$$

$$= 4\pi r^2 \cdot \frac{q}{r^2} = 4\pi q$$

Size of sphere ($r$) didn't matter!

Also, didn't matter that charge was at center.

Shape of surface didn't matter.

That surface closed and around charge did.
\[ \frac{\vec{E}(R) \cdot \vec{A}}{\vec{E}(r) \cdot \vec{a}} = \frac{r^2}{R^2} \cdot \frac{r^2}{R^2} \cdot \frac{1}{r^2 \cos \theta} \cdot \cos \theta \]

\[ = 1 \]

Flux through the first little patch equals the flux through the outer, bigger patch! Sum up...

\[ \int \vec{E} \cdot d\vec{a} = 4\pi \sum q_i \]

only the charges in side...

Closed surface
Charges Outside?

\[ \Phi_{\text{total}} = 4\pi q \]

\[ \Phi_1 \to 4\pi q \]

\[ \Phi_2 = \Phi_{\text{total}} - \Phi_1, \to 0 \]

So

\[ \Phi = 0! \]

This is not true for all vector functions.
For example, what if \( D = \frac{q}{R^3} \) ?

Then \( \Phi_D = \int \vec{E} \cdot d\vec{a} = \left( \frac{q}{R^3} \right) \cdot 4\pi R = \frac{4\pi q}{R} \)

which is not independent of \( R \).

**Spherical Charge Distribution**

\[
p(x, y, z) \text{ depends only on } r = \sqrt{x^2 + y^2 + z^2}
\]

\( \vec{E} \) must only have radial component; Gauss's law really useful here:

at \( p_2 \)

\[
\int \vec{E} \cdot d\vec{a} = 4\pi q(\text{inside})
\]

\( 4\pi r_2^2 E_2 = 4\pi q(\text{inside}) \)

\[
E_2 = \frac{q(\text{inside})}{r_2^2}
\]

\( E_1 = \frac{q(\text{total})}{r_1^2} \)

same as if \( q \) concentrated at center!
Example:

\[ Q = \frac{4\pi}{3} \rho R_0^3 \]

Inside:

\[ E(r) = \frac{q_{\text{inside}}}{r^2} = \frac{\frac{4\pi}{3} \rho R_0^3}{r^2} \quad r < R_0 \]

Outside:

\[ E(r) = \frac{\frac{4\pi}{3} \rho R_0^3}{r^2} = \frac{Q}{r^2} \quad \text{also} \]

\[ = \frac{4\pi}{3} \rho \cdot \left( \frac{R_0}{r} \right)^2 R_0 \]

\[ \frac{Q}{r^2} = 4\pi \int \frac{\rho}{r} \left( \frac{R_0}{r} \right)^2 \cdot R_0 \]

just like point charge \( Q \)!

not true for a cubical charge distribution