Electric Potential Chapter 24

1. Electric Potential Energy
2. Electric Potential
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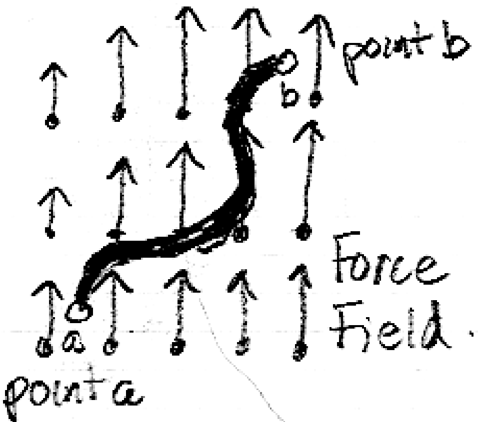
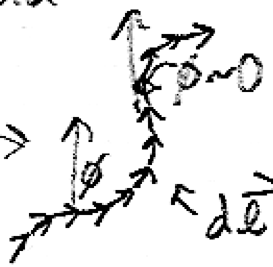
Work in a Force Field



in moving a particle that feels force field $\vec{E}$, THAT FIELD does work

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b F\cos\phi \, dl$$



ϕ varies along path since direction of $d\vec{l}$ varies

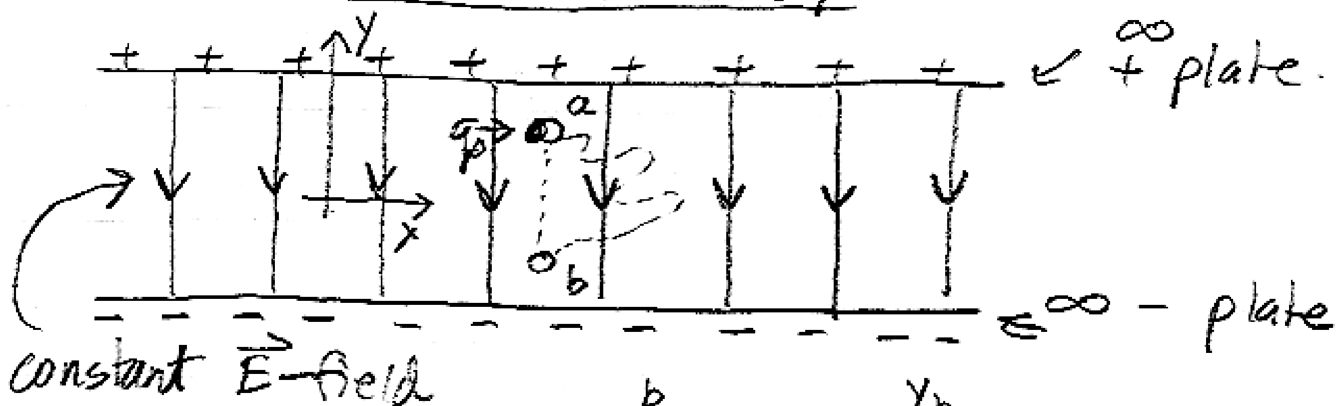
as shown, this work is positive .... a particle traveling this path will gain energy... meaning it will lose potential energy

$$W_{a\to b} = -(U_b - U_a) = -\Delta U$$

$$= K_b - K_a \quad \text{(change in kinetic energy)}$$

so $K_a + U_a = K_b + U_b$

Electric Potential Energy



$$\vec{F} = -q_0 E\,\hat{y}\,;\ W_{b\to a} = \int_a^b \vec{F}\cdot d\vec{l} = \int_{y_a}^{y_b} (-q_0 E)\,dy$$

$$= -q_0 E\,(y_b - y_a)$$

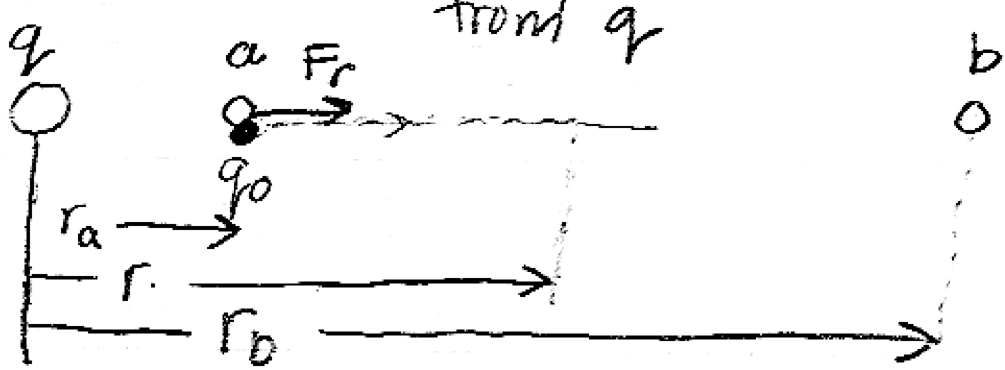
$$W_{b\to a} = q_0 E\,(y_a - y_b) = -(U_b - U_a) = -\Delta U$$

$$U_a = q_0 E y_a \qquad U_b = q_0 E y_b$$

$U = q_0 E y$ ... U higher near the + charge.

Remember... answer is path-independent

Point Charges: move q_0 from r_a to r_b away from q



$$F_r(r_a) = \frac{q\,q_0}{4\pi\varepsilon_0 r_a^2}$$ at $r = r_a$; radial component

+ : repulsive

− : attractive

then $W_{a\to b} = \int_a^b dr\, F_r(r) = \frac{q q_0}{4\pi\varepsilon_0}\int_{r_a}^{r_b} \frac{dr}{r^2}$

$$W_{a\to b} = \frac{q q_0}{4\pi\varepsilon_0}\left(-\frac{1}{r}\Big|_{r_a}^{r_b}\right) = \frac{q q_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

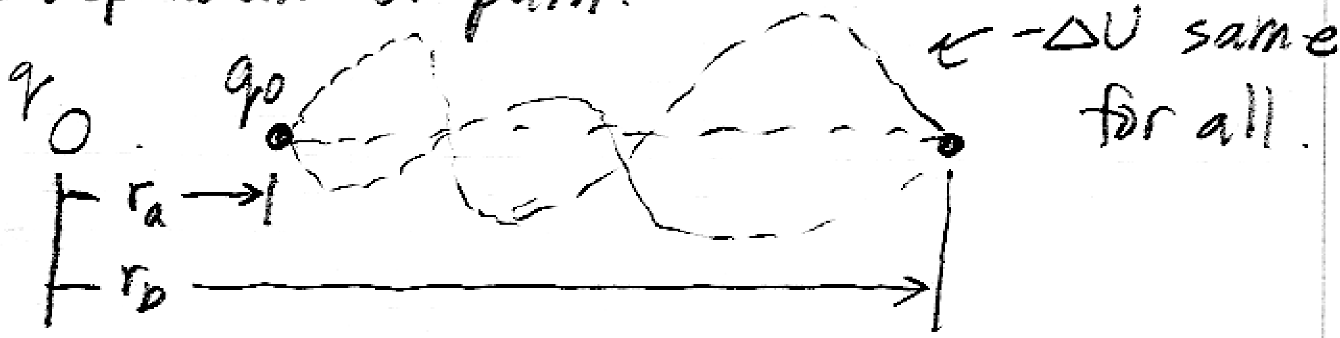
$$= -(U_b - U_a) = -\Delta U$$

$U_a = \frac{q q_0}{4\pi\varepsilon_0}\frac{1}{r_a}$ $\qquad$ $U_b = \frac{q q_0}{4\pi\varepsilon_0}\frac{1}{r_b}$

$$U(r) = \frac{q q_0}{4\pi\varepsilon_0}\frac{1}{r}$$

$\propto 1/r$ (not $1/r^2$).

The change in potential energy is independent of path:



<u>Systems</u>: potential energies of all pairs add up. Example 24-2:

Point Charges: #1 $q_1 = -e$ $\quad x = 0$

#2 $q_2 = +e$ $\quad x = a$

(a) how much work to bring charge #3 to $q_3 = +e$, $x = 2a$?

(i) $U(\infty) = 0$ (convention)

either #1 or #2

(2) $U(2a) = \underbrace{U_{13}(2a)}_{\text{energy of \#1+\#3}} + \underbrace{U_{23}(2a)}_{\text{energy of \#2,\#3}}$

$$= \frac{(-e)(+e)}{4\pi\varepsilon_0|2a-0|} + \frac{(+e)(+e)}{4\pi\varepsilon_0|2a-a|}$$

$$= \frac{e^2}{4\pi\varepsilon_0}\frac{1}{a}\left(-\frac{1}{2}+1\right) = \boxed{\frac{e^2}{8\pi\varepsilon_0}\frac{1}{a}}$$

(b) Total Potential Energy.

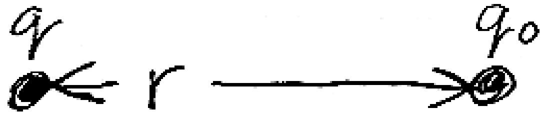
How many pairs? $\underbrace{1\text{-}3, 2\text{-}3}_{\text{done in (a)}}, \underbrace{1\text{-}2}_{\text{remaining}}$

$$U_{12} = \frac{(-e)(e)}{4\pi\varepsilon_0|a-0)} = -\frac{e^2}{4\pi\varepsilon_0 a}$$

$$U_{12} + U_{13} + U_{23} = \frac{e^2}{4\pi\varepsilon_0 a}\left(-1+\frac{1}{2}\right) = \boxed{\frac{-e^2}{8\pi\varepsilon_0 a}}$$

Electric Potential... is Electric Potential Energy per unit charge

$$\text{Electric Potential} = V = \frac{U}{q_0} \begin{array}{l}\leftarrow \text{electric potential} \\ \leftarrow \text{test charge.}\end{array}$$



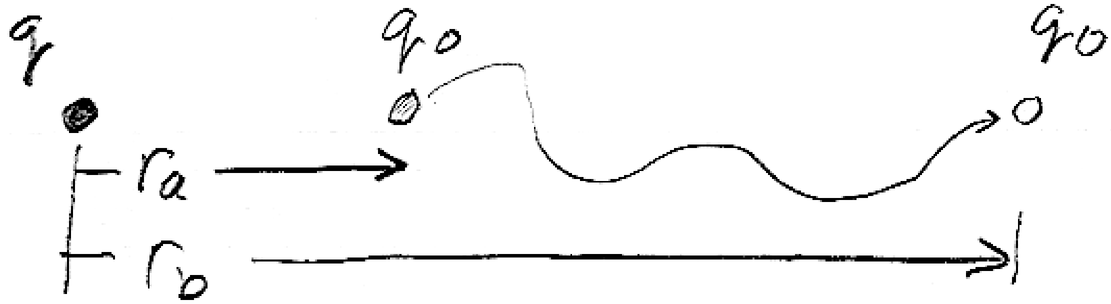
$$U = \frac{q q_0}{4\pi\varepsilon_0 r} \quad \left(\begin{array}{c}\text{taking} \\ U(\infty)=0\end{array}\right)$$

$$V = \frac{U}{q_0} = \frac{q}{4\pi\varepsilon_0 r} \quad \underline{\text{due}}\ \underline{\text{to}}\ q$$

units: $\frac{\text{Joules}}{\text{Coulomb}}$ also known as a "Volt"

So, Potential Difference is the difference in Electric Potential Energy, per unit charge... Potential Difference is also known as... **VOLTAGE**!

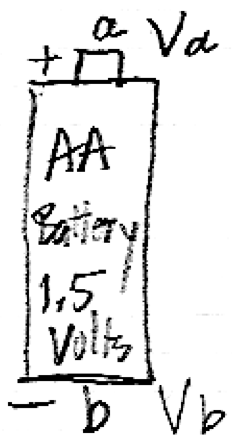
Example:



move "test charge" from r_a to r_b away from charge q ...

$$\underbrace{\frac{W_{a\to b}}{q_0}}_{\text{property of } q} = \frac{-\Delta U}{q_0} = -\left(\frac{U_b}{q_0} - \frac{U_a}{q_0}\right) = \frac{U_a}{q_0} - \frac{U_b}{q_0}$$

$$\frac{W_{a\to b}}{q_0} = \underbrace{V_a - V_b}_{\text{"Voltage"}}$$



$$V_a - V_b = 1.5\,V$$

meaning take a charge of 1 Coulomb... in moving from a to be, will pick up +1.5 Joules

−1 coulombs will lose 1.5 Joules

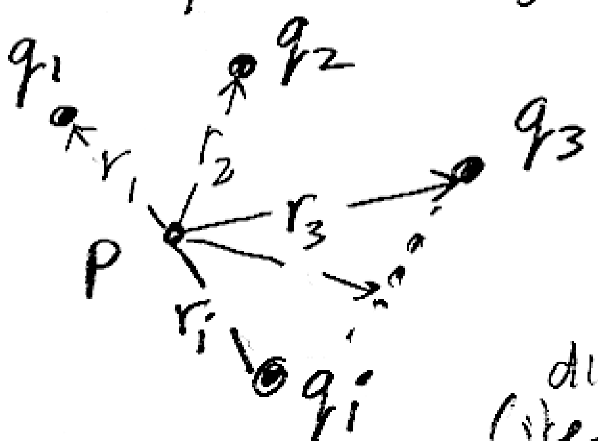
(path independent) but $\frac{-1.5}{-1} = \frac{1.5}{1} = 1.5\,V$

$\boxed{\text{VOLTAGE} \Leftrightarrow \text{always difference between two points.}}$

Potential: can exist at one point (with convention for extra constant).

point charge q: $V = \dfrac{q}{4\pi\varepsilon_0 r}$ ← r = distance from q to observation point

Many Point Charges:



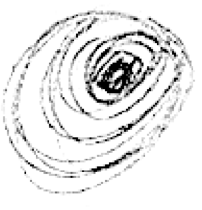
$$V(P) = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i}$$

for a continuous distribution of charge (like charge per unit length, or per unit area, or per unit volume)

$dq = \lambda dx$

$dq = \sigma dA$

$dq = \rho dV$



$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}$$

Electric Potential and $\vec{E}$ field

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = q_0 \int_a^b \vec{E}\cdot d\vec{l} = U_a - U_b$$

so
$$\frac{U_a}{q_0} - \frac{U_b}{q_0} = \int_a^b \vec{E}\cdot dl$$

$$\boxed{V_a - V_b = -\int_b^a \vec{E}\cdot d\vec{l}}$$

Dimensions

volts   meters

$\therefore$ $[\vec{E}]$ is Volts/meter

also
$$\frac{1}{q_0}\int_a^b \vec{F}\cdot d\vec{l} = \int_a^b \vec{E}\cdot d\vec{l}$$

so
$$[\vec{E}] = \left[\frac{\vec{F}}{q_0}\right] = \frac{\text{Newtons}}{\text{Coulomb}}$$

Electron Volt

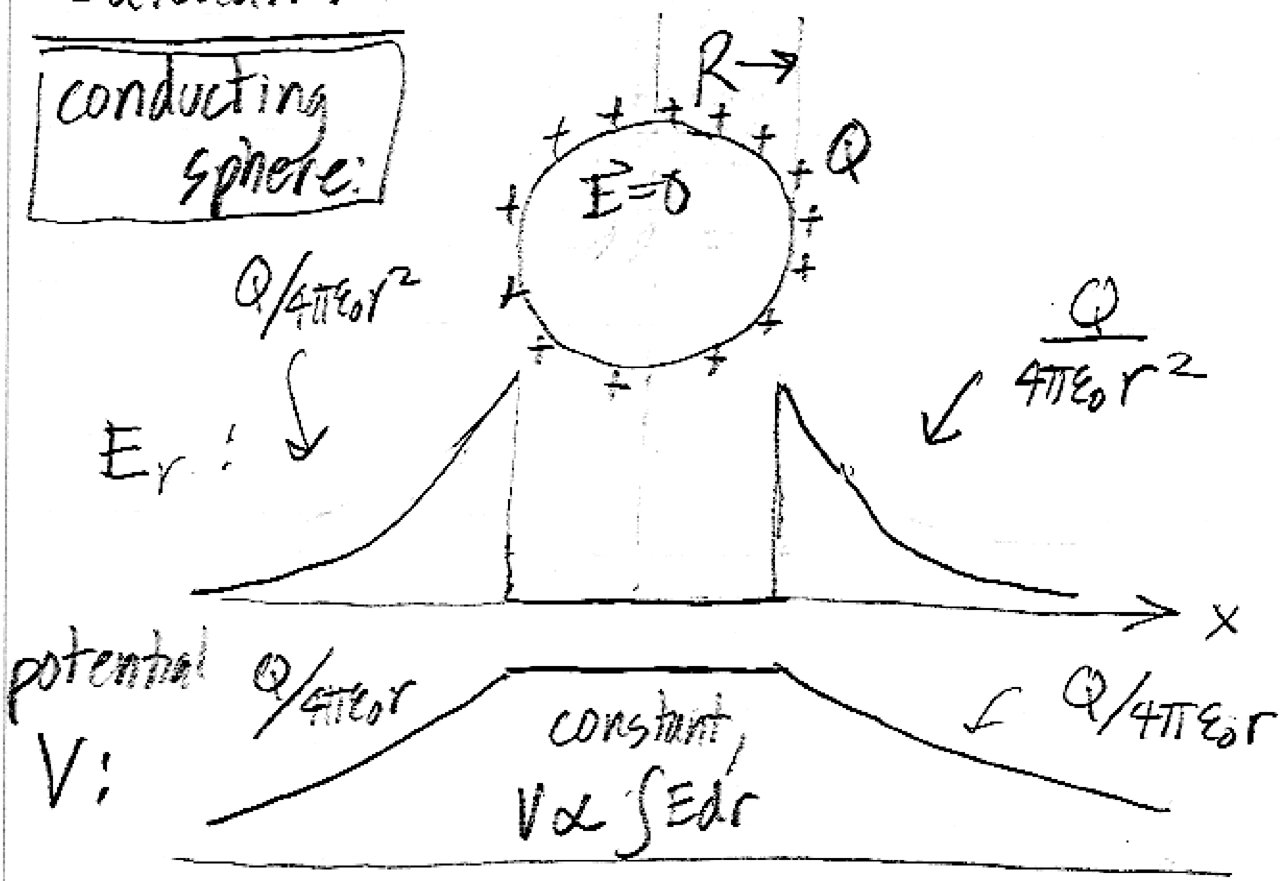
suppose one electron (charge $q = -e = -1.6\cdot 10^{-19}$ C) "falls" through -1 Volt. How much energy does it pick up?

$$\Delta U = q\cdot\Delta V = (-1.6\cdot 10^{-19})(-1\,V)$$
$$= 1.6\cdot 10^{-19} \text{ Joules} \equiv \text{"}\underline{1}\text{ eV"}$$

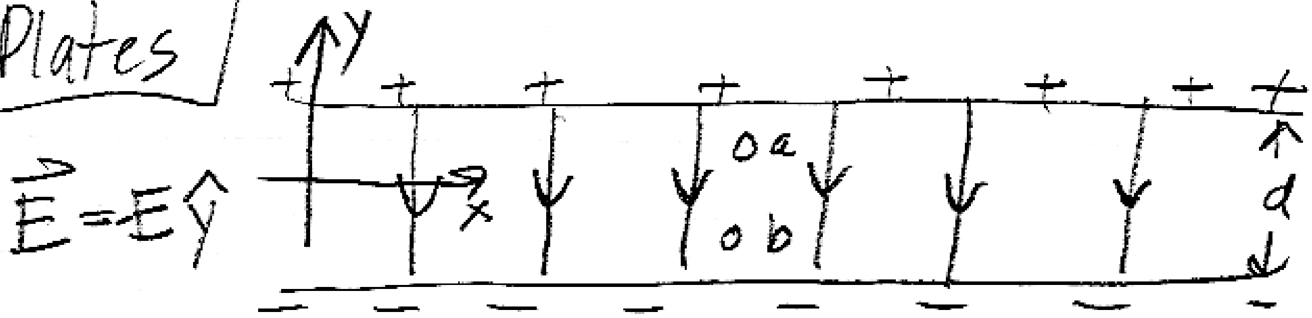
aka "electron volt"

Calculations

conducting sphere:



Plates



$$V_a - V_b = -\int_b^a (-E)\,dy = E(y_a - y_b)$$

make point a on upper plate
b on lower

$$\boxed{\Delta V = Ed}$$

← note, voltage independent of x