Waves (Chapter 19)

You know already about them - beach

Our study will include:

1. Types - transverse, longitudinal, mix
2. Periodic Waves
3. Wave function
4. Wave equation
5. Speeds
   a) Transverse
   b) Longitudinal
6. Gas Waves
7. Energy

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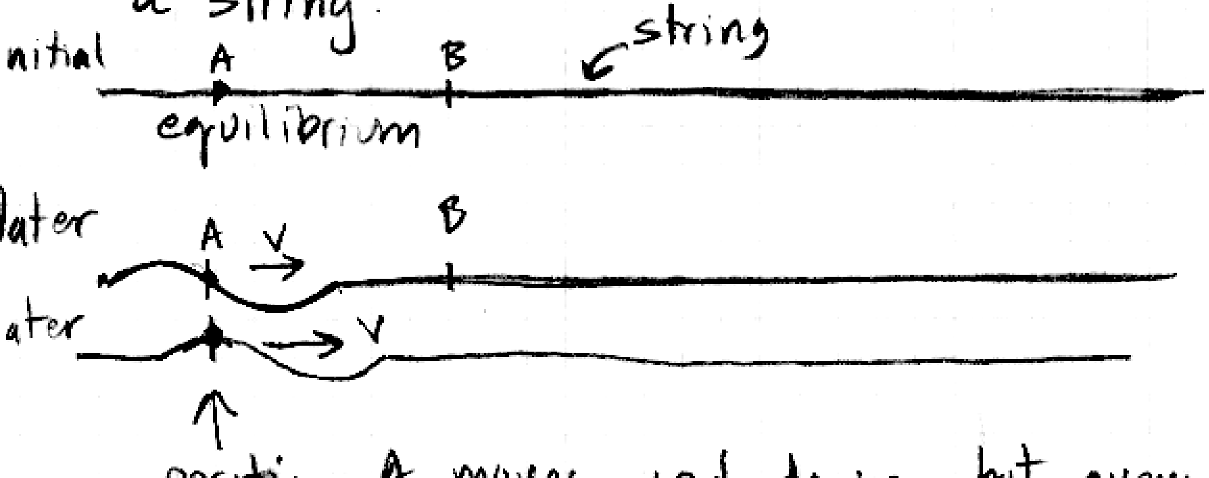
1. Types of waves

Waves move energy from one point to another; but there is no net motion of the medium carrying the wave.

→ look at equilibrium position of, say, a string:



position A moves up + down, but averaging

over a long time, has no net movement.
Energy flows from left to right, though

Look at: • instantaneous motion of medium
• direction of energy flow

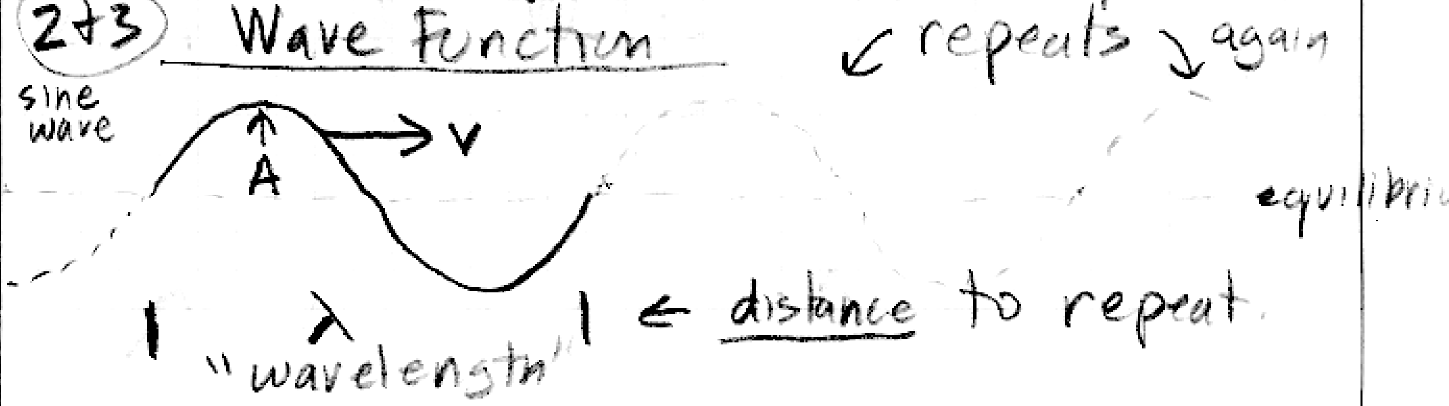
When they are perpendicular: it is a TRANSVERSE wave.
Example: string
(Light in free space)

When they are parallel: it is a LONGITUDINAL wave.
Example: sound

A little of both: OK too: Example: Ocean waves

Comment: some waves have no medium! • Light
• Matter itself
(quantum mechanics only)

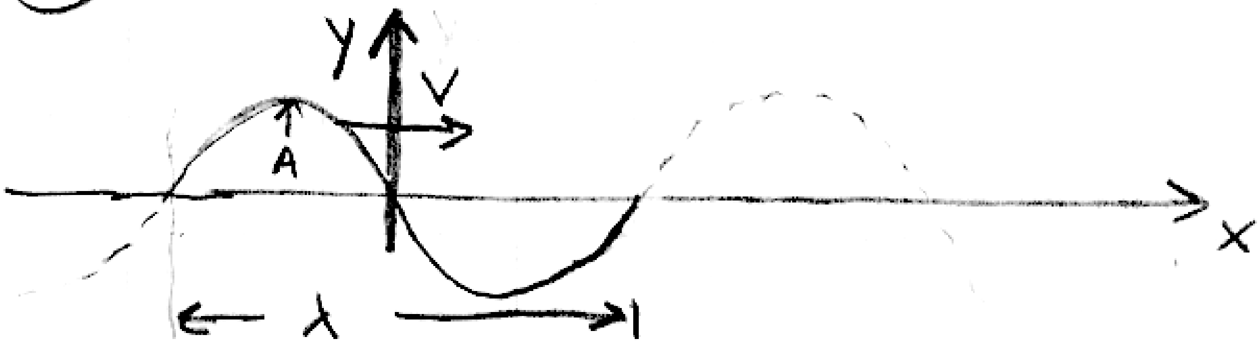
2+3 Periodic Waves
Wave Function



Mathematics to describe this ...

(A) that picture is frozen in time ... say, $t = 0$

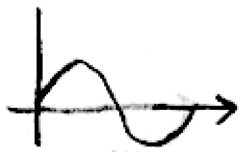
(B) need co-ordinate axes



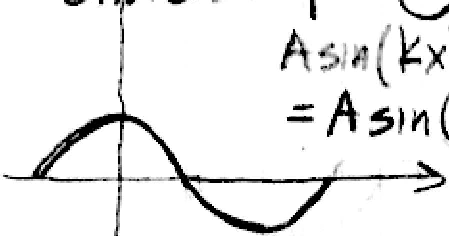
$$y(x, t=0) = -\underbrace{A\sin(kx)} = A\sin(-kx)$$

"pure sine" because <u>arbitrarily</u> chose origin of x-axis to coincide with a "zero"

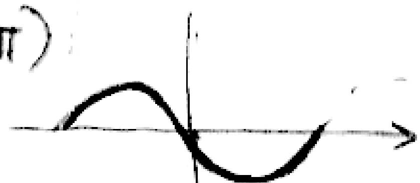
other choices:



$A\sin(kx)$
$= A\sin(-kx+\pi)$

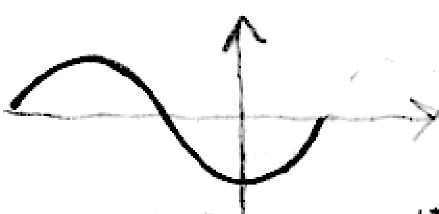


$A\cos(kx) = A\sin(-kx+\frac{\pi}{2})$



"Phase Advance"

$-A\sin(kx) = A\sin(-kx)$



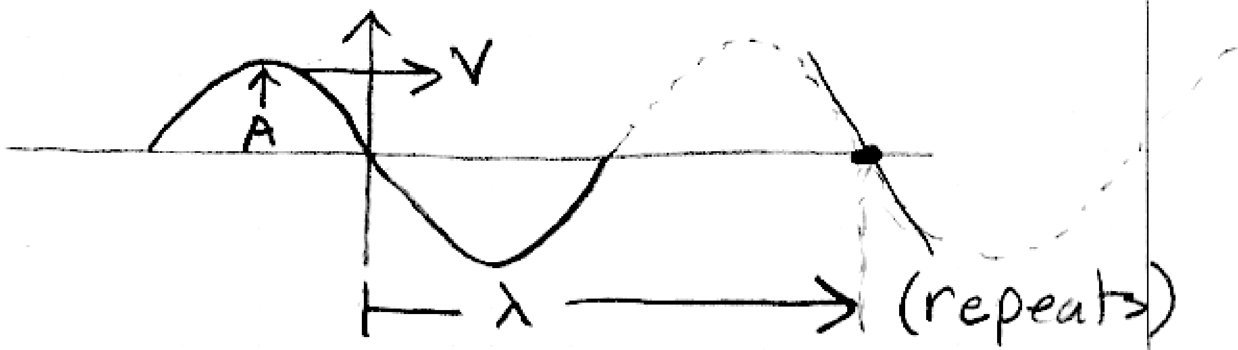
$-A\cos(kx) = -A\sin(\frac{\pi}{2}-kx)$
$= A\sin(-kx-\pi/2)$

$A\sin(kx) = A\sin(-kx-\pi)$

<u>Bottom Line</u>: <u>Arbitrary</u> choice of origin can be described by $A\sin(-kx+\phi)$, where the angle ϕ chosen to coincide with axis origin choice

Textbook Choice: $\phi = \pi$, $A\sin(-kx)$

(C) Condition on k:



look at level, slope

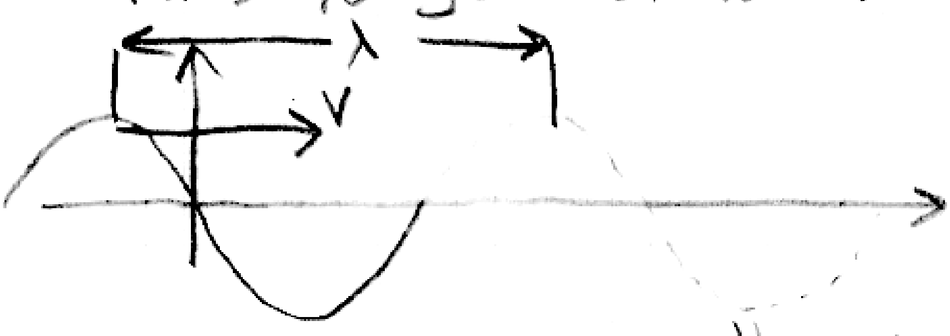
$$y(\lambda, t=0) = A\sin(\underbrace{-k\lambda}_{=-2\pi}) = 0$$

$= -2\pi$ (repetition)

$$\boxed{k = \frac{2\pi}{\lambda}}$$

name is "wave-number"

(D) Time: call T the time it takes to go distance λ..

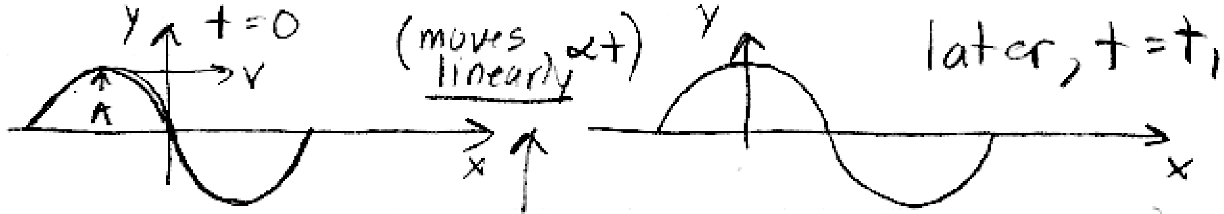


$v \cdot T = \lambda$ $\qquad$ $T \equiv$ "period"

$\boxed{v = \lambda \cdot f}$ $\qquad$ $\frac{1}{T} \equiv f =$ "frequency"

important

(E) What about intermediate times between $t=0$ and $t=T$



$y(x,t=0) = A\sin(-kx)$ $\qquad$ $A\cos(kx$ $\quad A \quad (\frac{\pi}{2}-kx)$

ansatz: $A\sin(\omega t - kx)$ $\qquad$ say... $\omega t_1 - \frac{\pi}{2}$

wait until $t=T$ then: $\omega T = 2\pi$

$$\omega = \frac{2\pi}{T} = = 2\pi f$$

Finally:

$$\boxed{\text{"Wave Function"} \quad \equiv y(x,t) = A\sin(\underbrace{\omega t - kx}_{\text{"the phase"}})}$$

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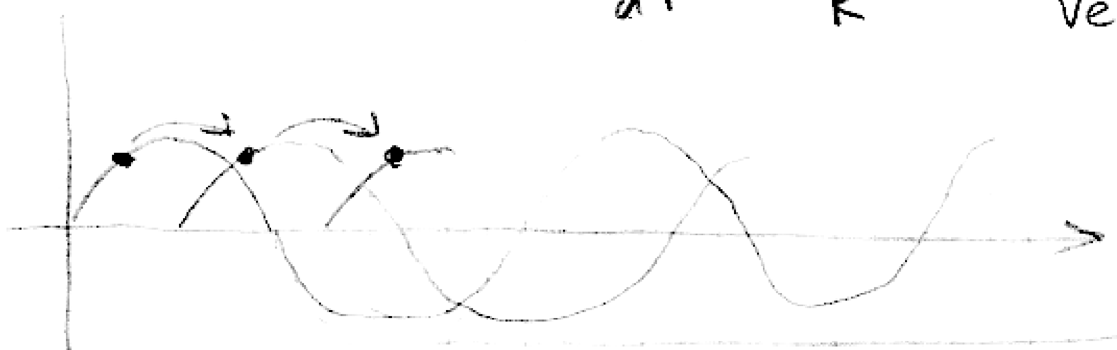
describes wave moving in the $+x$ direction

"constant phase" $\omega t - kx = \text{constant}$

$\Rightarrow$ differentiate w/r to t: $\omega - k\frac{dx}{dt} = 0$

$$v = \frac{dx}{dt} = \frac{\omega}{k} \leftarrow \text{"phase velocity"}$$



$-x$ direction: $\boxed{y(x,t) = A\sin(\omega t + kx)}$

4. Wave Equation

$$y(x,t) = A\sin(\omega t \pm kx)$$

$$\frac{\partial y}{\partial x} = \pm kA\cos(\omega t \pm kx) \qquad \frac{\partial y}{\partial t} = \omega A\cos(\omega t \pm kx)$$

$$\frac{\partial^2 y}{\partial x^2} = -k^2 A\sin(\omega t \pm kx) \qquad \frac{\partial^2 y}{\partial t^2} = -\omega^2 A\sin(\omega t \pm kx)$$

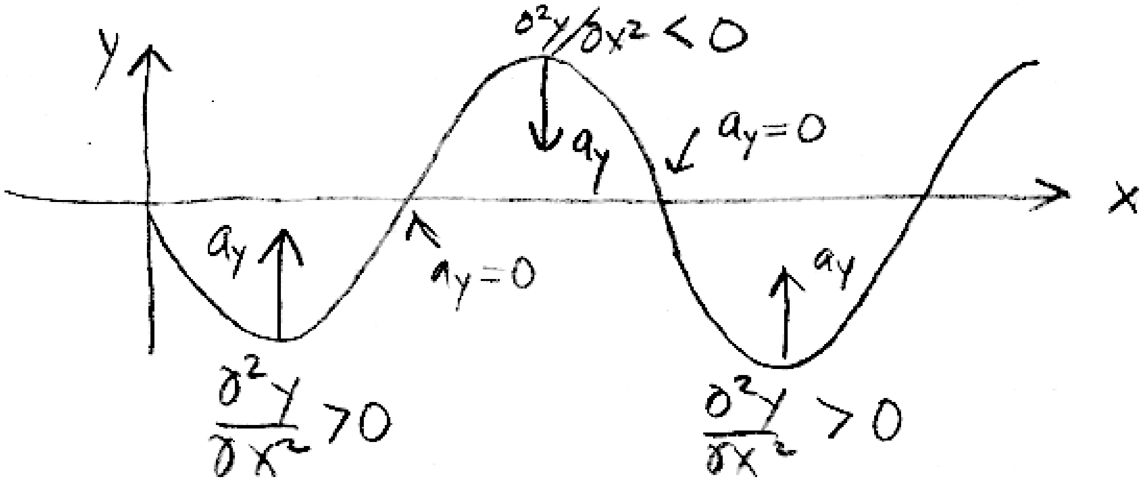
$$\frac{1}{k^2}\frac{\partial^2 y}{\partial x^2} = -A\sin(\omega t \pm kx) = \frac{1}{\omega^2}\frac{\partial^2 y}{\partial t^2}$$

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$$

wave equation

"better" $\Rightarrow \quad a_y = \frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$

relates acceleration $\left(\frac{\partial^2 y}{\partial t^2}\right)$ to curvature $\left(\frac{\partial^2 y}{\partial x^2}\right)$



note: $\frac{\partial y}{\partial t} = \pm \frac{\omega}{k}\frac{\partial y}{\partial x} = \pm v \frac{\partial y}{\partial x}$ too.

where is velocity high (low)?