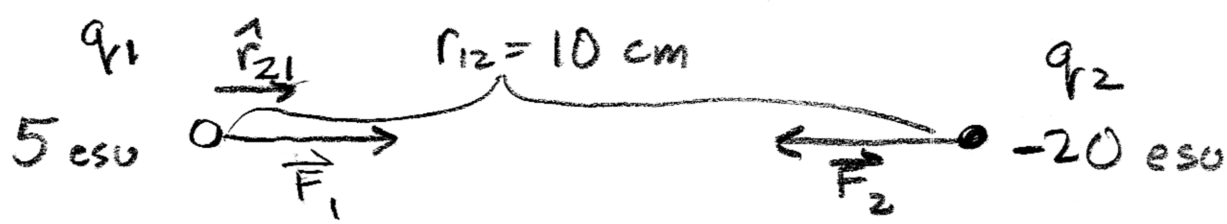
Also, in a hydrogen atom, the electron moves at $\simeq 10^{-2} \times$ speed of light, proton $\simeq 10^{-5} \times$ speed of light, but nonetheless their charges still cancel...

$\boxed{\text{Electric charge is independent of velocity}}$

Charges add up.

Electrostatic Forces add up... presence of a third charge does not influence behavior of first two.



$$\vec{F}_2 = \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21}$$ Gaussian c.g.s.

$$\boxed{\vec{F}_2 = \frac{5(-20)}{(10^2)^2} = (-10^{-2} \text{ dynes}) \cdot \hat{r}_{12}}$$

SI: $1 \text{ dyne} = 1 \frac{\text{gm} \cdot \text{cm}}{s^2} = 1 \cdot 10^{-3} \cdot 10^{-2} \frac{\text{kg m}}{s^2}$

$= 10^{-5}$ N

$$\boxed{\vec{F}_2 = (-10^{-7}) \hat{r}_{21} \text{ N}}$$

<u>Other way</u>:

$$5\text{ esu} = \frac{5}{3\cdot 10^9} = \frac{5}{3}\cdot 10^{-9}\text{ C}$$

$$-20\text{ esu} = \frac{-20}{3\cdot 10^{-9}} = -\frac{20}{3}\cdot 10^{-9}\text{ C}$$

$$10\text{ cm} = 0.1\text{ m}$$

$$\vec{F}_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

$$= -9\cdot 10^9\cdot\frac{5\cdot 20}{9}\cdot 10^{-18}\hat{r}_{12}$$

$$= -10^9\cdot 10^2\cdot 10^{-18}\hat{r}_{12}$$

$$\boxed{\vec{F}_2 = -10^{-7}\hat{r}_{12}\text{ N}}$$

$\vec{F}_1$? (1) Newton III ...

$$\vec{F}_1 = -\vec{F}_2$$

$$= (+10^{-2}\text{ dynes})\hat{r}_{12}$$

$\longleftarrow \vec{F}_2$

$\longrightarrow \vec{F}_1$

(2) Coulomb's Law

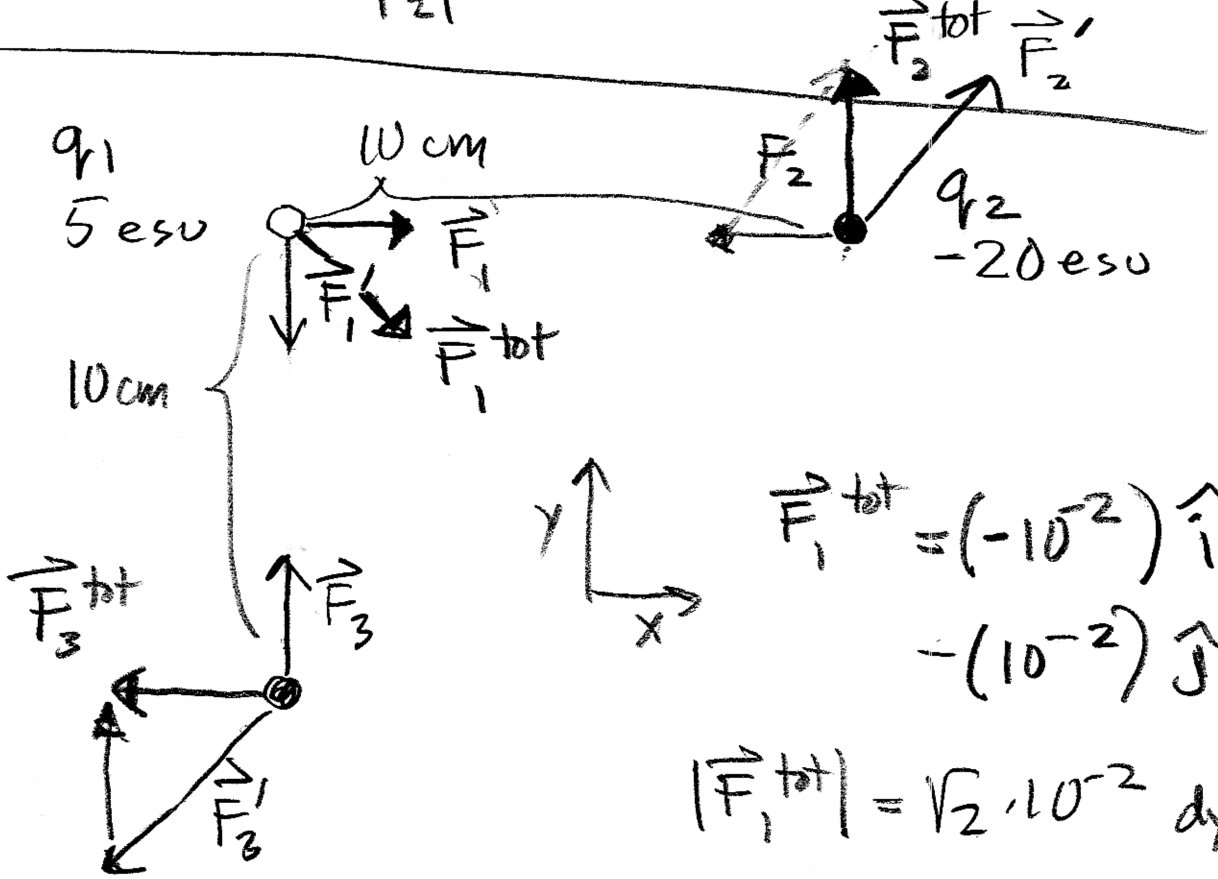
$$\vec{F}_1 = \frac{q_2 q_1}{r_{12}^2}\hat{r}_{12}$$ (note: 1 + 2 are <u>swapped</u>).

$$\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$$ (points from $\vec{r}_2$ to $\vec{r}_1$)

$$\hat{r}_{12} = \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} = -\hat{r}_{21}$$

$$r_{12}^2 = |\vec{r}_1 - \vec{r}_2|^2 = |\vec{r}_2 - \vec{r}_1|^2 = r_{21}^2$$

$$\vec{F}_1 = -\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} = -\vec{F}_2$$

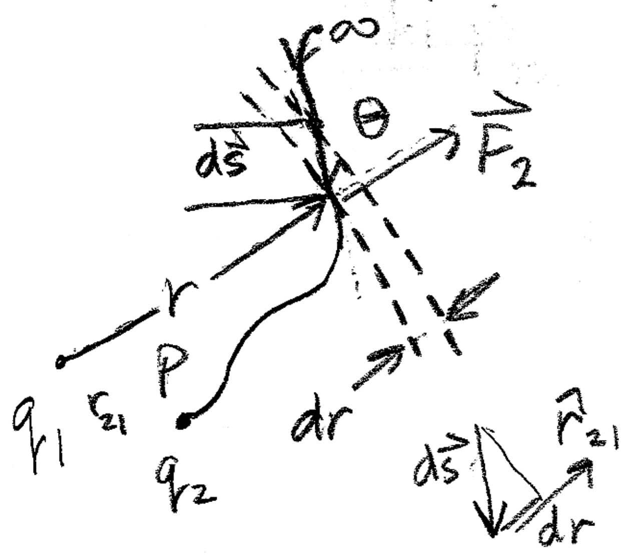


$$\vec{F}_1^{tot} = (-10^{-2})\hat{i} - (10^{-2})\hat{j}$$

$$|\vec{F}_1^{tot}| = \sqrt{2}\cdot 10^{-2} \text{ dynes}$$

<u>you do others.</u>

Potential Energy



$$dW = \vec{F}_2 \cdot \vec{ds}$$

$$= \left(\frac{q_1 q_2}{r^2}\right) \hat{r}_{21} \cdot \vec{ds}$$

$$\hat{r}_{21} \cdot \vec{ds} = -dr$$

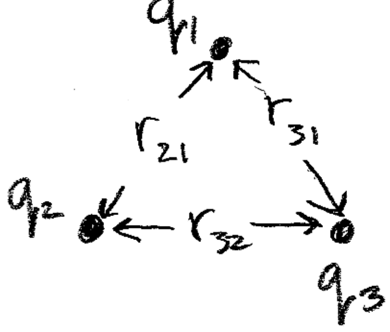
$$W = -\int_{\infty}^{r_{21}} \frac{q_1 q_2}{r^2}\, dr$$

$$W = \frac{q_1 q_2}{r}\Bigg|_{\infty}^{r_{21}} = \frac{q_1 q_2}{r} \text{ (gCGS)}$$

$W < 0$ when q_1, q_2 opposite sign (like gravity)

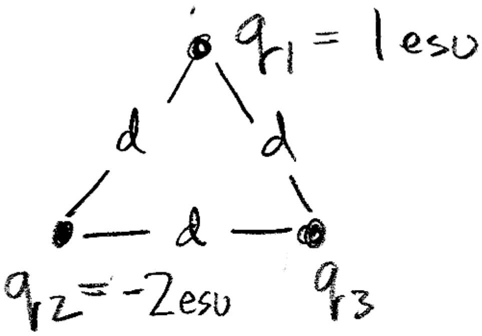
$W > 0$ when q_1, q_2 same sign

SI: $W = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$



$$U = \frac{q_1 q_2}{r_{21}} + \frac{q_1 q_3}{r_{31}} + \frac{q_2 q_3}{r_{32}}$$

Can U be zero for 3 non-zero charges, say, at the corners of an equilateral triangle



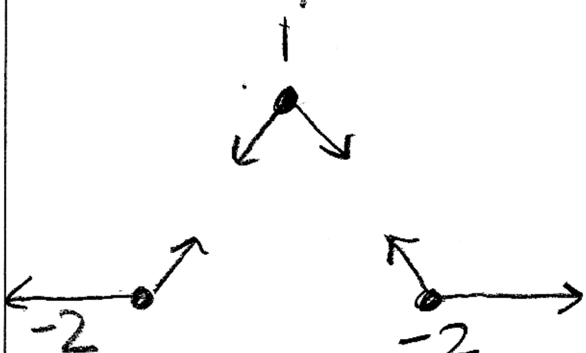
$$U = \frac{1}{d}\left(1\cdot(-2) + 1\cdot q_3 - 2q_3\right) = 0$$

$$= \frac{1}{d}\left(-2 - q_3\right) = 0$$

$$\boxed{q_3 = -2\ \mathrm{esu}}$$

Is this a stable situation ?

Check: if stable, no net force on any charge.



Net force on none is zero — not stable.

Generalization

$$U = \frac{1}{2}\sum_{j=1}^{N}\sum_{k\neq j}\frac{q_j q_k}{r_{jk}}$$

evaluate for $N = 3$

$$= \frac{1}{2}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_1}{r_{21}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}} + \frac{q_3 q_2}{r_{32}}\right)$$