Chapter 26

1. Current
   a. Definition
   b. +/- 
   c. real materials
   d. Current Density

2. Resistivity (Intrinsic)

3. Resistance (Geometric)

4. Batteries... EMF, internal resistance

5. Energy + Power

6. Theory of resistivity (or conductance)

7. Physiology

Up until now: static (not moving in time) charge configurations

Now: let charge move

Conceptually easiest:

- area of endcap
- $\sigma$ = charge density

in time $dt$,

$\frac{dQ}{dt} = \sigma A v A$

bunch of + charges, uniform distribution, all with same velo
\( \frac{dQ}{dt} \), the charge/time, is the current.

Units: \( \frac{\text{Coulombs}}{\text{sec}} = \text{Ampere} \)

In case above, sign of current defined as positive. What about...

\[ S_q < 0 \]

\[ I \]

\[ \frac{dQ}{dt} = (\text{vdt})A S_q < 0 \]
\[ \frac{dQ}{dt} = S_q VA < 0 \]

Sign flip \( \leftrightarrow \) direction flip

(this doesn't happen with fluid flow!)

Can ever have

\[ S_q = \text{net charge density} = 0 \]
\[ = \sigma + q + \sigma - q = 0 \]
\[ \sigma + q = -\sigma - q \]

Net current \( \neq 0 \):
\[ \frac{dQ}{dt} = A (\sigma + q \cdot V + \sigma - q (-V)) = 2AV \sigma \]
In real materials, charge does not get to go in straight lines (not very often, anyway). In real materials, often electrons do the moving, atoms stay still, and current looks like this:

\[ \Rightarrow I \]

\[ \Rightarrow E \]

Actually, this path far less random than real path. Further, the path above is the result of an applied $E$ field. In absence of an $E$ field, $\oplus$ wanders with no direction without drift. $E$ applied, each trajectory bent a little. Cumulative effect is drift in direction of $qE$.

$V_{\text{drift}} \ll V$ (instantaneous).
letting \( \rho = \frac{nq}{V} \) charge \( \frac{\text{charge}}{\text{volume}} \) \( \langle \text{charge carriers} \rangle \) \( \frac{\text{charge of each carrier}}{\text{volume}} \) usually \( -e \)

then \( I = \frac{dQ}{dt} = nqV_a \) \( A \) \( \text{depends on size} \)

"extrinsic" \( \text{current density} \)

"intrinsic" \( \frac{I}{A} = nqV_a = J \) \( \uparrow \) \( \text{component of } J \)

\( \vec{J} = nqV \) \( \text{vector current density} \)

(note) \( I \) never a vector, it is \( I = \int \text{d}A \hat{\mathbf{n}} \cdot \vec{J} \equiv \text{current "flux"} \)

\( \text{Surface} \)

\( \text{Resistivity} + \text{Resistance} \)

"extrinsic" depends on geometry.

\( \rho \left\langle \frac{\vec{J}}{\rho} \right\rangle \vec{E} \) \( \rho \) \( \text{large}, \) "high resistivity"

in \( \text{material} \)

\( \rho \) \( \text{small}, \)
Units:
\[
\frac{\text{Amps}}{\text{meter}^2} = \left( \frac{1}{\rho} \right) \left( \frac{\text{Volts}}{\text{meter}} \right)
\]

\[\{\rho\} = (\text{meter}) \cdot \left( \frac{\text{Volts}}{\text{Amp}} \right)\]

a.k.a. ohm

\[\{\rho\} = \text{ohm} \cdot \text{meters}\]

- **Silver**: \(\rho \sim 1.5 \times 10^{-8} \text{ ohm} \cdot \text{meter}\.\)
- **Copper**: \(\rho \sim 1.7 \times 10^{-8} \)
- **Mercury**: \(\rho \sim 95 \times 10^{-8} \)

Conductor: \(\sim (1-100) \times 10^{-8} \text{ ohm} \cdot \text{meter}\). "Free" electrons; the charge carrier.

- **Pure Carbon**: \(\rho \sim 4 \times 10^{-5} \text{ ohm} \cdot \text{m}\.\)
- **Silicon**: \(~ 2 \times 10^3 \text{ ohm} \cdot \text{m}\.\)
- **Germanium**: \(~ 0.6 \text{ ohm} \cdot \text{m}\.\)

Semiconductors: Electrons can "get" free, but are usually bound; when they get free they leave a divot or "hole".

\[\text{neutral}\]

\[\text{electron with ~ little excitation}\]

\[\text{hole} \rightarrow \text{can move too}\]
**Insulators**

\[ P : 10^8 \rightarrow 10^{18} \text{ J/m} \]

(26 orders of magnitude separate best conductors from best insulators)

**P is dependent on temperature**

**Conductors**

\[ P = P_0 \]

\[ \text{slope} = \alpha \cdot P_0 \]

\[ T \]

\[ T_0 \]

So

\[ P(T) = P_0 + (T - T_0) \cdot \alpha \cdot P_0 \]

\[ P(T) = P_0 (1 + \alpha (T - T_0)) \]

\[ \alpha : -5 \cdot 10^{-4} \text{ up to } 5 \cdot 10^{-3} \text{ }^1/\text{C} \]

**Semiconductors**

\[ P \]

\[ T \]

\[ T_{\text{runaway}} \]

**Superconductors**

\[ P \]

\[ T \]

really 0! \rightarrow \[ T_c \]
Resistivity + Geometry = Resistance

Example #1

\[ \text{resistivity } \rho \]

\[ A \rightarrow \rightarrow \rightarrow E \]

\[ \Rightarrow J \]

\[ V = EL \]

\[ J = \frac{1}{\rho} E \]

\[ I = JA = \frac{A}{\rho} \]

\[ V = \left[ \rho \cdot \frac{L}{A} \right] \cdot I = I \times \left[ \rho \cdot \frac{L}{A} \right] \]

\[ V = IR \]

\[ R = \rho \cdot \frac{L}{A} \]

\[ \text{dimensions } \left[ R \right] = \left[ \frac{V}{I} \right] \text{ volts per amperes} = \text{ohms} = \left[ \rho \cdot \frac{L}{A} \right] \]

\[ \Omega \cdot \text{m}, \frac{1}{\text{m}} \]
Example #2: Cylindrical

- Outer conductor radius $b$
- Region between $a$ and $b$ filled with resistive material
- Center conductor radius $a$

Slice thickness $dr$ shown contributes resistance:

$$dR = \rho \cdot \frac{\text{thickness}}{\text{Area} \perp \text{to J}}$$

$$dR = \rho \cdot \frac{dr}{L \times 2\pi r}$$

$$R_{\text{total}} = \int dR = \frac{\rho}{2\pi L} \int_a^b \frac{dr}{r} = \frac{\rho \ln(b/a)}{2\pi L}$$

$$R = \rho \times \frac{\ln(b/a)}{2\pi L}$$

$\rho$ - Resistivity

$R$ - Resistance

$2\pi L$ - Geometry