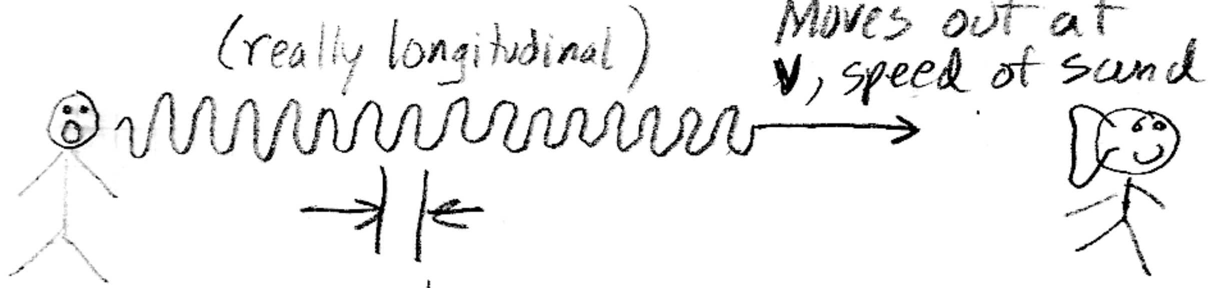
Doppler Effect



Sings a middle C C_4

$\nu = 261.6$ Hz

$T = \frac{1}{\nu}$

$\lambda = vT = \frac{v}{\nu}$

Hears 261.6 Hz (eventually)

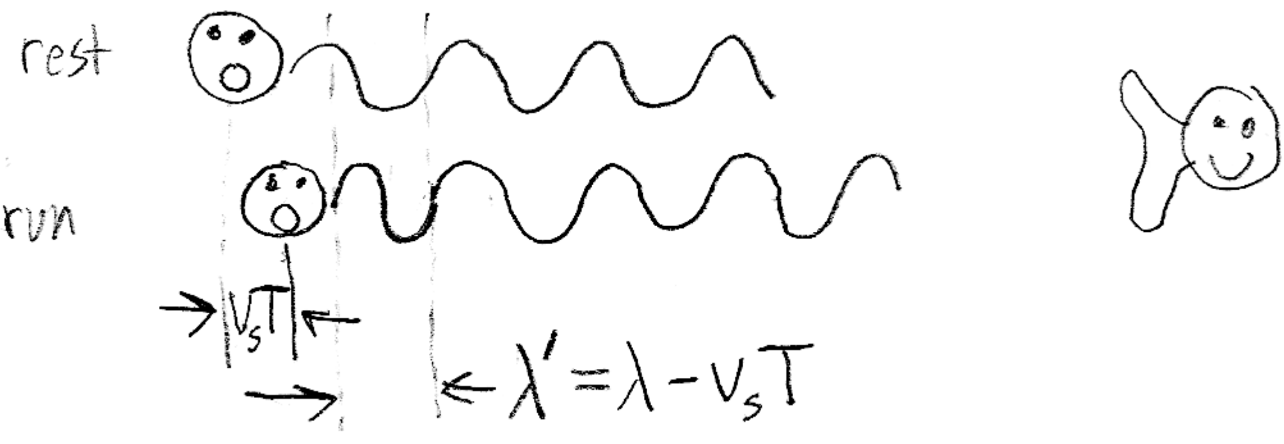
$v \simeq 343$ m/s air, 20°C

$$\lambda_{C_4} = \frac{343 \text{ m/s}}{261.6} = 1.5 \text{ meters}$$

Doppler Effect

What if you run toward the listener?

Think of sirens... frequency heard by listener increases, even though you keep singing a C_4 (to you)



Listener will hear ν, λ until the shorter wavelength moves out to their ear.. then they hear..

$$\nu' = \frac{V}{\lambda'} = \frac{V}{\lambda - V_s T} = \frac{\frac{V}{\lambda}}{1 - \frac{V_s}{(\lambda/T)}}$$

($\frac{V}{\lambda} \leftarrow \nu$; $(\lambda/T) \to V$)

$$\boxed{\nu' = \frac{\nu}{1 \mp \frac{V_s}{V}}}$$

$-$ = singer running toward listener

$+$ = away.

V_s = speed of singer

How fast to get a $\frac{1}{2}$ tone?

$$\frac{\nu'}{\nu} = 2^{1/12} = \frac{1}{1 - \frac{V_s}{V}}$$

$$1 - \frac{V_s}{V} = \frac{1}{2^{1/12}} \Rightarrow 1 - \frac{1}{2^{1/12}} = \frac{V_s}{V}$$

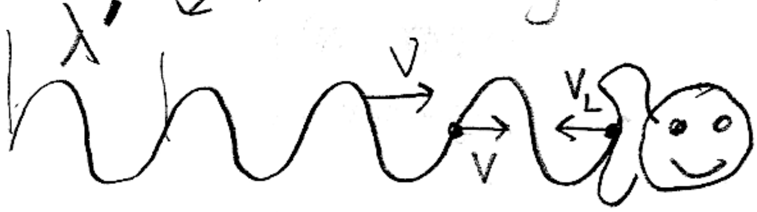
$$V_s = V \cdot \left(1 - \frac{1}{2^{1/12}}\right)$$

$$= 343 \cdot \left(1 - \frac{1}{2^{1/12}}\right) = 19 \frac{m}{s}$$

$$\boxed{V_s \simeq 43 \text{ mph}}$$

What if the listener starts moving?

$\lambda' = \lambda$ if singer at rest



$V \gg V_L$ usually $\quad (V + V_L) T'' = \lambda'$

$$\frac{(V + V_L)}{\lambda'} = \frac{1}{T''} = \nu''$$

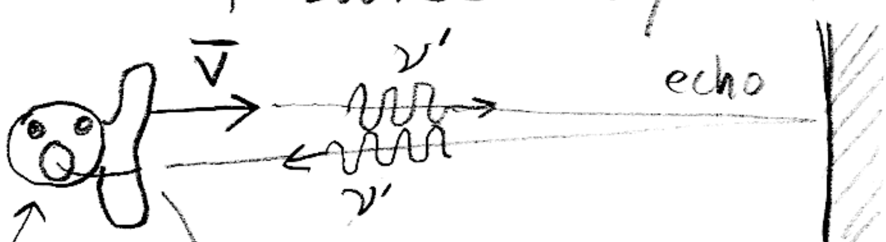
$$\nu' = \frac{V}{\lambda'} \cdot \left(1 + \frac{V_L}{V}\right) = \nu''$$

$$\nu'' = \left(1 + \frac{V_L}{V}\right) \cdot \nu'$$

$$\nu'' = \frac{\left(1 \pm \frac{V_L}{V}\right)}{\left(1 \mp \frac{V_S}{V}\right)} \nu$$

(top sign) + toward source, − away from source

(bottom sign) − source toward listener, + source away from listener



C_4, 261.6 Hz

hears beats with frequency $\Delta\nu$ ... what is speed $\bar{V}$?

hears even higher pitch!

$$\nu' = \frac{\nu}{1 - \frac{\bar{V}}{V}} \quad \text{higher}$$

$$\nu'' = \frac{1 + \frac{\bar{V}}{V}}{1 - \frac{\bar{V}}{V}}\, \nu$$

$$\Delta\nu = \nu'' - \nu = \left(\frac{1 + \frac{\bar{V}}{V}}{1 - \frac{\bar{V}}{V}} - 1 \right) \nu$$

$$= \left(\frac{\cancel{1} + \frac{\bar{V}}{V} - \cancel{1} + \frac{\bar{V}}{V}}{1 - \frac{\bar{V}}{V}} \right) \nu$$

$$\frac{\Delta\nu}{\nu} = \frac{2\bar{V}}{V - \bar{V}} \Rightarrow \frac{\Delta\nu}{\nu} V - \frac{\Delta\nu}{\nu} \bar{V} = 2\bar{V}$$

$$V \cdot \frac{\Delta\nu}{\nu} = \left(2 + \frac{\Delta\nu}{\nu}\right) \bar{V}$$

$$\boxed{\bar{V} = \frac{\Delta\nu/\nu}{2 + \Delta\nu/\nu}\, V}$$

Same phenomena affects light.

higher pitch ⟷ more blue

lower pitch ⟷ more red

redshift measures V.