Then, imagine a region of space filled with a charge distribution:

\[ \text{Volume } V \]

\[ \mathbf{dE} = \frac{\mathbf{p}(x', y', z') \mathbf{r}' \mathbf{r}}{|\mathbf{r} - \mathbf{r}'|^2} \]

\[ \mathbf{E} = \int_V \mathbf{p}(x', y', z') \mathbf{r}' \mathbf{r} \frac{dV}{|\mathbf{r} - \mathbf{r}'|^2} \]

This is easier than it looks....

Often, all 3-d not really needed.

Example:

Q put evenly on plastic rod bent in semicircle of radius R

Find \( \mathbf{E} \) field at center

\[ \lambda = \frac{Q}{\pi R} \leq \frac{1}{2} \text{ circumference} \]

\[ dq = \lambda ds \]

\( y \)-components of symmetric points (#1 + #2) cancel
\[ dE_x = \frac{dq}{R^2} \cdot \sin \theta \]

\[ = \frac{1}{R^2} \left( \frac{Q}{\pi R} \right) \cdot Rd\theta \cdot \sin \theta \]

\[ E_x = \frac{Q}{\pi R^2} \int_0^{\pi} d\theta \sin \theta \]

\[ E_x = \frac{Q}{\pi R^2} \left( -\cos \theta \right) \bigg|_0^\pi \]

\[ = \frac{Q}{\pi R^2} \left( -(-1) - (-1) \right) \]

\[ E_x = \frac{2}{\pi} \frac{Q}{R^2} \quad \text{(g CGS)} \]

\[ = \frac{1}{4\pi \varepsilon_0} \frac{2}{\pi} \frac{Q}{R^2} \quad \text{(SI)} \]

\[ \text{Flux} \]

\[ \text{given: a vector that is a function of location in space} \]

\[ \text{an area A} \]

\[ \text{the concept of flux quantifies how much of the vector goes through the area... so orientation of the area with respect to the vector field} \]
Easiest example: vector field that is constant, always in x-direction

\[ \vec{E} \]

Area A \perp to constant \( \vec{E} \)

This case \( \Phi = \vec{E} \cdot A \)

Area A \parallel to constant \( \vec{E} \)

This case \( \Phi = 0 \)

To describe flux... make a vector that describes A, that is \perp to the surface

\[ |\vec{a}| = A \]

direction \perp to surface.

In this simple case

\[ \Phi = \vec{E} \cdot \vec{a} = EA \cos \theta \]

\( A \perp \) to \( \vec{E} \): \( \theta = 0 \), \( \Phi = EA \cdot (\cos 0 = 1) \)

\( \parallel \) to \( \vec{E} \): \( \theta = \frac{\pi}{2} \), \( \Phi = E \cdot A \cdot (\cos \frac{\pi}{2} = 0) \)
Surface Integral for Flux

The E field.

"kidney"

segment surface into patches.

et cetera

Then... make a direction that is \( \mathbf{n} \) to each patch -- that will be the direction of the normal vector.

\[ \mathbf{a}_i = \mathbf{A}_i \cdot (\text{unit vector} \perp \text{to surface}) \]
Then at the center of each path, determine the value (magnitude and direction) of the vector field $E$. 

The velocity $\mathbf{V}$ is of some fluid. For a closed surface that does not have a sink or a source of fluid, that limit as $S$ goes to zero of the contribution to flux $\oint \mathbf{E} \cdot d\mathbf{a}$ of some fluid $\mathbf{S}$ will be zero.

Intuition: imagine changing $\mathbf{E}$ to $\overline{\mathbf{E}}$ going into the out of the surface. For a closed surface that does not have a sink or a source of fluid, 

$\bar{\Phi} = \mathbf{S} \cdot d\mathbf{a}$