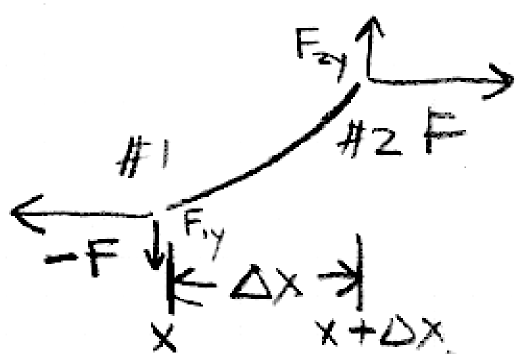
5. Speeds

Heart of the matter: for "linear" media, if I shake the end of a medium with arbitrary frequency f (circular frequency $\omega = 2\pi f$), the speed of the disturbance that propagates is always the same. Not every medium is linear: **ocean** (very non-linear) ... so waves break

glass (a little non-linear) ... so white light can be broken into a rainbow by a prism

Derivation (String)

BLOW UP → Look at forces



Ⓐ no net force in x direction (why one end labelled F, other −F)

Ⓑ $\left.\frac{\partial y}{\partial x}\right|_{x} = -\frac{F_{1y}}{F}$ $\qquad \left.\frac{\partial y}{\partial x}\right|_{x+\Delta x} = \frac{F_{2y}}{F}$

force in string ∥ to string

$$F\frac{\partial y}{\partial x}\Big|_{x+\Delta x} - F\frac{\partial y}{\partial x}\Big|_{x} = F_{2y} + F_{1y} = \text{net force in } y \text{ direction}$$

$$= \underbrace{(\text{mass in } \Delta x)}_{=\mu \Delta x} \frac{\partial^2 y}{\partial t^2}$$

$\mu = \frac{\text{mass}}{\text{length}}$ of string

so $$\frac{F\left(\frac{\partial y}{\partial x}\Big|_{x+\Delta x} - \frac{\partial y}{\partial x}\Big|_{x}\right)}{\Delta x} = \mu \frac{\partial^2 y}{\partial t^2}$$

taking limit as $\Delta x \to 0$, get

$$F \cdot \frac{\partial^2 y}{\partial x^2} = \mu \frac{\partial^2 y}{\partial t^2}$$

so $$\frac{\partial^2 y}{\partial t^2} = \frac{F}{\mu}\frac{\partial^2 y}{\partial x^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

so $$\boxed{v = \sqrt{\frac{F}{\mu}}}$$ (19-19, p. 605)

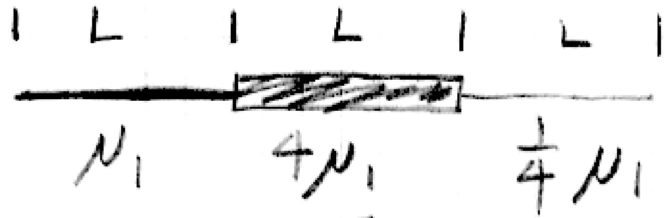
<u>dimensions</u>: $$\frac{[F] = \frac{kg \cdot m}{s^2}}{[\mu] = \frac{kg}{m}} = \frac{m^2}{s^2} = [v^2]$$

when F is a function of position (or μ is), v will depend on position.

19-33 Three pieces of string, each of length L, are joined together end to end to make a combined string of length $3L$.

The first piece of string has mass per unit length μ_1, the second piece has mass per unit length $\mu_2 = 4\mu_1$, and the third piece has mass per unit length $\mu_3 = \mu_1/4$. a) If the combined string is under tension F, how much time does it take a transverse wave to travel the entire length $3L$? Give your answer in terms of L, F, and μ_1. b) Does your answer to part (a) depend on the order in which the three pieces are joined together? Explain.

Diagram:



all with tension F

a)

$$v = \sqrt{\frac{F}{\mu_1}} \qquad \sqrt{\frac{F}{4\mu_1}} \qquad \sqrt{\frac{4F}{\mu_1}}$$

$$t = \frac{L}{v} = \sqrt{\frac{\mu_1}{F}} \qquad 2\sqrt{\frac{\mu_1}{F}} \qquad \frac{1}{2}\sqrt{\frac{\mu_1}{F}}$$

$$\text{total time} = \left(1 + 2 + \tfrac{1}{2}\right)\sqrt{\frac{\mu_1}{F}} = \boxed{\frac{7}{2}\sqrt{\frac{\mu_1}{F}}}$$

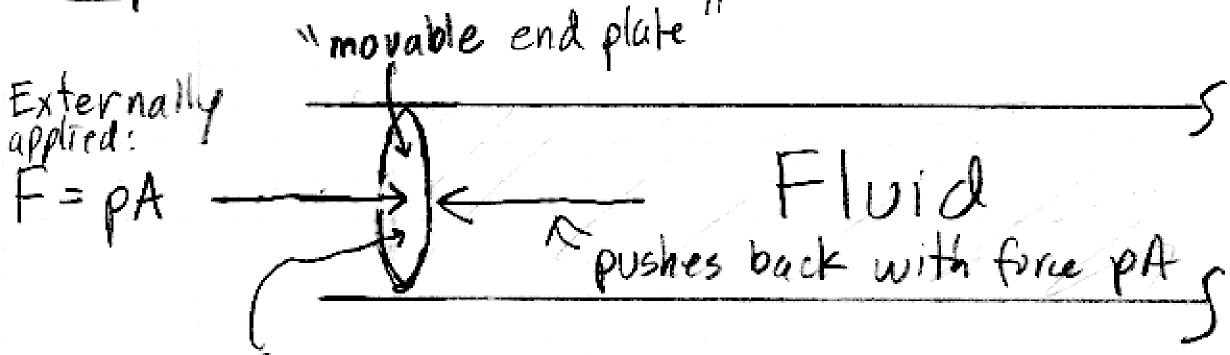
b) No; this would just change the order in which one adds the times together. Time to traverse doesn't depend on order.

Longitudinal Wave in Fluid

See fig. 19-10

no shear strength (a board made of water can't lift anything)

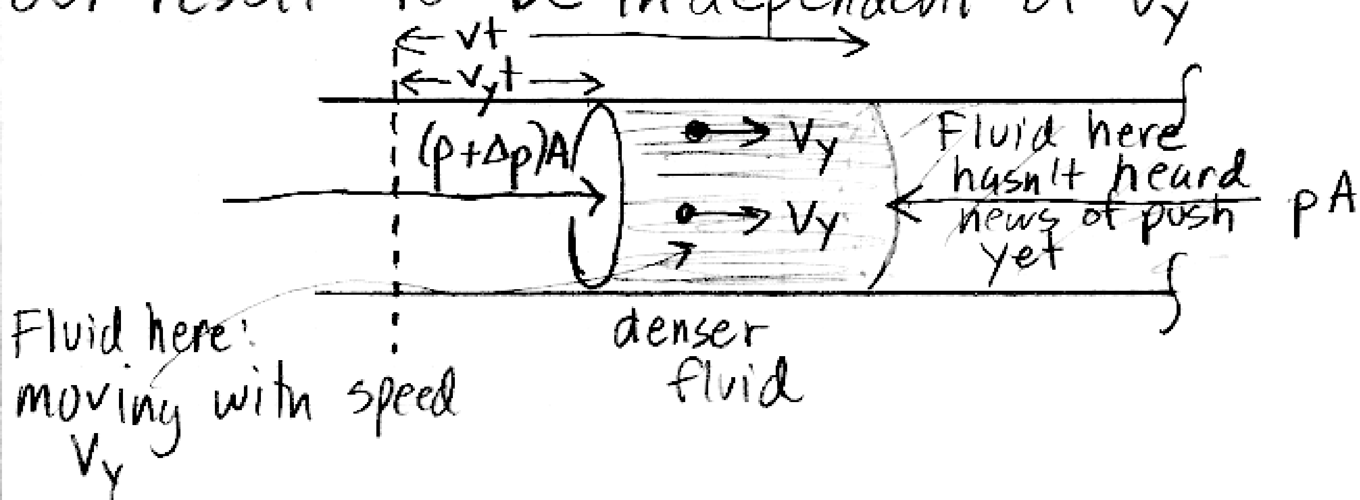
Equilibrium:



Area of face is A
Has pressure p on it

Now imagine that there is an increase in pressure of amount Δp on the external side, that persists for a time t.

The response of the fluid will be for a "wave front" to eminate from the end with speed v (the "speed of sound"). However, friction will limit the speed of the end plate to $v_y < v$; we expect our result to be independent of v_y



Goal: solve for v in terms of intrinsic poperties of fluid. First order of business: ELIMINATE Δp...

Bulk Modulus: $B = \dfrac{-\Delta p}{(\Delta V/V)}$ (p. 342, 11-13)

intrinsic property

$$\frac{\Delta V}{V} = \frac{-v_y t A}{v t A} = \frac{-v_y}{v}$$

so, $\Delta p = B \cdot \dfrac{v_y}{v}$ ($v_y = 0$ means no push, $\Delta p = 0$)

The impulse imparted by this increment of pressure is:

$$\Delta F \cdot t = (\Delta p \cdot A) \cdot t = B \frac{v_y}{v} A t$$

This impulse imparts a momentum in the "original" fluid, which had and has mass

$= \rho \cdot V_{original}$ $\qquad V_{original} = A \cdot (v \cdot t)$

not v_y, want original

and to get the momentum, multiply by v_y ... the speed of the portion of the fluid now moving:

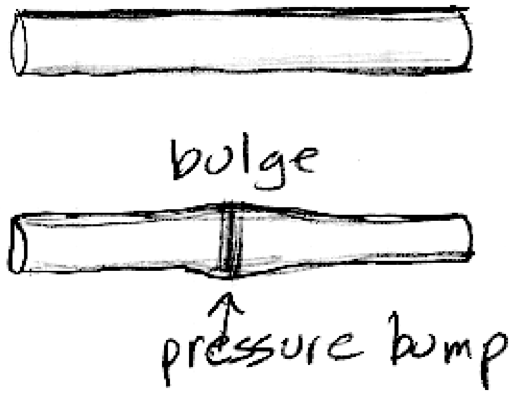
momentum imparted $= \rho V_{orig} v_y = \rho A v t v_y$

so, $B \dfrac{v_y}{v} \cancel{A}\cancel{t} = \rho \cancel{A} v \cancel{t} \cancel{v_y}$ $\qquad \boxed{v^2 = \dfrac{B}{\rho}}$

19-21 p. 607

In solid

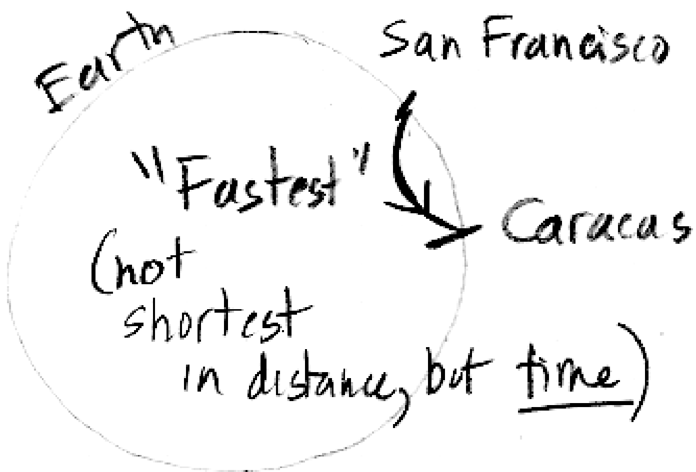
Case #1 : a bar



when wave pulse propagates, nothing opposes "bulge"...

Result : $\boxed{v^2 = \frac{Y}{\rho}}$ ← Young's Modulus

Case #2 : Earthquake (sound in an infinite media...)



$$\boxed{v^2 = \frac{B + \frac{4}{3}S}{\rho}}$$

B = bulk modulus

S = shear modulus.

(S < B, p. 33a).

Comment: $v = \frac{\omega}{k} =$ constant

or $k = \frac{\omega}{v\,[\text{constant}]}$ called a dispersion relation

This one is linear. Sometimes, non-linear dispersion relations arise....

<u>Linear</u> dispersion relation, $k(\omega)$, means wave pulses retain their shapes for all time as they travel

<u>Non-Linear</u> : wave pulses <u>disperse</u>, or tend to smear out, over time

6) Sound in Gases

Trick is to get proper bulk modulus; for most normal sound in gases, there is not sufficient <u>time</u> for energy to flow in or out of compressing/expanding volumes. So the expansion/compression is <u>adiabatic</u> (at least for $20-20\cdot10^3$ Hz frequencies)

<u>then</u> : $pV^\gamma = \text{constant}$ $\quad \gamma$ = ratio of heat capacities (p.548)

$= \frac{5}{3}$ (monatomic)

$= \frac{7}{5}$ (diatomic).

$$\text{so } \frac{dp}{dV}V^\gamma + p\gamma V^{\gamma-1} = 0$$

$$\left[\left(V\frac{dp}{dV}\right) + p\gamma\right]V^{\gamma-1} = 0$$

$$-V\frac{dp}{dV} = p\gamma = B_{adiabatic}$$

M = molecular weight, R = Heat constant/mole

$$\text{so } pV = nRT, \quad \rho = \frac{M\cdot n}{V} = \frac{pM}{RT}$$

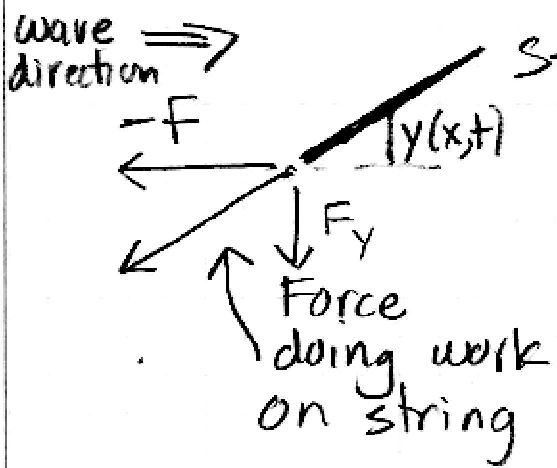
$$\boxed{v = \sqrt{\frac{B_{adiabatic}}{\rho}} = \sqrt{\frac{\gamma RT}{M}}}$$

Newton Missed the γ!

7) Energy in Waves

All waves: energy flow $\propto A^2$

mechanical: also $\propto \omega^2$



String portion getting worked on

$$\frac{\partial y}{\partial x} = -\frac{F_y}{F}$$

$$F_y = -F\frac{\partial y}{\partial x}$$

$\Rightarrow$ F does no work

$\Rightarrow$ F_y does, at a rate of:

$$\text{Power} = F_y \cdot v_y = -F\frac{\partial y}{\partial x}\frac{\partial y}{\partial t}$$

$$y(x,t) = A\sin(\omega t - kx)$$

$$\frac{\partial y}{\partial x} = -kA\cos(\omega t - kx) \qquad \frac{\partial y}{\partial t} = \omega A\cos(\omega t - kx)$$

$$= -\frac{\omega}{v}A\cos(\omega t - kx)$$

$$\text{Power} = (F/v)\cdot\omega^2 A^2 \underline{\cos^2(\omega t - kx)}$$

$= F\sqrt{\frac{\mu}{F}}$ average value is ½

$$\boxed{\langle \text{Power} \rangle = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2}$$

Fluid $\Rightarrow$ $\dfrac{\langle \text{Power} \rangle}{\text{Area}} = \frac{1}{2}\sqrt{\rho B}\,\omega^2 A^2$ | Solid Bar $\Rightarrow$ $\dfrac{\langle \text{Power} \rangle}{\text{Area}} = \frac{1}{2}\sqrt{\rho Y}\,\omega^2$