Chapter 21

1) Sound Waves - You Hear Pressure
2) \( I \propto A^2 \propto P_{\text{max}}^2 \), \( \beta = (10 \text{db}) \log(I/I_0) \)
3) Beats: Interference in time
4) Doppler Effect
5) Shock Waves

Sound: \( 20 \text{Hz} \leq f \leq 20,10^3 \text{Hz} \) in air

\[ y \rightarrow \text{now displacement from equilibrium II to direction of wave} \]

\[ y(x,t) = A \sin(\omega t - kx) \quad \omega = 2\pi f = \frac{2\pi}{T} \]

\[ k = \frac{2\pi}{\lambda}, \quad v = \frac{\lambda}{T} = \frac{\omega}{k} \]

but the ear really hears increments, decrements in pressure

so absolute pressure = \( p_a + p(x,t) \)

ambient, constant pressure \( \approx 10^5 \text{ Pa at one atmosphere} \)

"gauge" pressure

Goal then: relate \( p(x,t) \) to \( y(x,t) \)

Imagine a little cylinder, length \( \Delta x \), endcap area \( S \),

\[ V = \frac{p}{S} \Delta x \]
NO WAVE

\[ V = S \Delta x \]

WITH WAVE

\[ V' = S(\Delta x + y(x+\Delta x,t) - y(x,t)) \]

\[ \Delta V = V' - V = S (y(x+\Delta x,t) - y(x,t)) \]

\[ \frac{\Delta V}{V} = \frac{S (y(x+\Delta x,t) - y(x,t))}{S \Delta x} \]

In the limit as \( \Delta x \to 0 \)

\[ \lim_{\Delta x \to 0} \frac{\Delta V}{V} = \frac{\partial y}{\partial x} \quad \text{call} = \frac{dV}{V} \]

**Bulk Modulus:** p. 342, Eq. 11-13; to achieve "volume strain" \( \frac{dV}{V} \), must apply pressure increment (there \( \Delta p \), here \( p(x,t) \))

\[ p(x,t) = -B \frac{dV}{V} = -B \frac{\partial y}{\partial x} \]

\[ p(x,t) = -B \frac{\partial}{\partial x}(As \sin(\omega t - kx)) \]

\[ y(x,t) \]

\[ p(x,t) = BkA \cos(\omega t - kx) \]

\[ V \ (\text{traveling wave}) \]

\[ x \]
not quite same as a standing wave -

\[ y(x, t) = \alpha \cos \left( \frac{\pi x}{L} \right) \cos(2\pi ft) \]

\[ p(x, t) = -B \frac{\partial y}{\partial x} \]

\[ = -\frac{B \alpha \pi}{L} \sin \left( \frac{\pi x}{L} \right) \cos(2\pi ft) \]

Back to travelling wave:

\[ P_{\text{max}} = BK \alpha \]

Book (Example 21-1): \[ P_{\text{max}} \approx 3 \times 10^{-2} \text{ Pa} \]

\[ A \approx \frac{P_{\text{max}}}{BK} \sim 10^{-8} \text{ m} \]

Compute from \( 10^3 \text{ Hz} = f \)

Can even hear: \( 3 \times 10^{-5} \text{ Pa} @ f = 10^3 \text{ Hz} \)

Sound Intensity

Power = force \cdot velocity

\[ \text{Power} = \frac{\text{force}}{\text{Area}} \cdot \frac{\text{velocity}}{\text{Area}} = \frac{\text{pressure}}{\text{area}} \cdot \text{velocity} \]

\[ p(x, t) = BK \alpha \cos(\omega t - kx) \]

\[ V = \frac{\partial y}{\partial t} = \omega A \cos(\omega t - kx) \]
\[
\frac{\text{Power}}{\text{Area}} = Bk\omega A^2 \cos^2 (\omega t - kx) \quad \text{average value is}\quad \frac{1}{2}
\]

\[
I = \langle \frac{\text{Power}}{\text{Area}} \rangle = \frac{1}{2} Bk\omega A^2
\]

\[
\frac{\omega}{k} = v = \sqrt{\frac{B}{p}} \quad ; \quad k = \sqrt{\frac{p}{B}} \omega
\]

then \quad \[ I = \frac{1}{2} B \cdot (\sqrt{\frac{p}{B}} \omega) A^2 \]

\[
I = \frac{1}{2} \sqrt{\frac{p}{B}} \cdot \omega^2 \cdot \frac{p_{\text{max}}}{B^2 k^2}
\]

\[
= \frac{1}{2} \sqrt{\frac{p}{B}} \cdot v^2 \cdot \frac{p_{\text{max}}}{B^2}
\]

\[
= \frac{1}{2} \sqrt{\frac{p}{B}} \cdot \frac{B}{p} \cdot \frac{p_{\text{max}}}{B^2}
\]

\[
I = \frac{1}{2} \frac{p_{\text{max}}}{\sqrt{\frac{p}{B}}}
\]

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**Speaker**

- **radius** \( r \)
- **area** \( 4\pi r^2 \)

**Sound intensity** \( I(r) = \frac{P}{4\pi r^2} \)

Want intensity down by a factor of 100? more a factor of 10 away in distance
Sound intensity is usually measured in decibels:

$$B = (10 \text{ dB}) \log \frac{I}{I_0}$$

10 decibels per decade... $I_0$ is the "reference" intensity:

$$I_0 = 10^{-12} \text{ W/m}^2$$

So...

$$I = 1 \text{ Watts/m}^2$$

is:

$$B = (10 \text{ dB}) \log \left( \frac{1}{10^{-12}} \right)$$

$$= 10 \times \log \cdot 10^{12}$$

$$B = 10 \times 12 = 120 \text{ dB}$$

This hurts!

<table>
<thead>
<tr>
<th>$\frac{B}{\text{dB}}$</th>
<th>$\frac{I}{10^{-12} \text{ W/m}^2}$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>$10^{-10}$</td>
<td>whisper</td>
</tr>
<tr>
<td>40</td>
<td>$10^{-8}$</td>
<td>quiet radio</td>
</tr>
<tr>
<td>60</td>
<td>$10^{-6}$</td>
<td>~ ordinary talking</td>
</tr>
<tr>
<td>70</td>
<td>$10^{-5}$</td>
<td>busy street</td>
</tr>
<tr>
<td>90</td>
<td>$10^{-3}$</td>
<td>train</td>
</tr>
</tbody>
</table>
Beats

1) \( f_1 \) is \( f_1 \)

2) \( f_2 \) is \( f_2 \)

\[
y_1(t) + y_2(t) = A \sin(2\pi f_1 t) + B \sin(2\pi f_2 t)
\]

simplify: take \( A = B \)

\[
f_1 = \frac{1}{2}(f_1 + f_2) + \frac{1}{2}(f_1 - f_2) = \bar{f} + \Delta f
\]

\[
f_2 = \frac{1}{2}(f_1 + f_2) - \frac{1}{2}(f_1 - f_2) = \bar{f} - \Delta f
\]

Then

\[
y_1(t) + y_2(t) = A \left\{ \sin(2\pi \bar{f} t + \bar{f} + \Delta f) + \sin(2\pi \bar{f} t + \bar{f} - \Delta f) \right\}
\]

\[
= A \left\{ \sin(2\pi \bar{f} t) \cos(2\pi \Delta f t) + \sin(2\pi \Delta f t) \cos(2\pi \bar{f} t)
+ \sin(2\pi \bar{f} t) \cos(-2\pi \Delta f t) + \sin(-2\pi \Delta f t) \cos(2\pi \bar{f} t) \right\}
\]

\[
= 2A \sin(2\pi \bar{f} t) \cos(2\pi \Delta f t)
\]

"carrier" \quad "envelope" \quad "beat"

\[
\left\langle \frac{1}{|\Delta f|} \right\rangle
\]

\[
\left\langle \frac{1}{2|\Delta f|} \right\rangle \left\langle \text{beat frequency} = 2\Delta f \right\rangle
\]
"Beat Frequency" = 2πf₁ = |f₁ - f₂|

Demonstration with tuning forks:
a little clay on one of them lowers its frequency; the smaller the amount of clay, the lower the (Δf), the longer the time between "beats."

**Doppler Effect**

Moving Listener... frequency changes

Time for fly to go from one wavecrest to next:
\[(v_L + v) \cdot t = \lambda\]

\[f'_L = \frac{1}{t} = \frac{v_L + v}{\lambda}\]

Recall: \[f_s = \frac{v}{\lambda}\]

\[f_L = \frac{v}{\lambda} (1 + \frac{v}{v_L}) = (1 + \frac{v}{v_L}) f\]

- higher pitch, higher frequency, move toward source
- lower pitch, lower frequency, move away from source
Moving Source... wavelength changes

\[ \lambda = \frac{V}{f_s} \]

Stationary emitter

\[ \lambda = (V + V_s) \frac{T}{f_s} = \frac{V + V_s}{f_s} (1 - V_s T) \]

Moving emitter

\[ \begin{bmatrix} \bar{P} \\ \bar{V}_L \end{bmatrix} \]

\[ f_L = \frac{V_L + V}{X} = \frac{V + V_L}{V + V_s} f_s \]

"Combined doppler"

Listener: \( V_L > 0 \), move toward source, \( f_L > f_s \) (higher pitch)

Source: \( V_s < 0 \), source moves toward listener, \( f_L > f_s \) too, (higher pitch)

Listener: \( V_L < 0 \), move away from source, \( f_L < f_s \) (lower pitch)

Source: \( V_s > 0 \), move away from listener, \( f_L < f_s \) (lower pitch)
Shock Waves

Moving source with speed $v_s$ GREATER THAN wavespeed $v$: 

$t=0$ source emits $v_s$ 

\[ \sin \alpha = \frac{v/f_s}{v/v_s} = \frac{v}{v_s} \]

when $v < v_s$, $\sin \alpha < 1$, $\alpha$ real valued

(step back further)

wavefronts pile up \( \text{``shock wave''} \)

1. Supersonic plane, rocket, missile

2. Particle going $v < c$ in medium where light goes $\approx n$ 

\[ n = \text{index of refraction} > 1 \]

(shock = Cerenkov radiation)