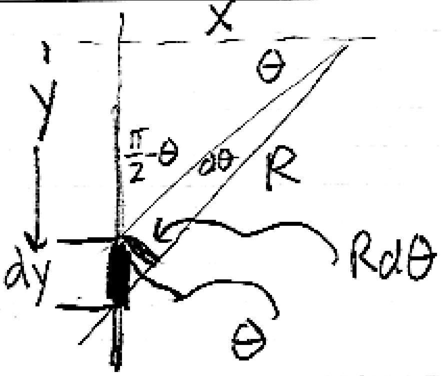


$$\cos\theta = \frac{x}{R} \qquad \cos\theta = \frac{R\,d\theta}{dy}$$

$$dy = \frac{R\,d\theta}{\cos\theta}$$

$$dE_x = k\frac{\lambda}{R^2}\cdot\frac{R\,d\theta}{\cos\theta}\cdot\cos\theta$$

$$dE_x = k\frac{\lambda}{R}\,d\theta$$

$$dE_x = k\frac{\lambda}{x}\cos\theta\,d\theta$$

$$E_x = \int dE_x = k\frac{\lambda}{x}\left(\int_{-\theta_a}^{\theta_a}\cos\theta\,d\theta\right) = 2k\frac{\lambda}{x}\sin\theta_a$$

where $\sin\theta_a = \dfrac{a}{\sqrt{a^2+x^2}}$

$$E_x = \cancel{2}\cdot k\cdot\frac{\frac{Q}{\cancel{2}a}}{x}\cdot\frac{a}{\sqrt{a^2+x^2}}$$

$$\boxed{E_x = k\frac{Q}{x\sqrt{a^2+x^2}} \qquad k = \frac{1}{4\pi\varepsilon_0}}$$

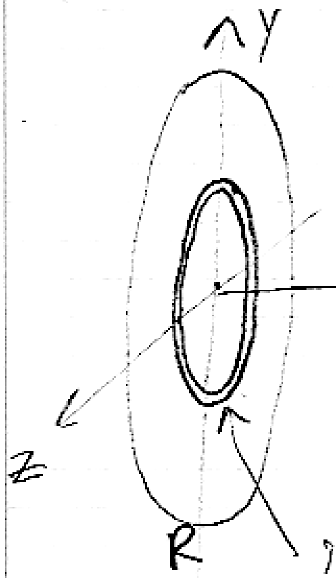
another way: set $a \to \infty$ with λ = constant

$$E_x = 2k\cdot\frac{\lambda}{x}\cdot\frac{a}{\sqrt{a^2+x^2}} \rightarrow 2k\frac{\lambda}{x}$$

$$\boxed{E_x = \frac{\lambda}{2\pi\varepsilon_0}\times\frac{1}{x}}$$ ← important.

$\propto$ 1/distance

Disk Charge : $\sigma = \frac{Q}{\pi R^2}$



$E_y = E_z = 0$
symmetry

$$dE_x = k \cdot \frac{(2\pi\sigma r\,dr)\,x}{(x^2+r^2)^{3/2}}$$

in annulus dr
area $2\pi r\,dr$
charge $dQ = 2\pi\sigma r\,dr$

$$E_x = k \cdot \pi \cdot \sigma x \int_0^R \frac{2r\,dr}{(x^2+r^2)^{3/2}}$$

$$= -2k\pi\sigma x \left.\frac{1}{(x^2+r^2)^{1/2}}\right|_0^R$$

$$= 2\pi k \sigma x \left[\frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}}\right]$$

$$\boxed{E_x = 2\pi k\sigma\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]}$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$$

imagine now! $R \to \infty$, σ = constant

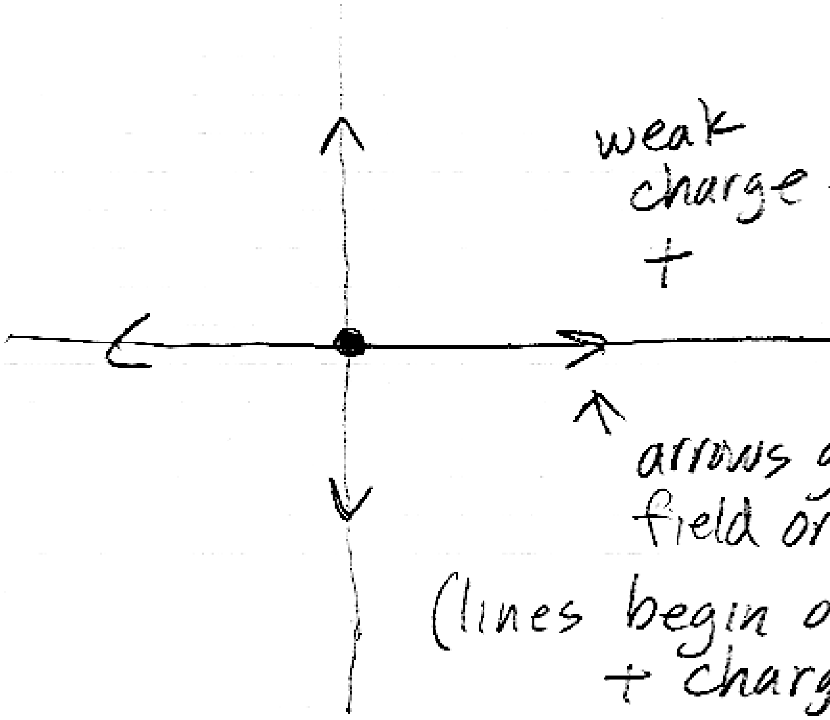
$$E_x \to \frac{\sigma}{2\varepsilon_0}$$

configuration	E
point charge	$\propto 1/(\text{distance})^2$
∞ line of charge	$\propto 1/(\text{distance})^1$
∞ sheet of charge	$\propto (\text{distance})^0$

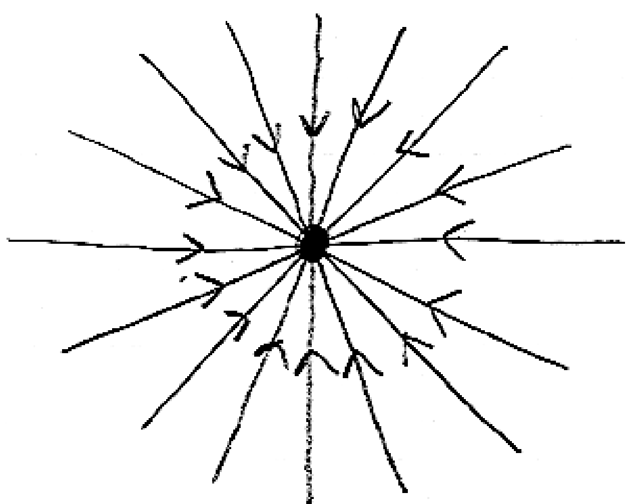
Electric Field Lines → visualization tool

Lines → represent direction of the electric field over a substantial region of space (intuitive)

Density of lines → represent magnitude of field (weird)

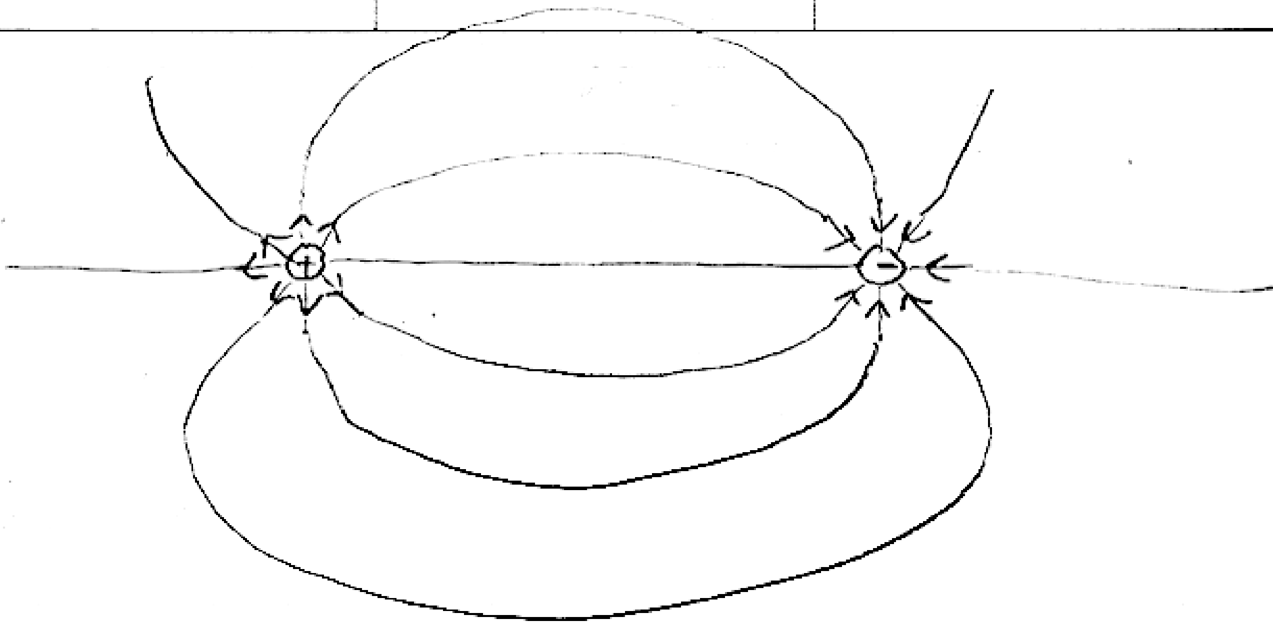


(lines begin on + charge)



(dense at center means: strong field)

(lines end on − charge)

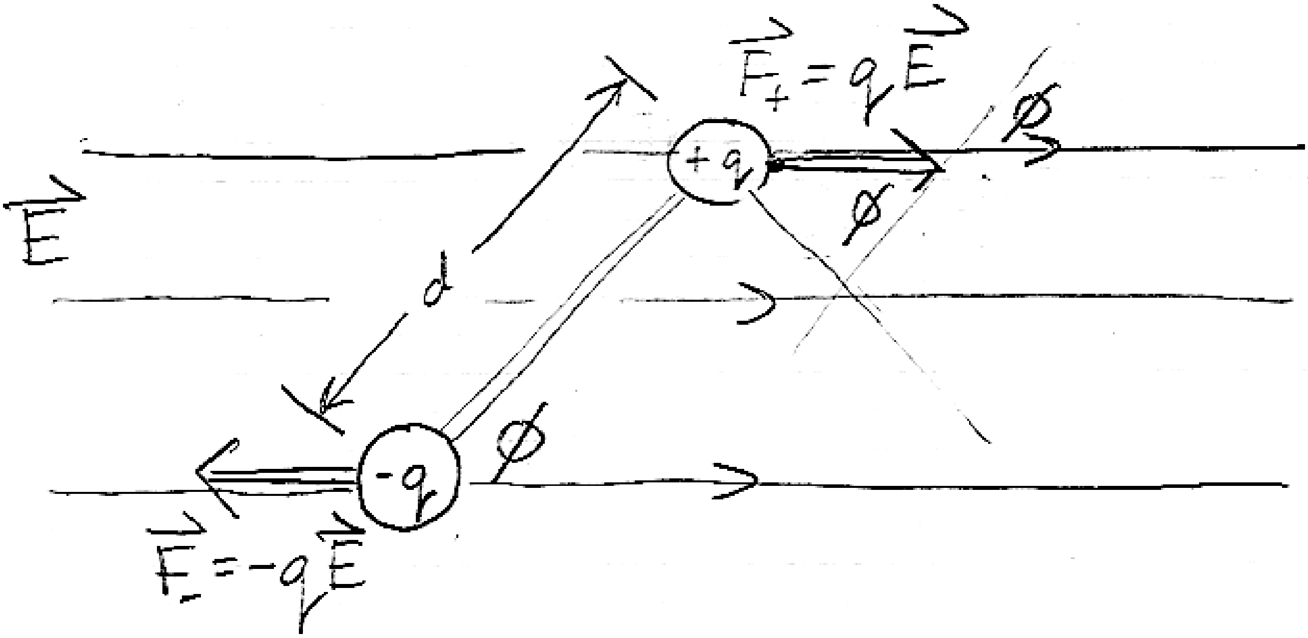


Electric Dipoles

In simplest form, an electric dipole has one positive charge and a second negative charge (of equal magnitude) a distance d apart, rigidly connected.

In constant $\vec{E}$ field, net force is 0 on an electric dipole, simply because the dipole has no net charge!

But, there will be a torque on the dipole (in a constant $\vec{E}$ field).

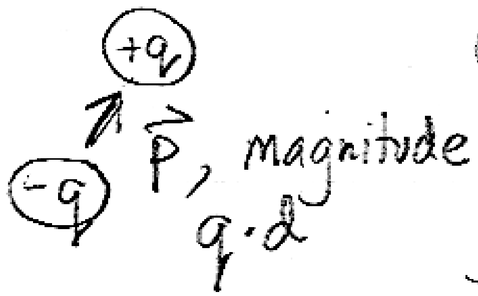


Compute torque about − charge

$$\tau = (qE\sin\phi)\cdot d$$

$$= \underbrace{(qd)}_{\text{magnitude of } p \text{, the dipole moment}} E\sin\phi$$

The dipole moment vector $\vec{p}$ goes from the − charge to the + charge.

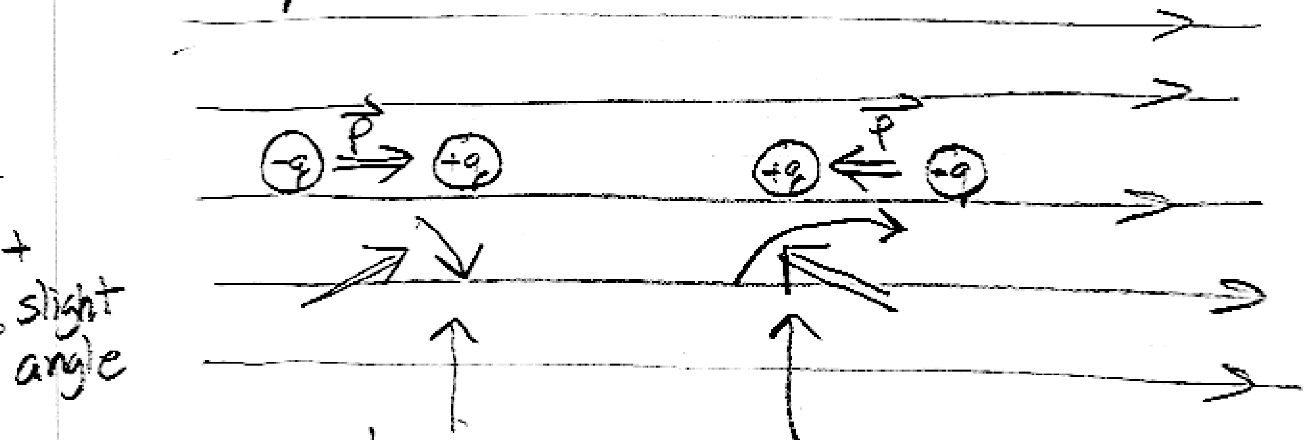


Then in constant $\vec{E}$ field.

$$\vec{\tau} = \vec{p}\times\vec{E}$$

Energy of the Dipole

Torque vanishes when $\phi = 0, \pi$



torque restores $(\tau \propto \vec{p} \times \vec{E})$ (stable equilibrium)

torque pushes away (unstable equilibrium)

$$\tau = -\frac{\partial U}{\partial \phi} = -pE\sin\phi \Rightarrow -pE\phi,\ \phi \to 0$$

$$\boxed{U(\phi) = -(+pE\cos\phi) = -\vec{p}\cdot\vec{E}}$$

$\vec{p} \parallel \vec{E}$ has lowest energy