Vector Addition of Coulomb Forces

$q_1 = -2 \mu C$
$q_2 = 1 \mu C$
$q_3 = 3 \mu C$

Net force on $q_3$?

"Magnitude" due to $q_1$: $F_{31} = k \frac{q_1 q_3}{r_{13}^2}$

$k = 9 \times 10^9 \frac{N \cdot m^2}{C^2}$

$r_{13}^2 = 0.25^2 + 0.2^2$

$r_{13}^2 = 0.0625 + 0.04$

$r_{13}^2 = 0.1025 \text{ m}^2$

$F_{31} = 9 \times 10^9 \frac{N \cdot m^2}{C^2} \cdot \frac{(-2 \times 10^{-6})(3 \times 10^{-6}) C^2}{0.1025}$

$F_{31} = -54 \cdot 10^9 \cdot 10^{-12} = -53 \cdot 10^{-2}$

$F_{31} \approx -0.53 N$ : - sign means attractive

"Magnitude" due to $q_2$: $F_{32} = k \frac{q_2 q_3}{r_{23}^2}$

$r_{23}^2 = 0.1^2 + 0.2^2 = 0.01 + 0.04 = 0.05 \text{ m}$

$F_{32} = 9 \times 10^9 \frac{N}{0.05}$

$F_{32} = 27 \times 20 \times 10^{-3} = 0.54 \text{ N}$ + sign means repulsive
\[ F_{31y} = |F_{31}| \cos \theta_1 = 0.53 \cdot \frac{0.2}{\sqrt{0.2^2 + 0.25^2}} = 0.33 \text{N} \]
\[ F_{32y} = -|F_{32}| \cos \theta_2 = 0.54 \cdot \frac{0.2}{\sqrt{0.2^2 + 0.1^2}} = -0.48 \text{N} \]
\[ F_{3N_{xy}} = -0.15 \text{N} \]
\[ F_{31x} = -|F_{31}| \sin \theta_1 = -0.53 \cdot \frac{0.25}{\sqrt{0.2^2 + 0.25^2}} = -0.41 \text{N} \]
\[ F_{32x} = -|F_{32}| \sin \theta_2 = -0.54 \cdot \frac{0.1}{\sqrt{0.2^2 + 0.1^2}} = -0.24 \text{N} \]
\[ F_{3N_{xy}} = -0.65 \text{N} \]
\[ |F_{3N_e}| = \sqrt{0.15^2 + 0.65^2} = 0.67 \text{N} \]
\[ \theta = \tan^{-1} \left( \frac{0.15}{0.65} \right) = 13.3^\circ \]

**Electric Field**

**Abstraction:** what sort of thing is present when just one charge is present?

⇒ "Electric Field"

⇒ confusion: every charge has its field...
Idea: $q_1 \leftrightarrow r_{12} \rightarrow q_2$

$F_{12} = k \frac{q_1 q_2}{r_{12}^2}$

(+) means repulsive)

$F_{12} = \left[ \frac{k}{r_{12}^2} \right] \cdot q_1$

depends on #2

$F_{21} = k \frac{q_1 q_2}{r_{12}^2}$

(diagram of direction)

$F_{21} = \left[ \frac{k}{r_{12}^2} \right] \cdot q_2$

depends on #1

$\Downarrow$ focus

$E_1 = k \frac{q_1}{r_{12}^2}$

$q_2 \rightarrow F_{21} = q_2 \cdot E_1$

$E_1$ caused by $q_1$

$F_{21}$ caused by $q_2$ interacting

$E_2 = \frac{k}{r_{12}^2}$

$E_2$ caused by $q_2$

$F_{12}$ caused by $q_1$ interacting with $E_2$

Note:

$|E_1| \neq |E_2|$

although

when $q_1 \neq q_2$

(as drawn, $q_1 > q_2$)

still $|\vec{F}_{12}| = |\vec{F}_{21}|$
"Test Charge" \( q_0 \)

Conceptually: region of space, pre-existing electric field from other charges.

Imagine plonking down "test charge" \( q_0 > 0 \)

- Measure force, \( \vec{F}_0 \)
- On this charge.

\[ \text{then } \vec{E} = \frac{\vec{F}_0}{q_0} \]

**Electric Field Direction**

1. **Away from a positive charge**
   - \( \vec{E} \) points away at random points
   - \( q > 0 \)

2. **Toward a negative charge**
   - \( \vec{E} \) points toward
   - \( q < 0 \)
   - \( q_0 \) would be attracted
**Electric Field Calculations**

Exercises in: symmetry, vector addition, calculus

**Charged Line, Semicircle, radius a**

**Total Charge** \( Q \), **linear charge density**: \( \lambda = \frac{Q}{\pi a} \)

\[ dQ = \lambda \, d\Theta \]

\[ \theta \]

\[ a \]

\[ \nu \]

\[ dE_x \]

\[ dE_y \]

\[ dE \]

\[ E_y = 0 \]

\[ E_x : dE_x = (k \frac{\lambda d\Theta}{a^2}) \cos \Theta \]

\[ E_x = \int dE_x = k \frac{\lambda}{a} \int_{-\pi/2}^{\pi/2} \cos \Theta \, d\Theta \]

\[ = k \frac{\lambda}{a} \left( \sin \Theta \right)_{-\pi/2}^{\pi/2} = k \frac{\lambda}{a} (1 - (-1)) \]

\[ = 2 k \frac{\lambda}{a} \]

\[ E_x = \frac{2}{\pi} k \frac{Q}{\lambda a^2} \]

\[ k = \frac{1}{4\pi \varepsilon_0} \]
Symmetry: at point $P$, $E_y = 0$
$E_z = 0$

\[ dQ = \lambda \, d\theta \]

\[ dE_x = k \, \frac{dQ}{r^2} \times \cos \alpha \]
\[ = k \, \frac{\lambda \, d\theta}{x^2 + a^2} \cdot \frac{x}{\sqrt{x^2 + a^2}} \]
\[ E_x = k \, \frac{\lambda \, ax}{(x^2 + a^2)^{3/2}} \int_0^{2\pi} d\theta = k \, \frac{2\pi \lambda \, ax}{(x^2 + a^2)^{3/2}} \]
\[ = k \, \frac{2\pi \lambda \, ox}{(x^2 + a^2)^{3/2}} \]

\[ E_x = k \, \frac{Q x}{(x^2 + a^2)^{3/2}} \quad k = \frac{1}{4\pi \varepsilon_0} \]

**Line Charge** \( \lambda = \frac{Q}{2a} \)

at point $P$, $E_y = 0$

\[ dE_x = k \, \frac{dQ}{R^2} \cos \theta \]
\[ = k \, \frac{\lambda \, dy}{R^2} \cos \theta \]