Chapter 21

1) Sound Waves - You Hear Pressure
2) $I \propto A^2 \propto p_{max}^2$, $\beta = (10\,db)\log(I/I_0)$
3) Beats : Interference in <u>time</u>
4) Doppler Effect
5) Shock Waves.

<u>Sound</u>: $20\,Hz \lesssim f \lesssim 20 \cdot 10^3\,Hz$ in ~air

$y \rightarrow$ now displacement from equilibrium $\parallel$ to direction of wave.

$y(x,t) = A\sin(\omega t - kx)$ $\omega = 2\pi f = \frac{2\pi}{T}$

$k = \frac{2\pi}{\lambda}$, $v = \frac{\lambda}{T} = \frac{\omega}{k}$

but the ear really hears <u>increments</u>, <u>decrements</u> in <u>pressure</u>

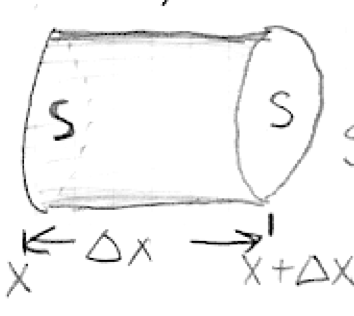
so absolute pressure $= p_a + p(x,t)$

ambient, constant pressure $\simeq 10^5$ Pa at one atmosphere

$\underbrace{\text{increment, decrement from ambient; }\sim 10}$ "gauge" pressure

Goal then: relate $p(x,t)$ to $y(x,t)$

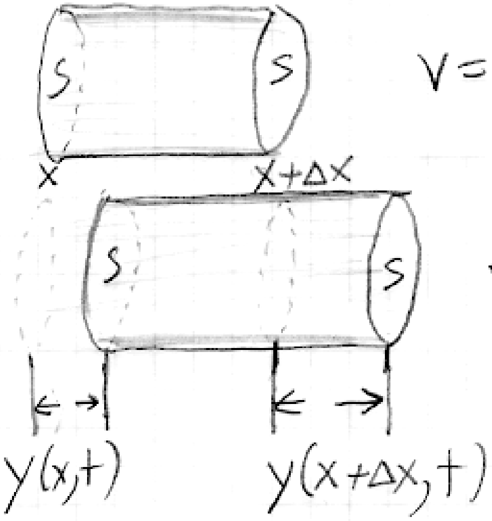
Imagine a little cylinder, length Δx, endcap area S. ; $\boxed{\text{NO WAVE}}$



$V = S\Delta x$

NO WAVE $V = S\Delta x$

WITH WAVE $V' = S(\Delta x + y(x+\Delta x, t) - y(x,t))$



so $\Delta V = V' - V = S(y(x+\Delta x, t) - y(x,t))$

$$\frac{\Delta V}{V} = \frac{S(y(x+\Delta x, t) - y(x,t))}{S\Delta x}$$

in the limit as $\Delta x \to 0$

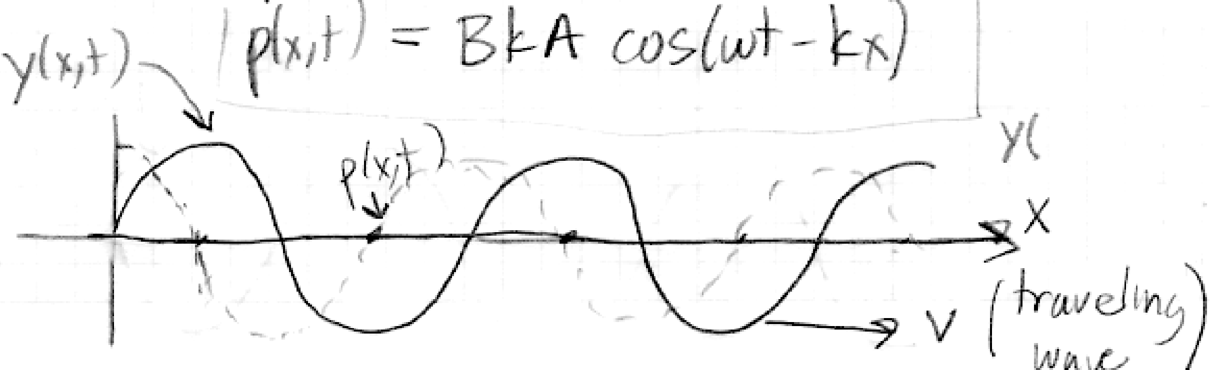
$$\lim_{\Delta x \to 0} \frac{\Delta V}{V} = \frac{\partial y}{\partial x}, \quad \text{call} = \frac{dV}{V}$$

Bulk Modulus: p. 342, Eq 11-13; to achieve "volume strain" dV/V, must apply pressure increment (there Δp, here $p(x,t)$)

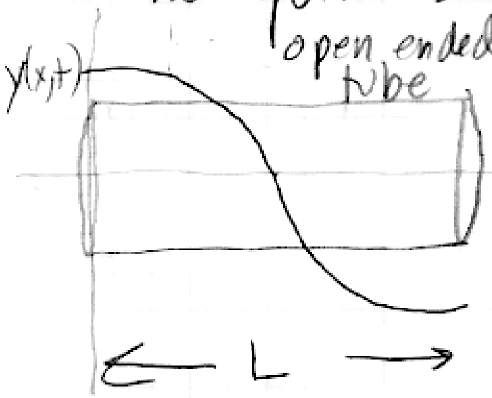
$$p(x,t) = -B\frac{dV}{V} = -B\frac{\partial y}{\partial x}$$

$$p(x,t) = -B\frac{\partial}{\partial x}(A\sin(\omega t - kx))$$

$$\boxed{p(x,t) = BkA\cos(\omega t - kx)}$$



not quite same as a standing wave.



$$y(x,t) = \alpha \cos\left(\frac{\pi x}{L}\right)\cos(2\pi f t)$$

$$p(x,t) = -B\frac{\partial y}{\partial x}$$

$$= -\frac{B\alpha\pi}{L}\sin\left(\frac{\pi x}{L}\right)\cos(2\pi f t)$$

Back to travelling wave:

$$\underline{p_{max} = BkA}$$

Book (Example 21-1): $p_{max} \simeq 3 \cdot 10^{-2}$ Pa

$$A \simeq \frac{p_{max}}{Bk} \sim 10^{-8}\,m\,!$$

compute from 10^3 Hz $= f$

Can even hear: $3 \cdot 10^{-5}$ Pa @ $f = 10^3$ Hz

Sound Intensity

Power = force · velocity

$$\frac{\text{Power}}{\text{Area}} = \frac{\text{force}}{\text{Area}} \cdot \text{velocity} = \text{Intensity}\left(\frac{W}{m^2}\right)$$

ear ≃ responds to this.

$$= \text{pressure} \cdot \text{velocity}.$$

$$p(x,t) = BkA\cos(\omega t - kx)$$

$$v = \frac{\partial y}{\partial t} = \omega A \cos(\omega t - kx)$$

$$\frac{\text{Power}}{\text{Area}} = BkwA^2 \underbrace{\cos^2(wt-kx)}_{\text{average value is } 1/2}$$

$$I = \left\langle \frac{\text{Power}}{\text{Area}} \right\rangle = \frac{1}{2} BkwA^2$$

$$\frac{w}{k} = v = \sqrt{\frac{B}{\rho}} \; ; \quad k = \sqrt{\frac{\rho}{B}}\, w$$

then $$I = \frac{1}{2} B \cdot \left(\sqrt{\frac{\rho}{B}}\, w\right) w A^2$$

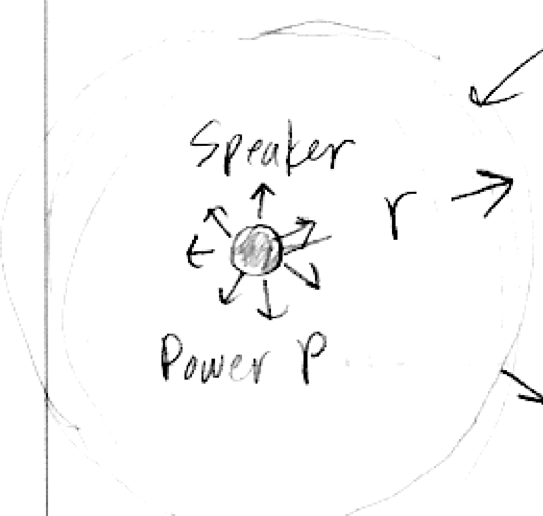
$$\boxed{I = \frac{1}{2}\sqrt{\rho B}\, w^2 A^2}$$

$$= \frac{1}{2}\sqrt{\rho B} \cdot w^2 \cdot \frac{p_{max}^2}{B^2 k^2} \qquad \left(\frac{w}{k} = v\right)$$

$$= \frac{1}{2}\sqrt{\rho B}\; v^2 \cdot \frac{p_{max}^2}{B^2}$$

$$= \frac{1}{2}\sqrt{\rho B} \cdot \frac{B}{\rho} \cdot \frac{p_{max}^2}{B^2}$$

$$\boxed{I = \frac{1}{2} \frac{p_{max}^2}{\sqrt{\rho B}}}$$



Sphere, radius r, area $4\pi r^2$

sound intensity $I(r) = \frac{P}{4\pi r^2}$

want intensity down by a factor of 100? move a factor of 10 away in distance

Sound intensity: usually measured in decibels:

$$\beta = (10\,\text{dB}) \log \frac{I}{I_0}$$

10 deci bels per decade...

I_0 is the "reference" intensity, $I_0 \equiv 10^{-12} \frac{W}{m^2}$

so... $I = 1$ Watts/m^2 is:

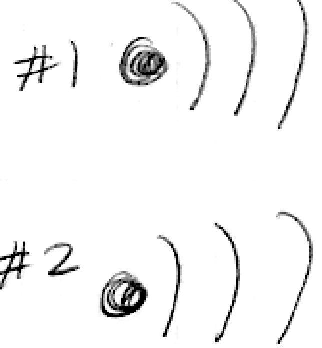
$$\beta = (10\,\text{dB}) \log \left(\frac{1}{10^{-12}}\right)$$

$$= 10 \times \log \cdot 10^{12}$$

$$\beta = 10 \times 12 = 120\,\text{dB}$$ } this hurts!

β	I	comment
0 dB	$10^{-12} \frac{W}{m^2}$	Threshold of hearing.
20	10^{-10}	whisper
40	10^{-8}	quiet radio
60	10^{-6}	~ ordinary talking
70	10^{-5}	busy street
90	10^{-3}	train

"Beats" – Inte



frequency is f_1

is f_2

neglect *spatial* interference pattern; focus on *time*.

$$y_1(t) + y_2(t) = A\sin(2\pi f_1 t) + B\sin(2\pi f_2 t)$$

simplify: take $A = B$

note: $f_1 = \underbrace{\tfrac{1}{2}(f_1+f_2)}_{\text{call } = \bar{f}} + \underbrace{\tfrac{1}{2}(f_1-f_2)}_{\text{call } = \Delta f} = \bar{f} + \Delta f$

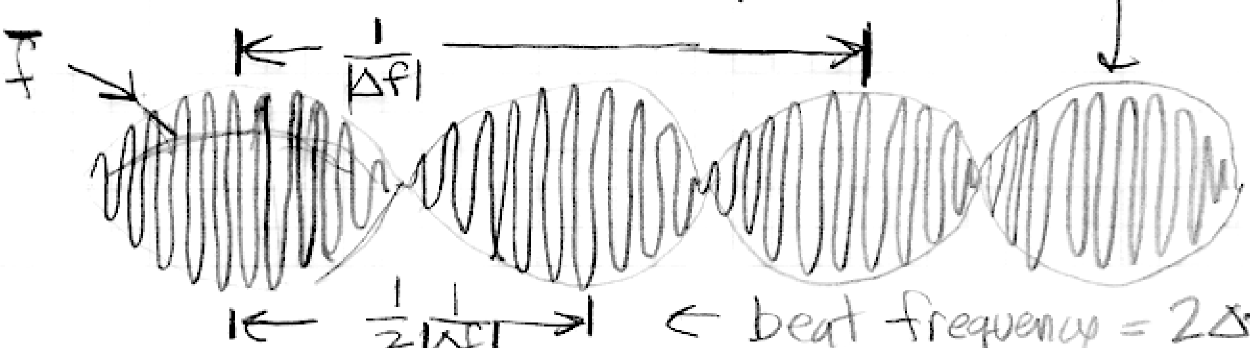
$$f_2 = \tfrac{1}{2}(f_1+f_2) - \tfrac{1}{2}(f_1-f_2) = \bar{f} - \Delta f$$

then

$$y_1(t) + y_2(t) = A\{\sin(2\pi t(\bar{f}+\Delta f)) + \sin(2\pi t(\bar{f}-\Delta f))\}$$

$$= A\{\sin(2\pi \bar{f} t)\cos(2\pi\Delta f t) + \cancel{\sin(2\pi\Delta f t)\cos(2\pi \bar{f} t)}$$
$$+ \sin(2\pi \bar{f} t)\cos(2\pi(-\Delta f t)) + \cancel{\sin(-2\pi\Delta f t)\cos(2\pi \bar{f} t)}\}$$

$$= 2A\underbrace{\sin(2\pi \bar{f} t)}_{\text{"carrier"}}\underbrace{\cos(2\pi\Delta f t)}_{\text{"envelope"}}$$



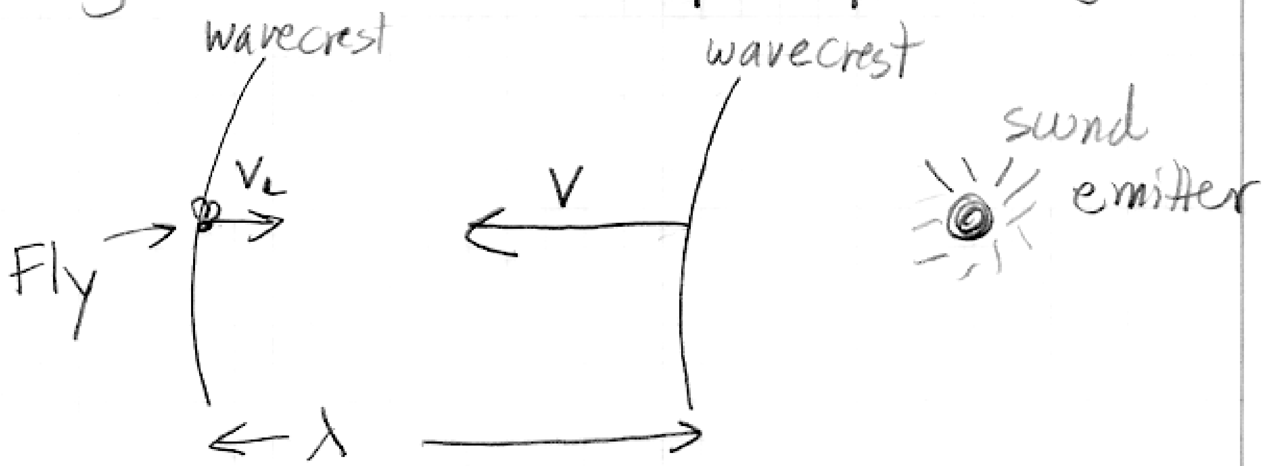
← beat frequency $= 2\Delta f$

"Beat Frequency" = $2|\Delta f| = |f_1 - f_2|$

Demonstration with tuning forks:
a little clay on one of them lowers its frequency; the smaller the amount of clay, the lower the $|\Delta f|$, the longer the time between "beats.

Doppler Effect

☆ Moving Listener... frequency changes



Time for fly to go from one wavecrest to next:

$$(V_L + V)T = \lambda$$

$$f_L = \frac{1}{T} = \frac{V_L + V}{\lambda} \qquad \text{recall: } f_s = \frac{V}{\lambda}$$

$$f_L = \frac{V}{\lambda}\left(1 + \frac{V_L}{V}\right) = \left(1 + \frac{V_L}{V}\right) f$$

- higher pitch, higher frequency, move toward source
- lower pitch, lower frequency, move away from source

Moving Source ... wavelength changes

$\lambda = vT = \dfrac{v}{f_s}$

stationary emitter

$\lambda = (v + v_s)T = \dfrac{v + v_s}{f_s}$

$-v_s T$

moving emitter

note $v_s < 0 \longrightarrow$

$v_L \rightarrow$ $\leftarrow v$

$$\boxed{f_L = \frac{v_L + v}{\lambda} = \frac{v + v_L}{v + v_s} f_s}$$

"combined doppler"

listener: $v_L > 0$: move toward source, $f_L > f_s$ (higher pitch)

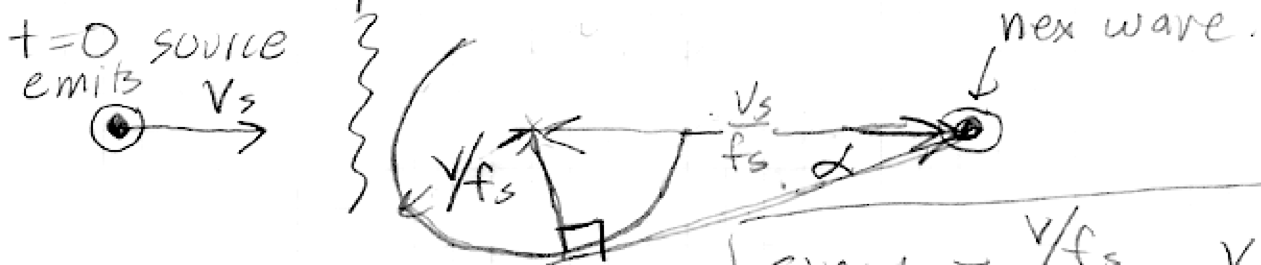
source $v_s < 0$, source moves toward listener, $f_L > f_s$ too. (higher pitch)

listener: $v_L < 0$, move away from source, $f_L < f_s$ (lower pitch)

source: $v_s > 0$, move away from listener, $f_L < f_s$ (lower pitch)

Shock Waves

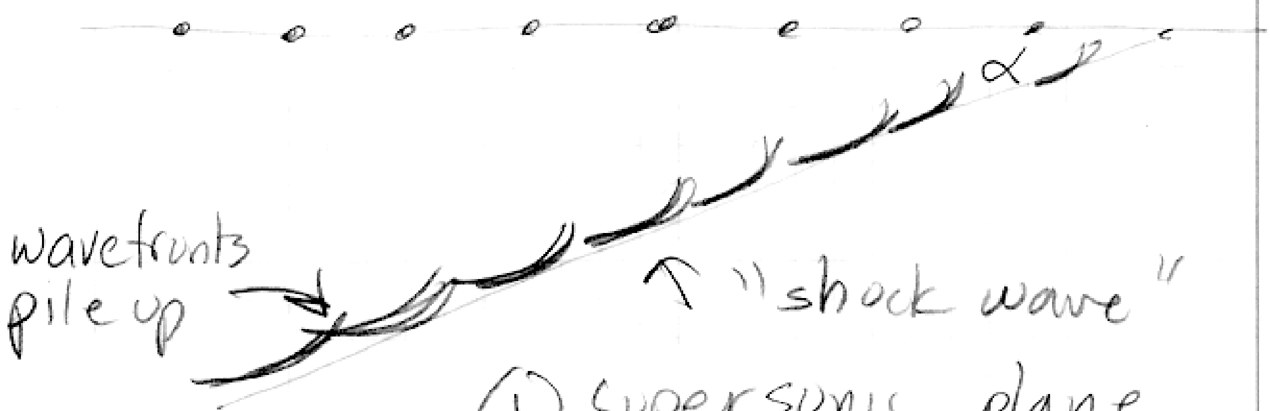
Moving source with speed v_s GREATER THAN wavespeed v:



$$\boxed{\sin\alpha = \frac{v/f_s}{v_s/\cancel{f_s}} = \frac{v}{v_s}}$$

when $v < v_s$, $\sin\alpha < 1$; α real valued

(step back further



① supersonic plane, rocket, missile

② particle going $\sim c$ in medium where light goes c/n

n = index of refraction > 1

(shock ≡ Cerenkov radiation)