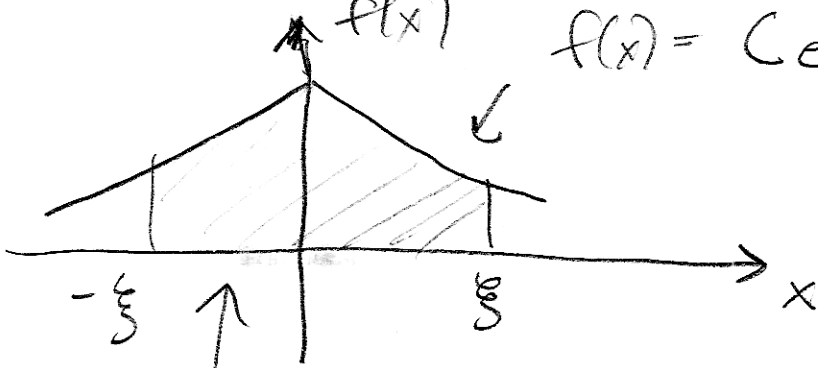
Volume Integrals...

1d:



$f(x) = Ce^{-\frac{2|x|}{a_0}}$

how much inside.

$$\int_{-\xi}^{\xi} Ce^{-\frac{2|x|}{a_0}}\,dx = 2C\int_{0}^{\xi} e^{-\frac{2x}{a_0}}\,dx$$

$$\cancel{2}\cdot C\cdot \frac{a_0}{\cancel{2}}\int_0^{\xi} e^{-\frac{2x}{a_0}}\left(\frac{2\,dx}{a_0}\right)$$

$z = \frac{2x}{a_0} \Rightarrow \frac{2\xi}{a_0}$ is limit

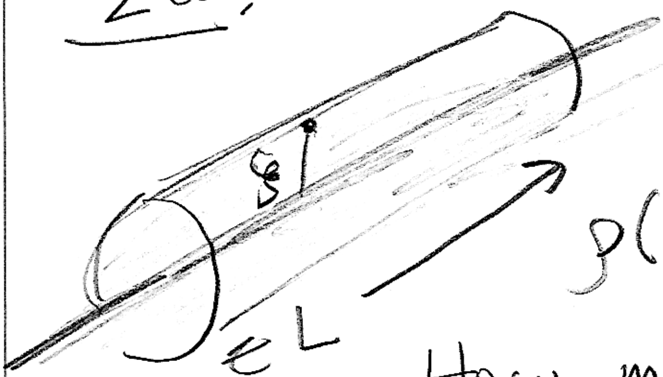
$$= a_0\cdot C\cdot\int_0^{2\xi/a_0} e^{-z}\,dz$$

$$= a_0\cdot C\left(-e^{-z}\Big|_0^{2\xi/a_0}\right)$$

$$= a_0 C\left(-e^{-\frac{2\xi}{a_0}} - -1\right)$$

$$= a_0\cdot C\cdot\left(1 - e^{-\frac{2\xi}{a_0}}\right) \rightarrow a_0 C \text{ as } \xi \rightarrow \infty$$

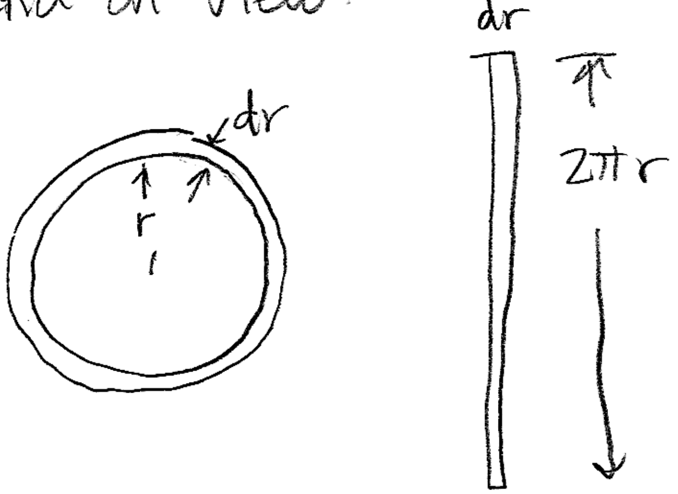
"2d":



$$\rho(r) = D e^{-\frac{2r}{a_0}}$$

How much charge in a cylinder, length L, radius ξ?

end-on view:



$$dA = 2\pi r\, dr$$

$$dV = L \cdot dA = 2\pi r L\, dr \;!$$

$$\int_0^{\xi} D e^{-\frac{2r}{a_0}} dV = 2\pi L D \int_0^{\xi} e^{-\frac{2r}{a_0}} r\, dr$$

$$= 2\pi L D \cdot \left(\frac{a_0}{2}\right)^2 \cdot \int_0^{\xi} e^{-\frac{2r}{a_0}} \left(\frac{2r}{a_0}\right)\left(\frac{2dr}{a_0}\right)$$

$$= \frac{\pi}{2} D L a_0^2 \int_0^{2\xi/a_0} e^{-z} z\, dz$$

$$\int e^{-z} z\,dz = -ze^{-z} - -\int e^{-z}\,dz$$

$v = z \quad du = e^{-z}\,dz$

$dv = dz \quad u = -e^{-z}$

$$= -(1+z)e^{-z}$$

$$-e^{-z} - ze^{-z}$$

$$e^{-z} - e^{-z} + ze^{-z} = ze^{-z}$$

$$\text{Integral} = \frac{\pi}{2} D L a_0^2 \left(-(1+z)e^{-z} \Big|_0^{\frac{2\xi}{a_0}} \right)$$

$$\left(-\left(1+\frac{2\xi}{a_0}\right)e^{-\frac{2\xi}{a_0}} - -1 \right)$$

$$= \left(\frac{\pi}{2} L a_0^2\right) \cdot D \cdot \left(1 - \left(1+\frac{2\xi}{a_0}\right)e^{-\frac{2\xi}{a_0}}\right)$$

gauss:

$$2\pi\xi \cdot \cancel{L} \cdot E_r = \frac{\pi}{2}\cancel{L} a_0^2 \cdot D \left(1 - \left(1+\frac{2\xi}{a_0}\right)e^{-\frac{2\xi}{a_0}}\right)$$

$$E_r = \frac{1}{4}\frac{a_0^2}{\xi} D \left(1 - \left(1+\frac{2\xi}{a_0}\right)e^{-\frac{2\xi}{a_0}}\right)$$