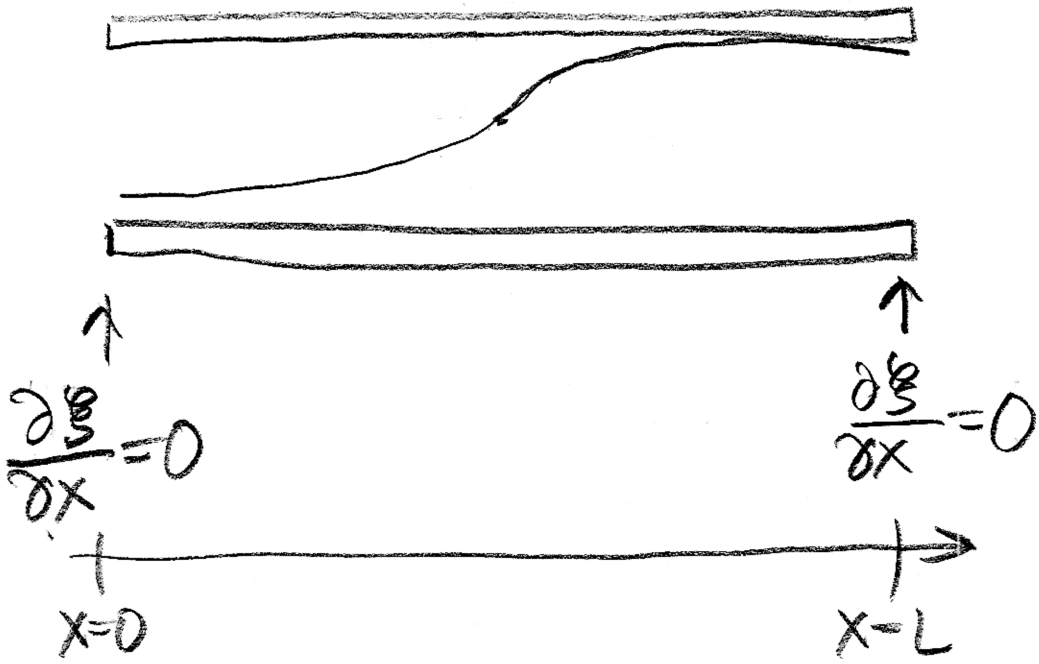
Double Open Pipe



Fundamental: $L = \frac{1}{2}\lambda, \quad \lambda = 2L$

$$\cos\left(\frac{2\pi}{2L}x\right)\cdot \sin(\omega_1 t)$$

$$\omega_1 = v \cdot k = v \cdot \frac{\pi}{L}, \quad \nu_1 = \frac{\omega_1}{2\pi} = \frac{v}{2L}$$

Recorder: $\nu_1 \simeq 520\ \text{Hz} = \frac{v}{2L}$

$$L = \frac{343\ \text{m/s}}{2 \cdot 520\ 1/\text{s}}$$

$$L = 0.330\ \text{m}$$

$$= 33.0\ \text{cm}$$

(actually, 32.5 cm, mouth too...)

<u>Harmonics</u>: Just like closed string...

$$\boxed{\omega_n = n \cdot \frac{v}{2L}}$$

Extension to more dimensions...

(1) Membranes... string, skin on a drumhead

$\Rightarrow$ forces add, in dimension $\perp$ to surface.

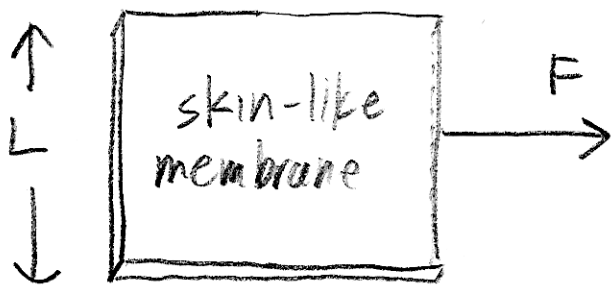
$\Rightarrow$ acceleration same.

$\Rightarrow$ quantities, 2-d skin, $z \to$ displacement, $x, y \to$ position.

$$\sigma = \frac{\text{mass}}{\text{area}}$$

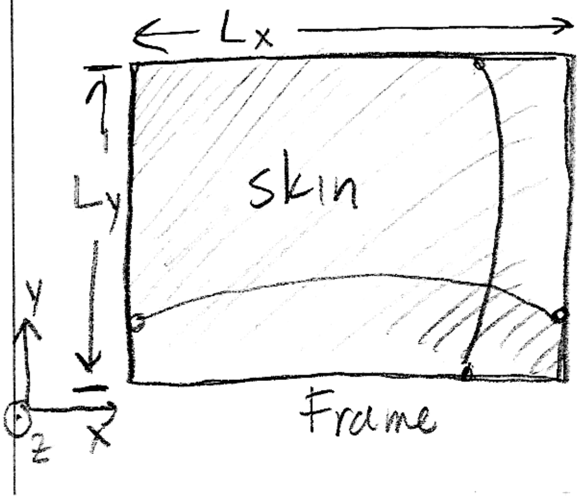
$$\underset{\uparrow \atop \frac{\text{Force}}{\text{Length}}}{S} \underbrace{\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right)}_{\frac{1}{\text{Length}}} = \underbrace{\sigma \cdot \frac{\partial^2 z}{\partial t^2}}_{\text{from acceleration}}$$

$\frac{1}{\text{Area}} \cdot$ mass $\cdot$ acc.



$$\frac{F}{L} = S$$

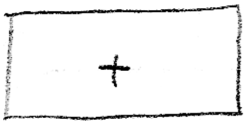
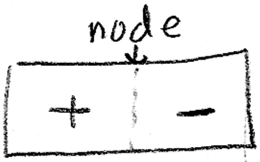
0's at ends



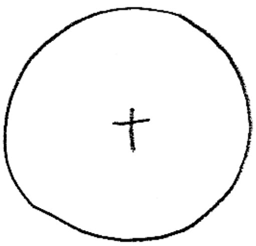
$$z(x,y,t) = A \sin(k_x x) \sin(k_y y) \times \sin(\omega_{xy} t)$$

$k_x \cdot L_x = n_x \pi \qquad k_y \cdot L_y = n_y \pi$

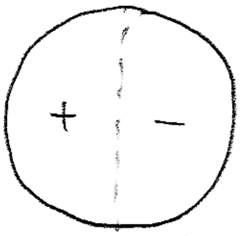
$k_x = n_x \frac{\pi}{L_x} \qquad k_y = n_y \frac{\pi}{L_y}$

n_x	n_y	picture	$\nu\left(\frac{S}{2\sigma}\right)$
1	1		$\sqrt{\frac{1}{L_x^2}+\frac{1}{L_y^2}}$
2	1		$\sqrt{\frac{4}{L_x^2}+\frac{1}{L_y^2}}$
1	2		$\sqrt{\frac{1}{L_x^2}+\frac{4}{L_y^2}}$
2	2		$2\sqrt{\frac{1}{L_x^2}+\frac{1}{L_y^2}}$

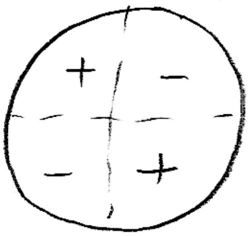
Other Shapes:



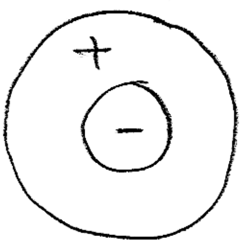
ν_1



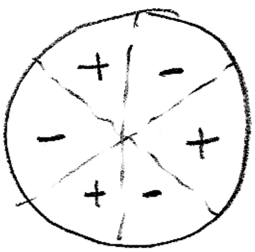
$\nu_2 = 1.56\,\nu_1$



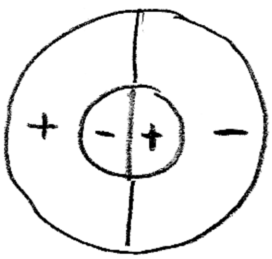
$\nu_3 = 2.13\,\nu_1$



$\nu_4 = 2.30\,\nu_2$



$\nu_5 = 2.65\,\nu_1$



$\nu_6 = \nu$

... put wave equation

Fourier Series

Cannot "pluck" a string in a sinusoidal pattern:

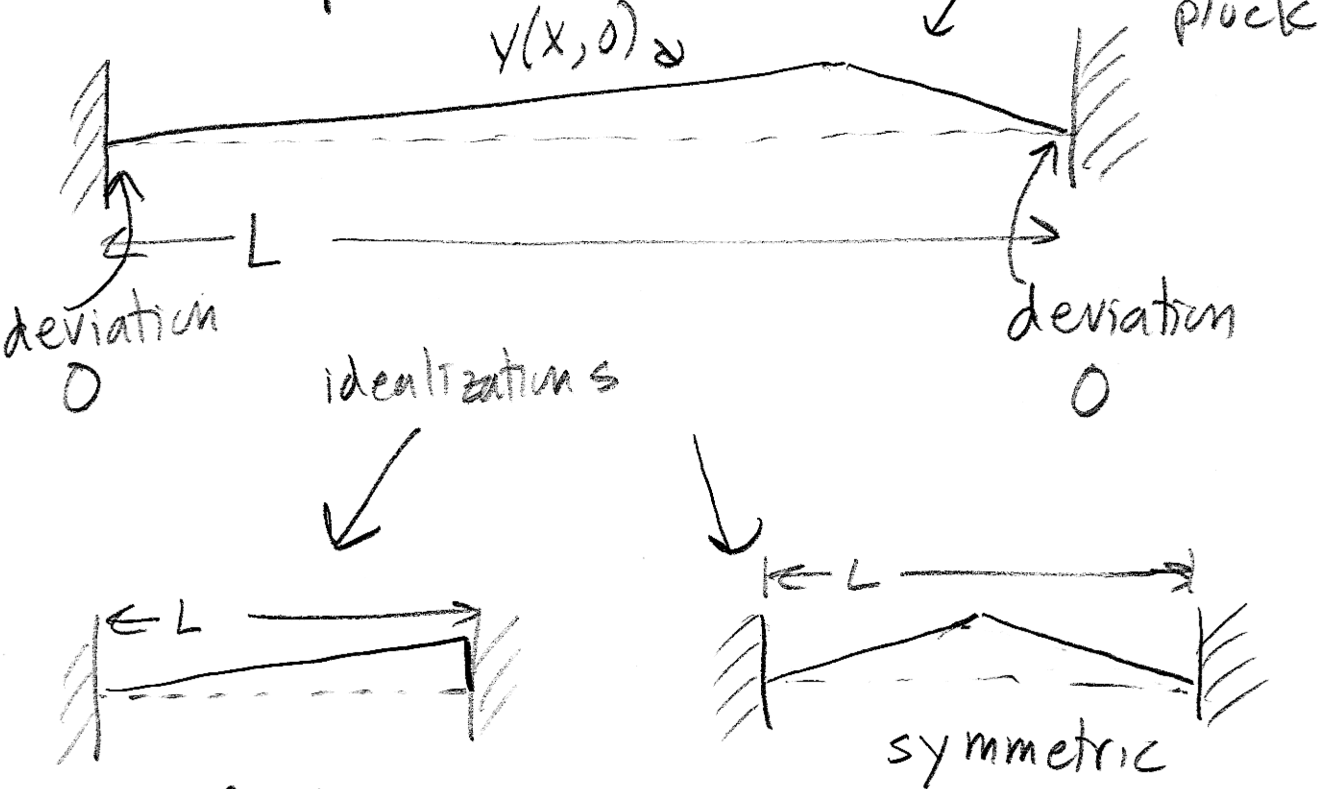


Fig 6-16 in book.

$$y(x,0) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right)$$ Fourier Series

then

$$y(x,t) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right) \underline{\cos(\omega_n t)}$$

normal modes

$$k_n = \frac{n\pi}{L} = \frac{2\pi}{\left(\frac{2L}{n}\right)} = \frac{2\pi}{\lambda_n}$$

oscillate in time with

$$\omega_n = v \cdot k_n \qquad v = \sqrt{\frac{T}{\mu}}$$

why cos? ($t=0$ condition)