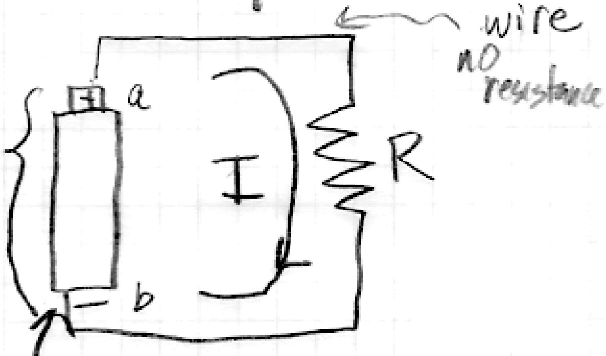
"Electromotive Force"

Force that moves charge in the "wrong" direction, that is, from low voltage to high voltage, is electromotive force

⇒ battery is best example.

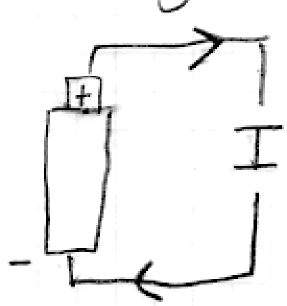
$V_{ab} = \mathcal{E}$ (ideal)

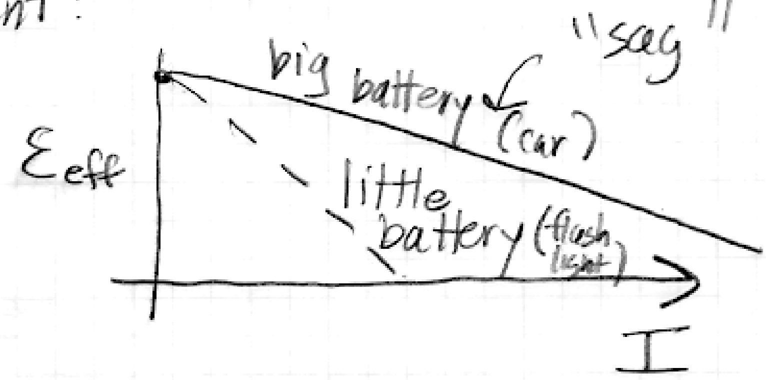


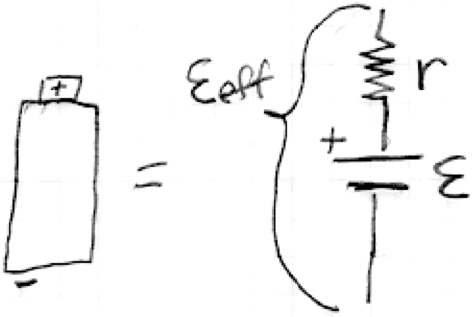
$$I = \frac{V_{ab}}{R} = \frac{\mathcal{E}}{R}$$
(ideal)

Chemistry inside pushes charge back up to the higher potential

Battery voltage "sags" as it pumps out increasing current:







← "internal resistance" makes

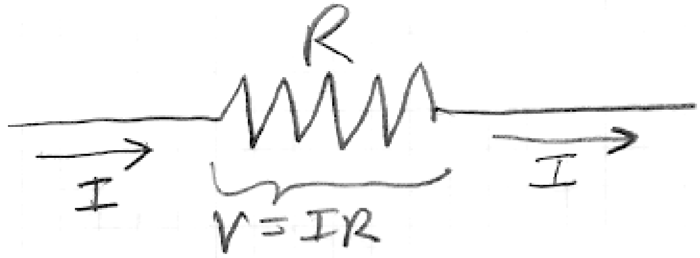
$$\mathcal{E}_{eff} = \mathcal{E} - Ir$$

r = larger: littler battery

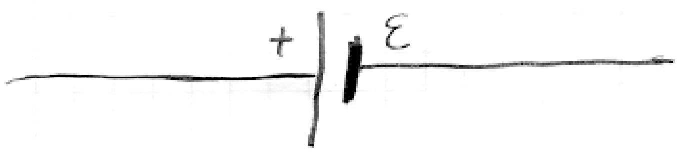
= smaller: bigger battery

Circuit Diagrams... symbols

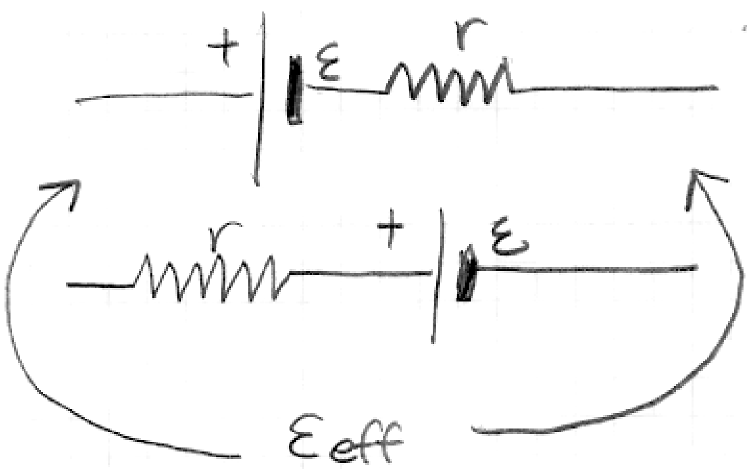
wire, negligible resistance



wires leading to resistor



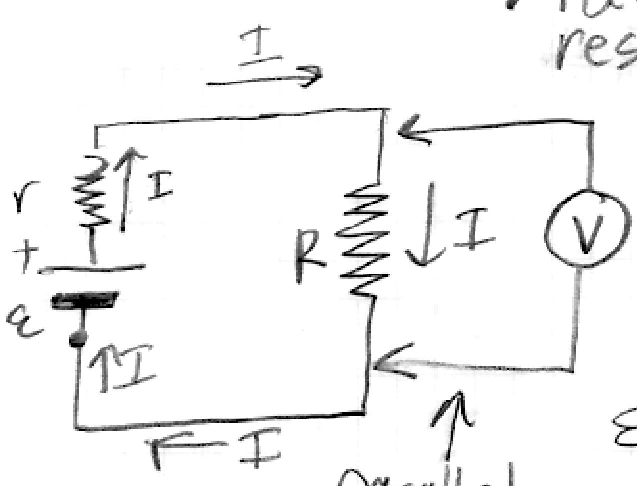
ideal "battery"



real batteries

$\varepsilon_{eff} = \varepsilon - Ir$

Voltmeters :
- measure voltage
- ideally, itself has infinite resistance
- put in parallel



(Symbol)

Volt meter, no current goes into it.

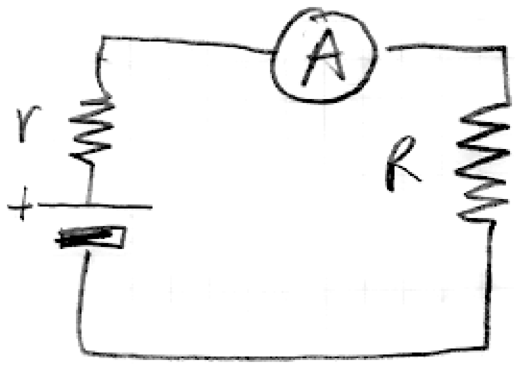
measures here

parallel

$\varepsilon_{eff} = \varepsilon - Ir$,

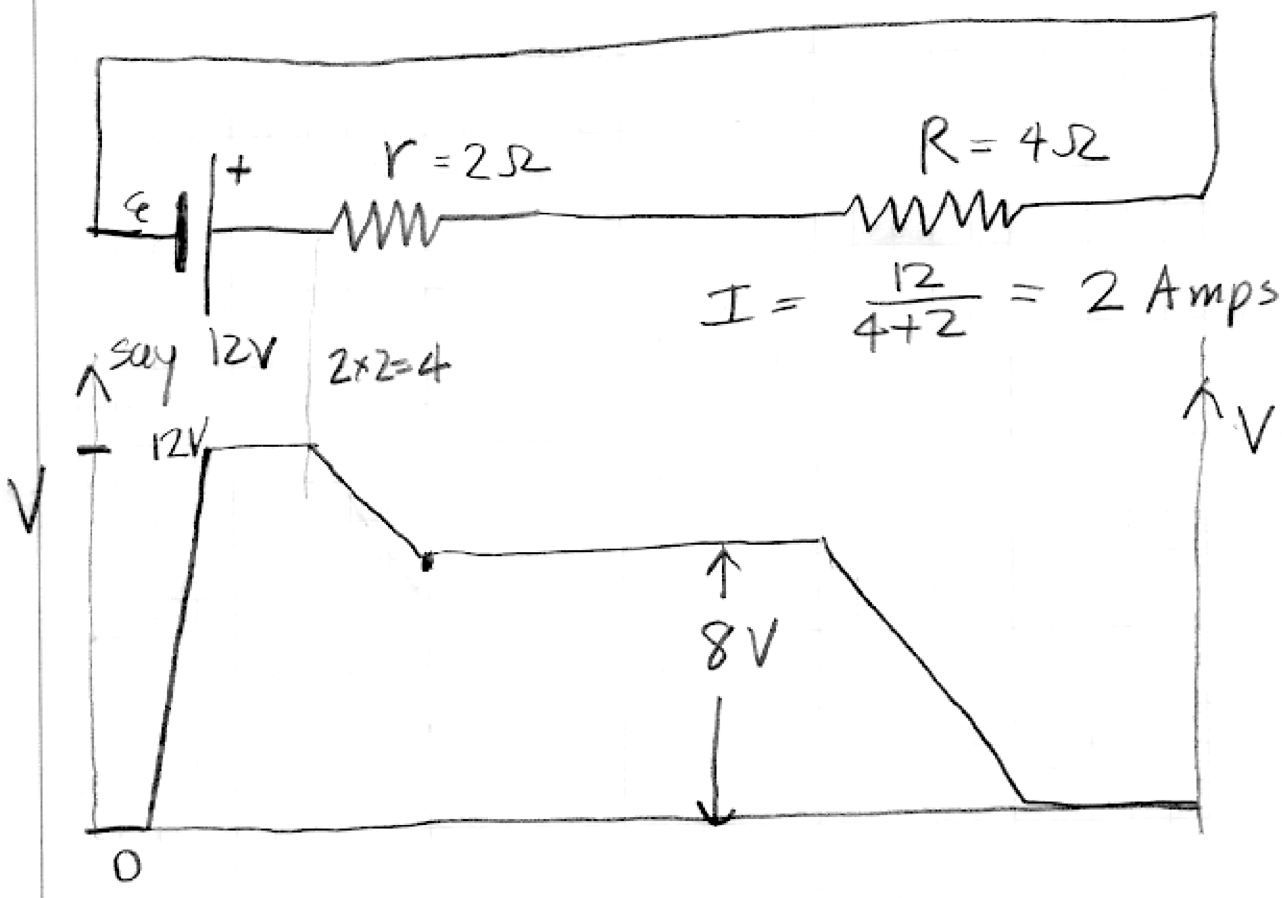
$= IR \Rightarrow I = \frac{\varepsilon}{r+R}$

Ammeters :
- measure current
- ideally, itself has 0 resistance
- put in series



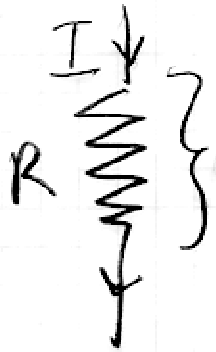
no new resistance.
allows measurement of current

Nice Visualization :

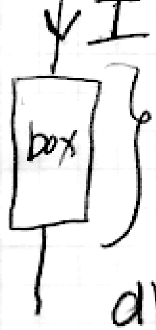


$I = \frac{12}{4+2} = 2$ Amps

Energy & Power



V or more generally



V

$$\frac{dQ}{dt} = I$$

$$dQ = I\,dt$$

$$dW = V\,dQ = IV\,dt$$

$$\text{power} = \frac{dW}{dt} = IV \quad \text{(important)}$$

Resistor: $V = IR$

$$P = I^2 R = \frac{V^2}{R}$$

Problem 26-59

Wire Gauge	diameter (cm)	I_{max} (A)	$\left(\frac{I^2}{d^3}\right)$
14	0.163	18	$\equiv 1$
10	0.259	30	0.69
8	0.326	40	0.62
6	0.412	60	0.69
4	0.519	85	0.69

(a) look at power

$$I^2R \propto R = \rho \cdot \frac{L}{A} = \rho \cdot \frac{L}{\pi r^2} = \frac{4\rho}{\pi} \frac{L}{d^2}$$

this will heat wire; wire will cool too, in proportion to the area of the outer surface of the wire, $A_{outer} = (2\pi r) \cdot L = \pi d \cdot L$

$$\frac{\text{Power}}{\text{outer area}} = \frac{I^2 R}{A_{outer}} = \frac{4\rho}{\pi}\frac{L}{d^2}\frac{I^2}{\pi d L}$$

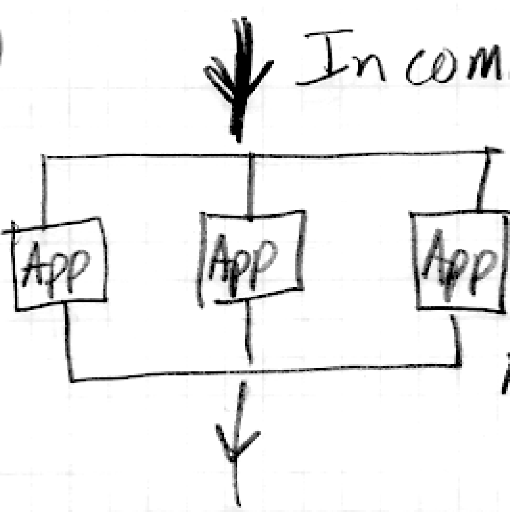
$$\frac{\text{Power}}{\text{outer area}} = \frac{4\rho}{\pi^2}\cdot\frac{I^2}{d^3}$$

$\frac{I^2}{d^3} \Rightarrow$ wire gauge 14 $\quad \left(\frac{I^2}{d^3}\right)_{14} = \frac{18^2}{0.163^3} = 74814\ \frac{A^2}{cm^3}$

others, normalize to this value:

$$\frac{\left(\frac{I^2}{d^3}\right)_{10}}{\left(\frac{I^2}{d^3}\right)_{14}} = \frac{51802}{74814} = 0.69$$

(b) Incoming Current



$\Big\}$ V = 120 Volts

4200 Watts = IV

$$I = \frac{4200}{120} = 35\ A$$

$\boxed{\text{Gauge 8-wire}}$

(c) $L = 42\,m$

$$R = \frac{\rho \cdot L}{A} = \frac{\rho \cdot L}{\pi (D/2)^2}$$

$$= (1.7 \cdot 10^{-8}\,\Omega \cdot m) \cdot \frac{42\,m}{\pi \left(\frac{0.326 \cdot 10^{-2}\,m}{2}\right)^2}$$

$$R = 8.6 \cdot 10^{-2}\,\Omega$$

$$\boxed{P = I^2 \cdot R = 35^2 \cdot 8.6 \cdot 10^{-2} = 106 \text{ Watts}}$$

(d) two ways: (i) compute R'

$$R' = \frac{\rho \cdot L}{\pi (D'/2)} = 5.4 \cdot 10^{-2}\,\Omega$$

$$P' = I^2 \cdot R' = 35^2 \cdot 5.4 \cdot 10^{-2} = 66 \text{ Watts}$$

(ii) $$\frac{P'}{P} = \left(\frac{D}{D'}\right)^2 = \left(\frac{0.326}{0.412}\right)^2 = 0.626$$

$$P' = 0.626 \cdot P = 0.626 \times 106$$
$$= 66 \text{ Watts}$$

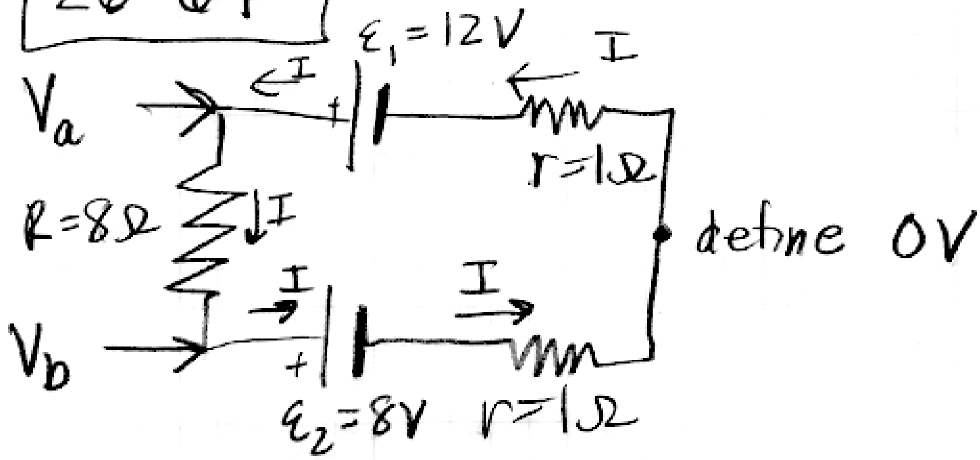
$$\Delta E = \Delta P \cdot t = (P - P') \cdot t$$

$$t = 12 \frac{\text{hours}}{\text{day}} \cdot 365 \frac{\text{days}}{\text{year}} = 4380 \frac{\text{hours}}{\text{year}}$$

$$\Delta E = \frac{(106 - 66) \cdot 4380}{1000} \text{ kw-h} = 172 \text{ kw-h}$$

$$\boxed{\$ = \Delta E \cdot (\text{Price/kwh}) = 172 \cdot 0.11 = \$18.8}$$

26-61



(a) $V_a = -r \cdot I + \varepsilon_1$

$V_b = +r \cdot I + \varepsilon_2$

$$I = \frac{V_a - V_b}{R} = \frac{-rI + \varepsilon_1 - rI - \varepsilon_2}{R}$$

$$\left(1 + 2\frac{r}{R}\right) I = \frac{\varepsilon_1 - \varepsilon_2}{R}$$

$$I = \frac{\varepsilon_1 - \varepsilon_2}{R\left(1 + \frac{2r}{R}\right)} = \frac{\varepsilon_1 - \varepsilon_2}{R + 2r}$$

$$\boxed{I = \frac{12 - 8}{8 + 2 \cdot 1} = \frac{4}{10} = 0.4\,A}$$

(b) $P_8 = I^2 \cdot R = 0.4^2 \cdot 8 = 1.28\,W$

$P_{batt} = I^2 \cdot r = 0.4^2 \cdot 1 = 0.16\,W$ (2 of these)

Total dissipation $= 1.28 + 2 \cdot 0.16$

$= 1.6\,W$

(c) #1 pumps out, $P_1 = I\mathcal{E}_1$
(big voltage)

$$= 0.4 \cdot 12 = 4.8\,W$$

(d) #2 gets pumped up, $P_2 = -I\mathcal{E}_2$

$$= -0.4 \cdot 8 = -3.2\,W$$

(e) (#1 out) $= 4.8\,W$

(#2) $= -3.2\,W$ (– "lost")

dissipation $= -1.6\,W$ (– "lost")

$0\,W$ energy conserved