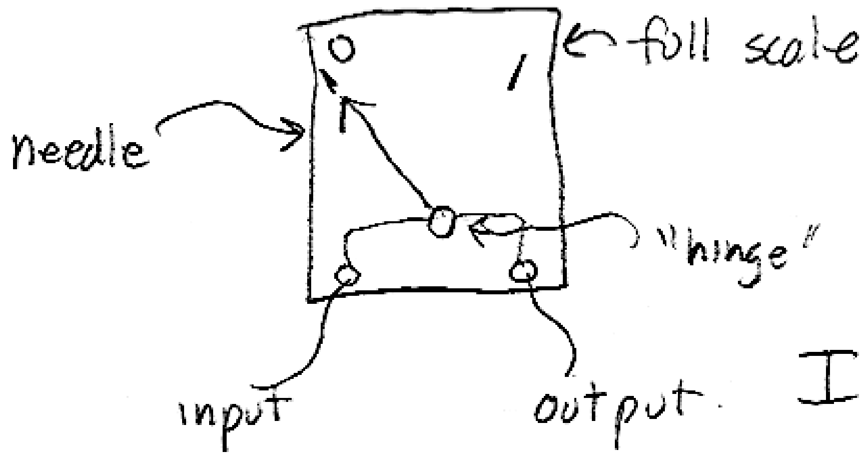
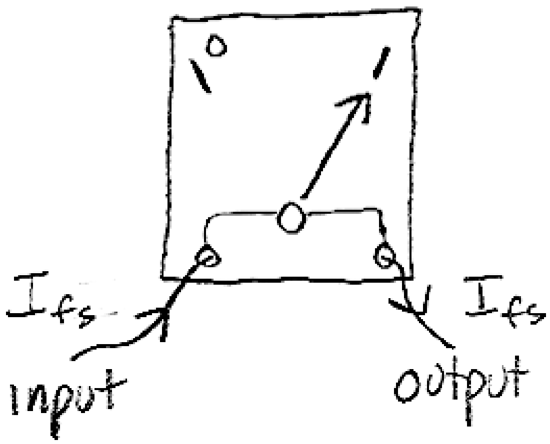
Old-Style Meters

Are usually "galvanometers," which convert electrical current into angular deflection of a needle..



$I = 0$

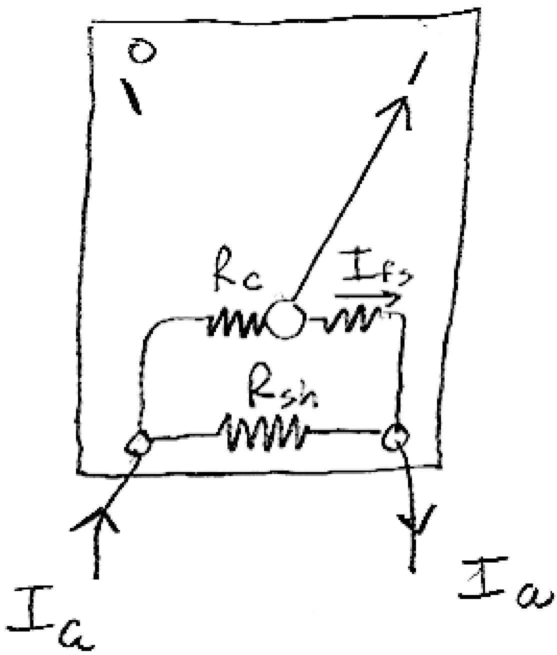


$$I = \underbrace{I_{fs}}_{\text{"full scale"}}$$

1) Physical mechanism is: current causes a magnetic field, magnetic field causes a torque
2) Field caused by a "coil," which actually has a non-zero resistance R_c ... gives non-ideal behavior, $R_c \approx$ ohms
3) Art is to use parallel and series resistances to make full scale deflection correspond to desired behavior.

Example... make full scale correspond to less than I_{fs} by "bleeding" current off through a "shunt" resistor, R_{sh}

goal: make $I_a > I_{fs}$ give the full scale reading



Ammeter

$$R_c I_{fs} = R_{sh}(I_a - I_{fs})$$

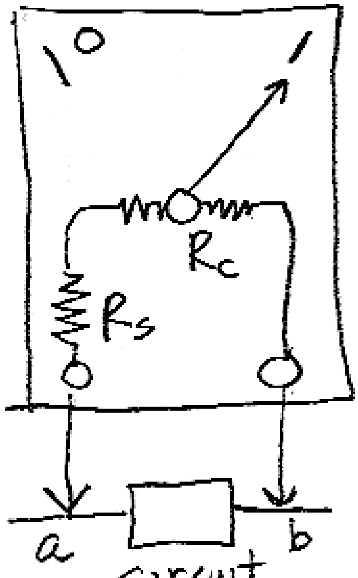
$$R_{sh} = \frac{I_{fs}}{I_a - I_{fs}} \cdot R_c \quad , \quad > R_c \text{ when } I_a > I_{fs}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_c} + \frac{1}{R_{sh}}$$

gives equivalent resistance, $< R_c$ but not 0.

Voltmeter

put resistor R_s in series



$V_{ab\,max} \equiv V_V$

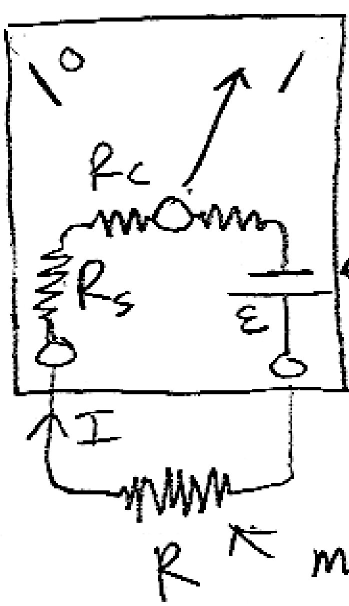
need R_{eq} for circuit $\ll (R_s + R_c)$

$$\frac{V_V}{R_s + R_c} = I_{fs}$$

$$R_s = \frac{V_V}{I_{fs}} - R_c \,, \quad \text{usually} \gg R_c$$

Ohmmeter

pick this



new battery introduced

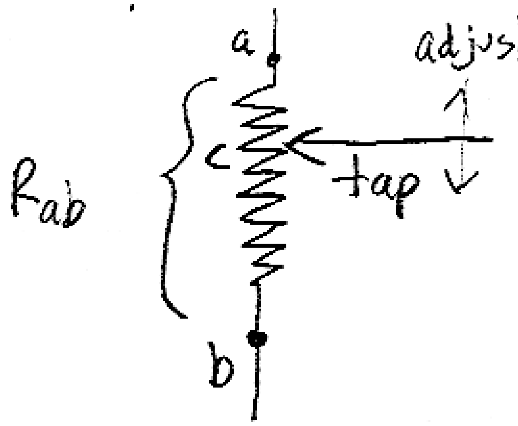
measure this

(scale looks funny)

$$I = \frac{\varepsilon}{R + R_s + R_c}$$

Potentiometer: Measure the voltage of an unknown battery

↓ resister with a "tap"

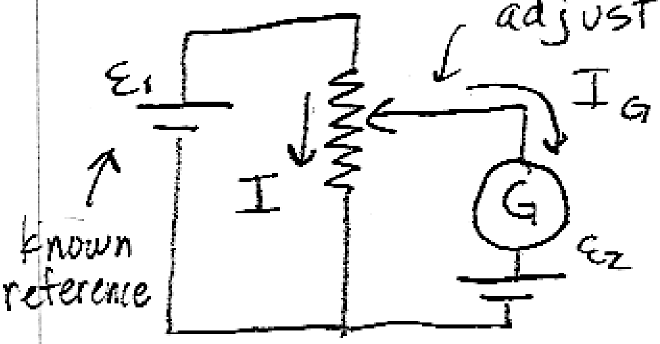


$R_{ac} < R_{ab}$

$R_{cb} = R_{ab} - R_{ac}$

Use this, with a galvanometer, to make more accurate measurement of the voltage across a battery

adjust until $I_G = 0$



$I R_{cb} = \mathcal{E}_2$

$I \cong \dfrac{\mathcal{E}_1}{R_{ab}}$ (or $\dfrac{\mathcal{E}_1}{R_{ab}+r}$)

so $\mathcal{E}_2 = \dfrac{R_{cb}}{R_{ab}} \cdot \mathcal{E}_1$

only works for $\mathcal{E}_2 < \mathcal{E}_1$

- No current need flow in/out of $\mathcal{E}_2$ battery ...

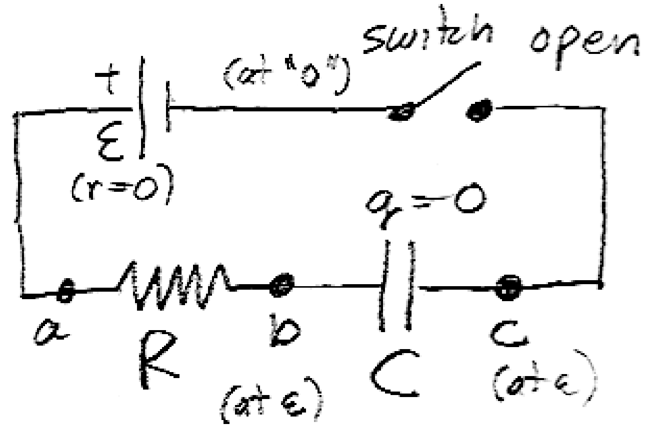
Resistance-Capacitance Circuits

V, I, Q : capital letters mean constant in time.

v, i, q : small letters mean varying in time (possibly)

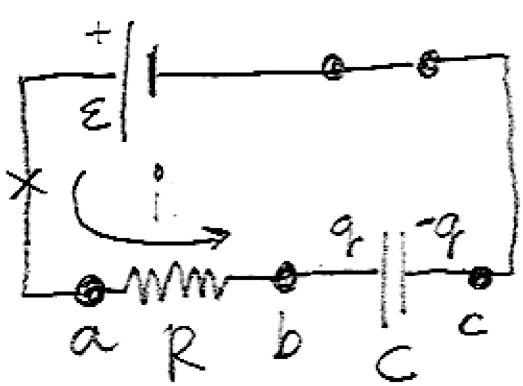
Initial:

from $t=-\infty$ up to $t=0$



switch open: no current flows
capacitor uncharged: $q=0$
$V_{ab} = V_{bc} = 0$ so all at "ε"

Now close switch at $t=0$ (this instant in time is singular); analyze voltages, currents, charges as a function of subsequent time t



charge conservation:

$$i = \frac{dq}{dt}$$

loop:

$$-iR - \frac{q}{C} + \varepsilon = 0$$

$$-R\frac{dq}{dt} - \frac{q}{C} + \varepsilon = 0$$

Qualitative Reasoning

① $t=0$, $q(0)=0$ so $-R\left.\frac{dq}{dt}\right|_{t=0} - \frac{0}{C} + \mathcal{E} = 0$

$$\boxed{q(0) = 0} \quad \boxed{i(0) = \left.\frac{dq}{dt}\right|_{t=0} = \frac{\mathcal{E}}{R}}$$

② $t \Rightarrow \infty$, all voltage drop is across the capacitor now: charge has built up to the point where a static situation has been achieved:

$$i(t=\infty) = 0 = \left.\frac{dq}{dt}\right|_{t=\infty}$$

$$-R\cdot 0 - \frac{q(\infty)}{C} + \mathcal{E} = 0$$

$$\boxed{i(\infty) = 0} \quad \boxed{q(\infty) = C\mathcal{E}}$$

what is "transition time"

Quantitative Solution

(change $t \to t'$, $q \to q'$) just notation

$$-R\frac{dq'}{dt'} - \frac{q'}{C} + \mathcal{E} = 0$$

$$-R\frac{dq'}{dt} = -\mathcal{E} + \frac{q'}{C}$$

$$\frac{dq'}{dt} = \frac{\mathcal{E}}{R} - \frac{q'}{RC} = \frac{C\mathcal{E} - q'}{RC}$$

dimensionless $\quad \frac{dq'}{q' - C\mathcal{E}} = -\frac{dt'}{RC}$ $\quad$ (RC has dimensions of time)

$$\int_0^q \frac{dq'}{q'-C\mathcal{E}} = -\frac{1}{RC}\int_0^t dt'$$

$$\ln(q'-C\mathcal{E})\Big|_0^q = -\frac{t}{RC}$$

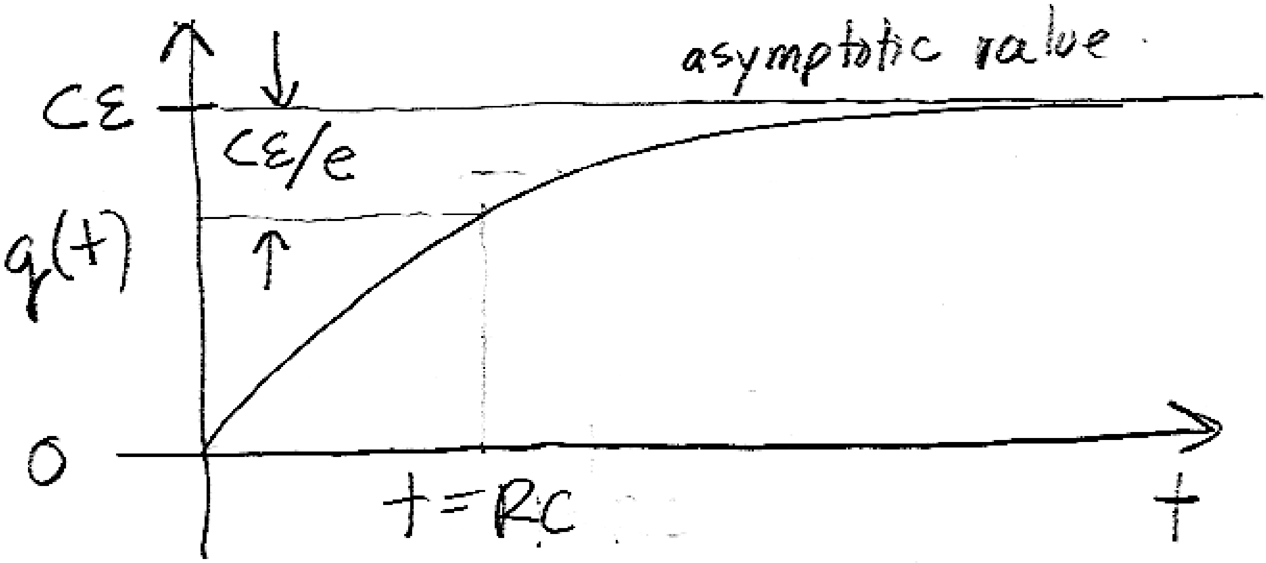
$$\ln\left(\frac{q-C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC}$$

$$\frac{q-C\mathcal{E}}{-C\mathcal{E}} = e^{-(t/RC)}$$

$$\boxed{q(t) = C\mathcal{E}\left(1-e^{-(t/RC)}\right)}$$

$$q(0) = C\mathcal{E}(1-e^{-0}) = C\mathcal{E}(1-1) = 0$$

$$q(\infty) = C\mathcal{E}(1-e^{-\infty}) = C\mathcal{E}$$



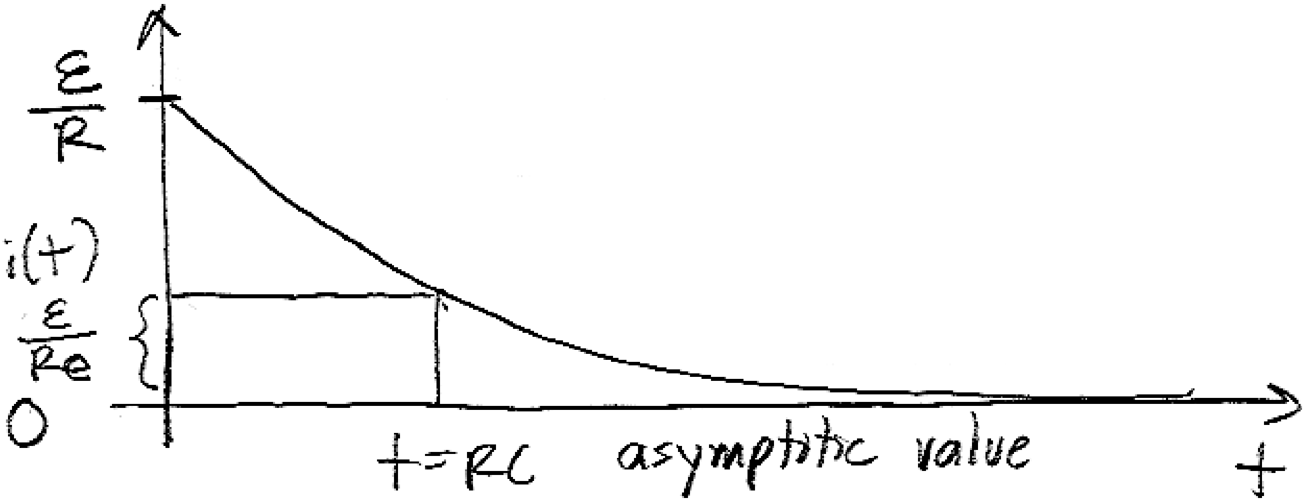
$$q(t=RC) = C\mathcal{E}(1-e^{-RC/RC}) = C\mathcal{E}(1-e^{-1})$$
$$= C\mathcal{E}\left(1-\frac{1}{e}\right)$$

$$i(t) = \frac{dq}{dt} = \frac{d}{dt}\left(C\varepsilon\left(1 - e^{-(t/RC)}\right)\right)$$

$$= -C\varepsilon e^{-t/RC} \times \left(-\frac{1}{RC}\right) = \frac{\varepsilon}{R} e^{-t/RC}$$

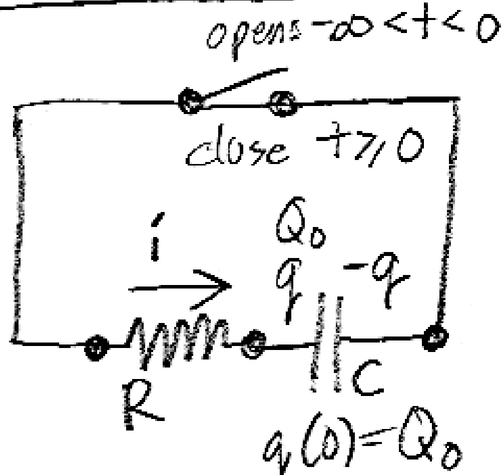
$$\boxed{i(t) = \frac{\varepsilon}{R} e^{-t/RC}}$$

$$i(0) = \frac{\varepsilon}{R} \qquad i(\infty) = 0 \qquad i(RC) = \frac{\varepsilon}{Re}$$



$\tau = RC$ is the "time constant" of the circuit.

Discharge



$$-iR - \frac{q}{C} = 0$$

$$\frac{dq'}{dt'} = -\frac{q'}{RC}$$

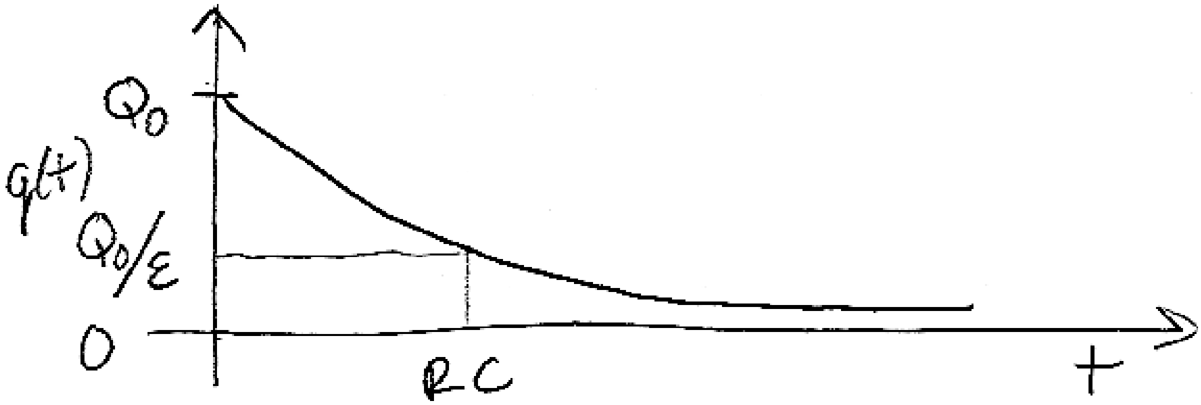
$$\frac{dq'}{q'} = -\frac{dt'}{RC}$$

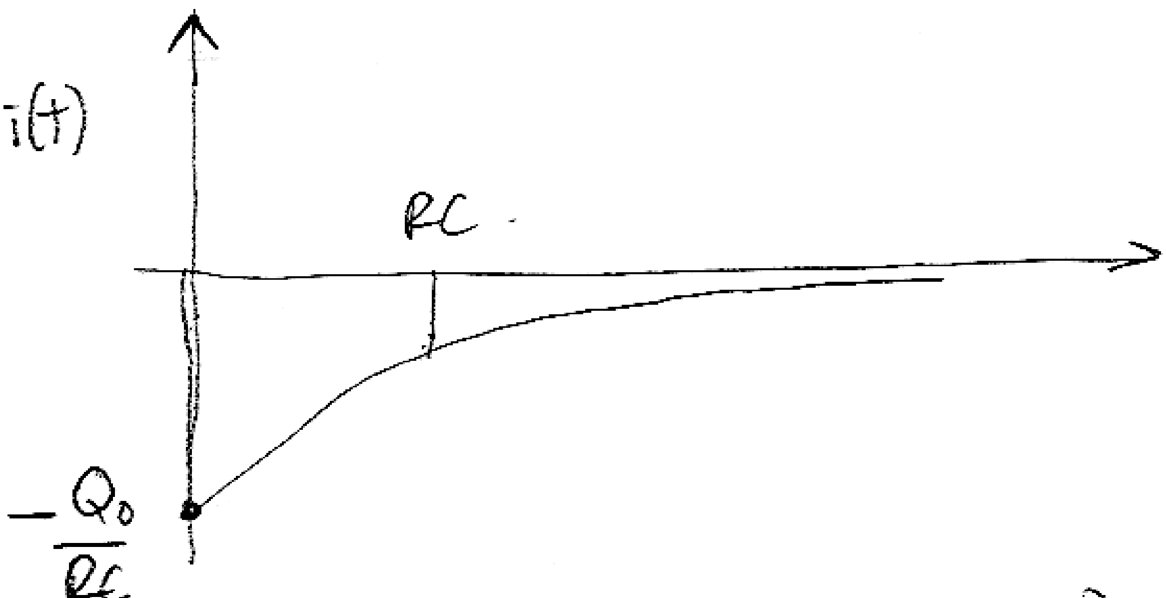
$$\int_{Q_0}^{q} \frac{dq'}{q'} = -\frac{1}{RC}\int_0^t dt'$$

$$\ln \frac{q}{Q_0} = -\frac{t}{RC}$$

$$q(t) = Q_0 e^{-t/RC}$$

$$\boxed{i(t) = \frac{dq}{dt} = -\frac{Q_0}{RC} e^{-t/RC}}$$





<u>Energy</u> (Charging up capacitor)

Power Dissipated on Resistor: (charging up)

$$= i^2(t) \cdot R = \frac{\varepsilon^2}{R^2} \cdot R \cdot e^{-2t/RC}$$

Energy Lost:

$$\int_0^\infty \left(\frac{\mathcal{E}^2}{R}\right) e^{\frac{-2t}{RC}}\,dt = \frac{\mathcal{E}^2}{R}\left(-\frac{RC}{2}\right)\left(e^{-\frac{2t}{RC}}\Big|_0^\infty\right)$$

$$= \frac{1}{2} C\mathcal{E}^2 = \frac{1}{2}\underbrace{(C\mathcal{E})}_{\text{Q on capacitor}}\cdot\mathcal{E}$$

Energy Stored on Capacitor

$$= \frac{1}{2} QV = \frac{1}{2}(C\mathcal{E})\,\mathcal{E}$$

Energy Provided by Battery

$$= \underbrace{\frac{1}{2}(C\mathcal{E})\,\mathcal{E}}_{\text{heats battery}} + \underbrace{\frac{1}{2}(C\mathcal{E})\,\mathcal{E}}_{\text{stored on capacitor}}$$

$$= C\mathcal{E}^2 \quad \text{total}$$