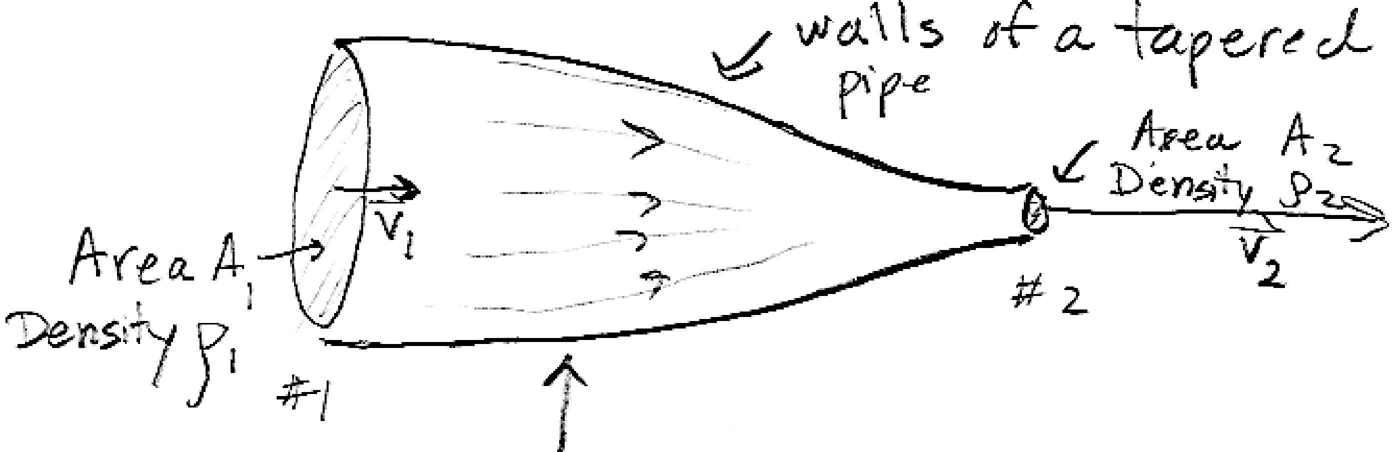
Chapter 23 (Gauss's Law)

1. The Main Idea
2. Electric Flux
3. Gauss's Law
4. Applications

The Main Idea – develop an analogy to the continuity equation (which is for fluids)... analogy is for electric field...

Continuity Equation was:



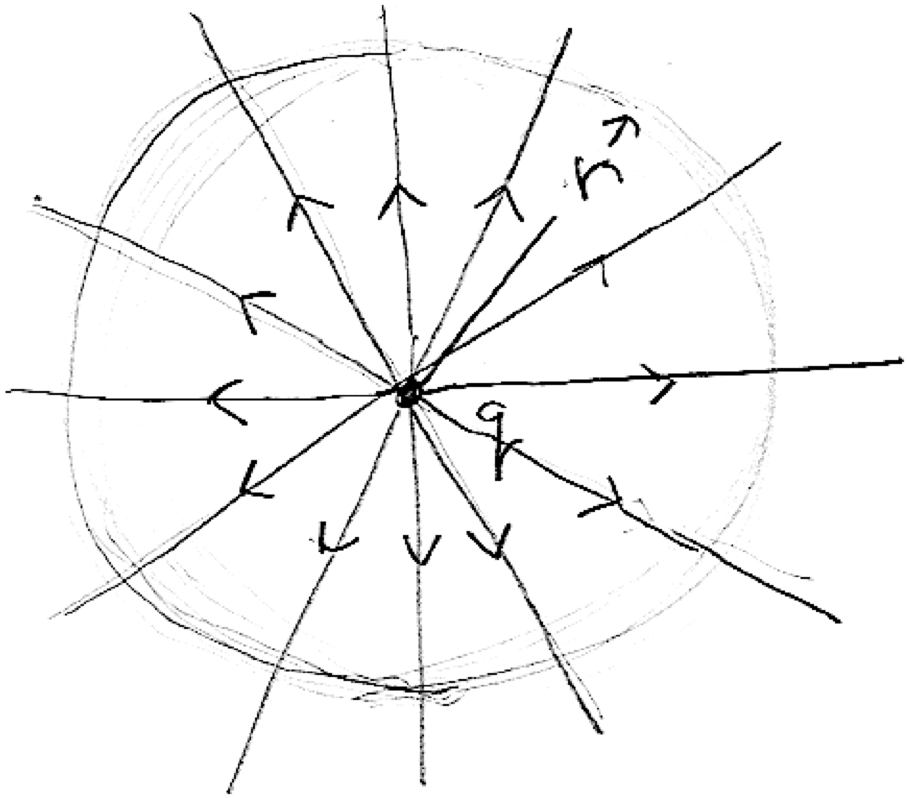
in time dt, same mass flowing in through area A_1 must flow out through surface A_2:

$$(\rho_1)(A_1 v_1 dt) = (\rho_2)(A_2 v_1 dt)$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

A similar thing happens for the correctly chosen quantity involving the electric field.

To develop that quantity, imagine the electric field surrounding a + point charge: Imagine a spherical surface centered on the point charge too:



The radius of the surface is r

On the surface of the sphere, the magnitude of electric field is:

$$E_{surface} = k\frac{q}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Now note that the surface area of the sphere is $4\pi r^2$, so,

$$\underbrace{(\text{Surface Area}) \times E_{surface}}_{\text{"electric flux"}} = 4\pi r^2 \times \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

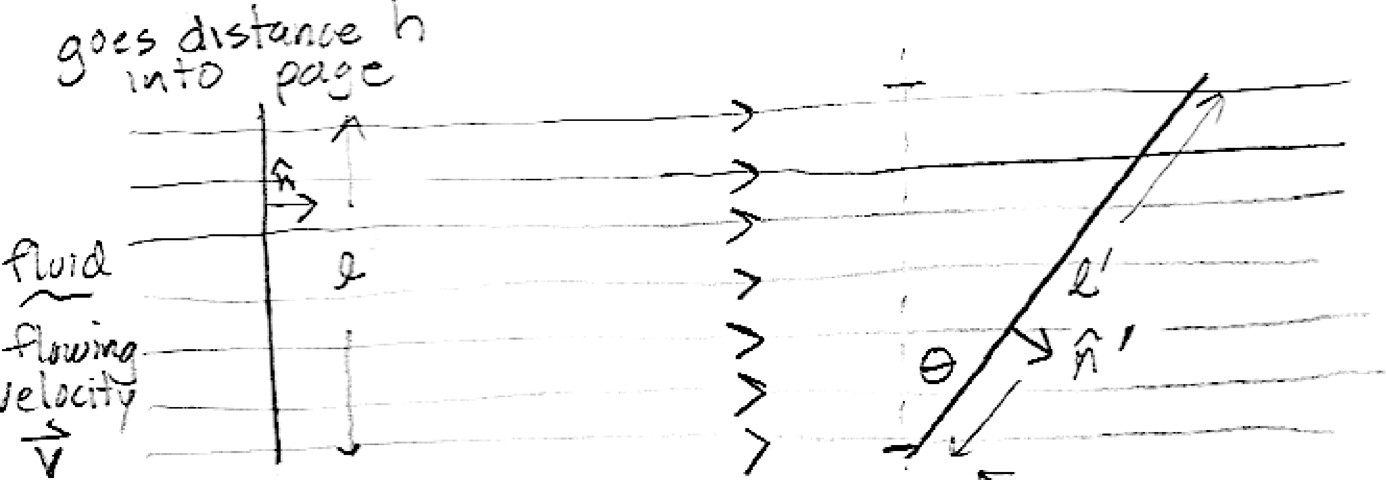
$$= \frac{q}{\varepsilon_0}$$ (Independent of Radius of surface!)

To further the analogy, identify:

$$\vec{E} \Longleftrightarrow \rho\vec{v}$$

and note that mass flow (described by $\rho\vec{v}$) is the same even when the surface it flows through is tilted w/r to $\vec{v}$:

Look down from above. $\rho, \vec{v}$ constant:



area $A = \ell h$ seen from above

"tilted" area $A' = \ell' h$ seen from above.

$A' > A$

$$\rho v A \neq \rho v A'$$

but: $\rho v A = \rho v A \cos\theta$ since $\ell = \ell' \cos\theta$

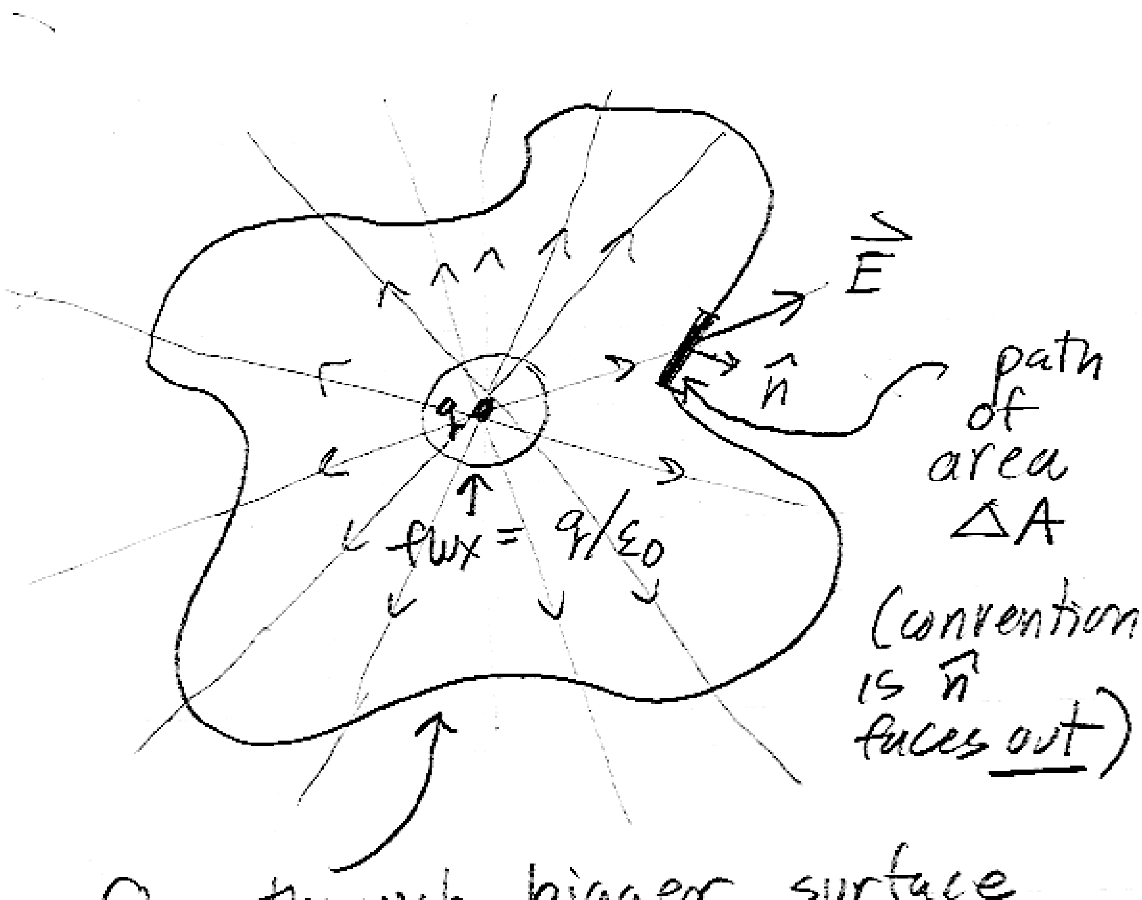
$$= \rho (v\cos\theta) A$$

$\uparrow$ $\vec{v}$'s component $\perp$ to surface.

$\hat{n} \equiv$ unit vector normal to surface

then $\rho A \vec{v}\cdot\hat{n} = \rho A' \vec{v}\cdot\hat{n}'$ "mass flux"

Now (try) to imagine situation for electric field.. idea is that the electric flux through a small sphere that surrounds a point charge will also "flow" through an irregular surface that also fully surrounds the charge:



flux through bigger surface (closed, enclosing charge) ALSO $\frac{q}{\varepsilon_0}$

How to define the flux for the larger, irregular surface?

$$\Phi_E \equiv \sum \vec{E} \cdot \hat{n} \Delta A \Rightarrow \oint \vec{E} \cdot \hat{n}\, dA$$

$$= \oint \vec{E} \cdot d\vec{A} \qquad d\vec{A} \equiv \hat{n}$$

$$= 4\pi r^2 \cdot \frac{q}{4\pi\varepsilon_0 r^2} = \frac{q}{\varepsilon_0} \Leftarrow$$

Back to nice spherical surface

note that if $E(r) \not\propto \frac{1}{r^2}$ for point charge, flux would vary as a function of the size of enclosing surface.

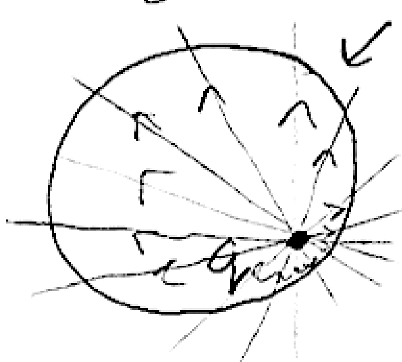
Since $E(r)$ is $\propto 1/r^2$,

$$\boxed{\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}}$$

$\oint$ means over a closed surface

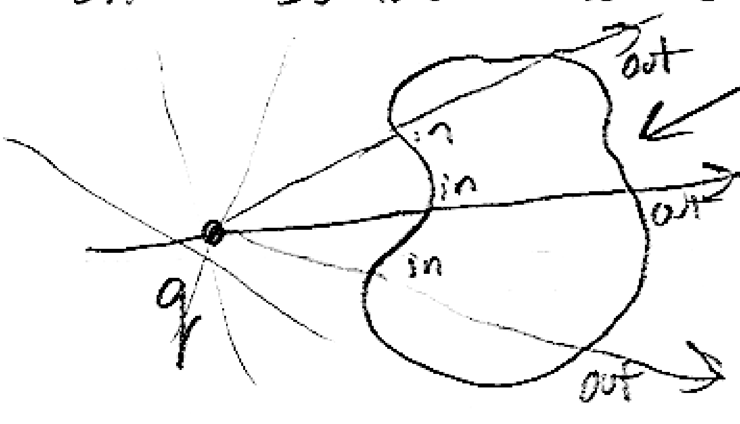
Gauss's Law

1) Charge need not be in center!



surface, $\Phi_E = \frac{q}{\varepsilon_0}$

2) When all charge is outside, the net flux is 0... equal amounts enter surface as exit.



$\oint \vec{E} \cdot d\vec{A} = 0$

3) Static charge on an ideal piece of a conductor resides on surface.

$\Rightarrow$ key insight: no electric field <u>inside</u> an ideal conductor (or else current would flow, pushed by field) at static equilibrium.

boundary of block of ideal conductor

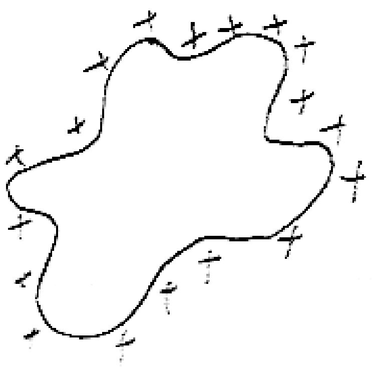
$\rightarrow$ arbitrary surface fully <u>inside</u> conductor

$\rightarrow$ $\Phi_E = 0$ <u>because</u> $\vec{E}=0$ everywhere inside.

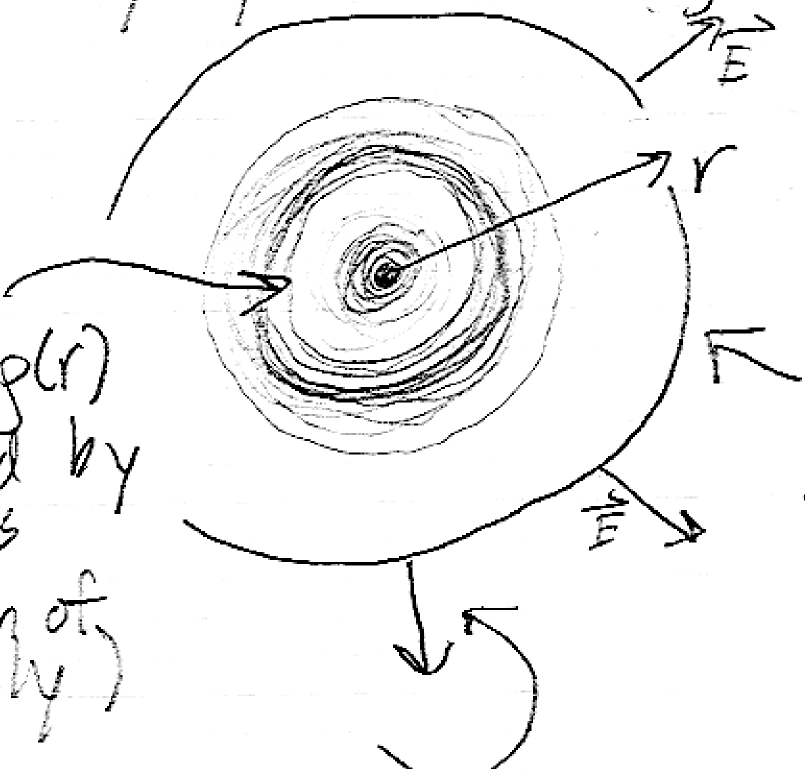
$\rightarrow$ $\therefore$ $Q_{enclosed} = 0$ for abitrary surface.

$\rightarrow$ $\therefore$ no charge ever inside coductor.

$\rightarrow$ but you can charge up a conductor! so all charge lives on the surface!



4) Spherically symmetric charge distributions:



$$\overbrace{4\pi r^2 E_r}^{\text{Flux}} = \frac{(\text{Charge Enclosed})}{\varepsilon_0}$$

radial component of r

$$E_r = \frac{(\text{Charge Enclosed})}{4\pi\varepsilon_0}$$

This means that for small r, the charge OUTSIDE r has no effect on E_r!

This charge doesn't influence E_r

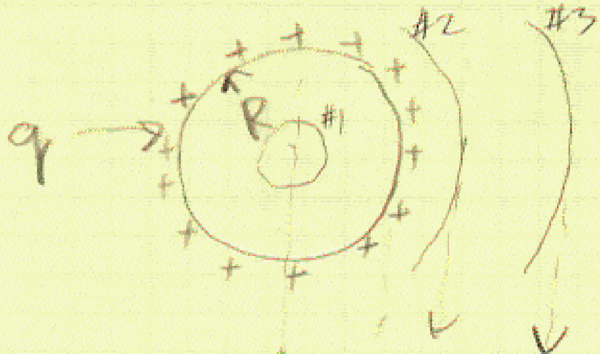
~~5) Hole inside a spatial charge distribution~~

Whoops! WRONG

~~Hole: $\vec{E} = 0$ inside, because every gaussian surface inside encloses 0 charge, so $\Phi_E = 0$ for arbitrary surface, $\vec{E} = 0$~~

~~darkness indicates $\rho(x)$~~

6) Field of a charged conducting sphere:

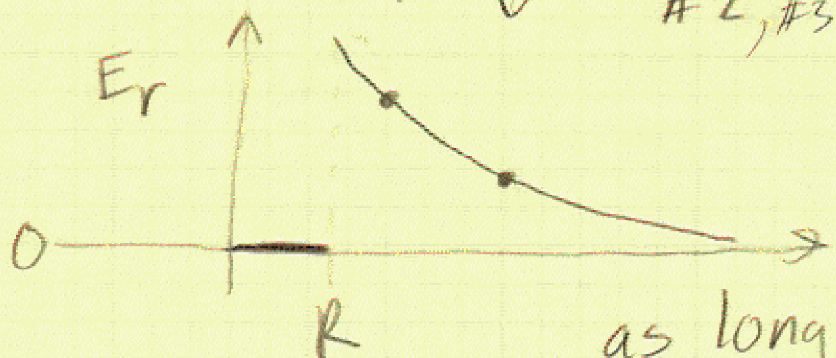


Gaussian Surfaces (portions shown)

#1: no enclosed charge $E_r = 0$ ... up to $r = R$

#2, #3: $4\pi r^2 E_r = \frac{q}{\varepsilon_0}$

$$E_r = \frac{q}{4\pi\varepsilon_0 r^2}$$



as long as $r > R$, field just like a point charge!

(Newton knew this too, but he didn't know Gauss's law.)