# Chapter 26

1. Current
   a. Definition
   b. +/−
   c. real materials
   d. Current Density.
2. Resistivity (Intrinsic)
3. Resistance (Geometric)
4. Batteries... EMF, internal resistance.
5. Energy + Power
6. Theory of resistivity (or conductance)
7. Physiology

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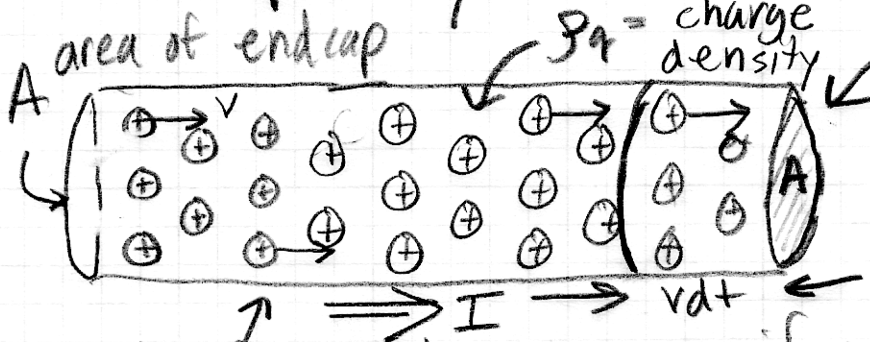
Up until now: <u>static</u> (not moving in time)

charge configurations

<u>NOW</u>: let charge <u>move</u>

Conceptually easiest:




in time dt, how

$$dQ = (v\,dt)A\rho_q$$

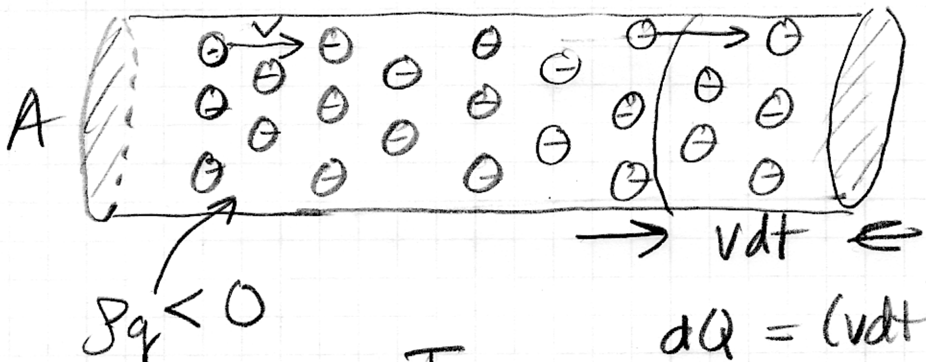
$$\boxed{\frac{dQ}{dt} = \rho_q v A}$$

bunch of + charges, uniform distribution, all with same velo

$\frac{dQ}{dt}$, the charge/time, is the current

units: $\frac{\text{Coulumbs}}{\text{sec}} \equiv$ Ampere

In case above, sign of current defined as positive. What about..




$dQ = (v\,dt) A \rho_q < 0$

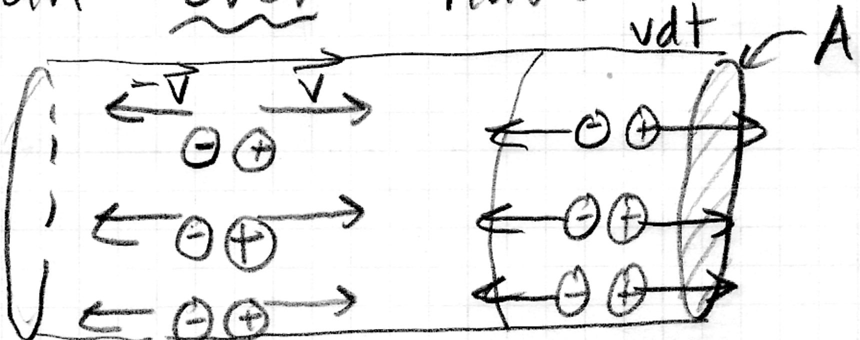
$\frac{dQ}{dt} = \rho_q v A < 0$

$I$ ⟸

sign flip ⟺ direction flip

(this doesn't happen with fluid flow!)

Can even have

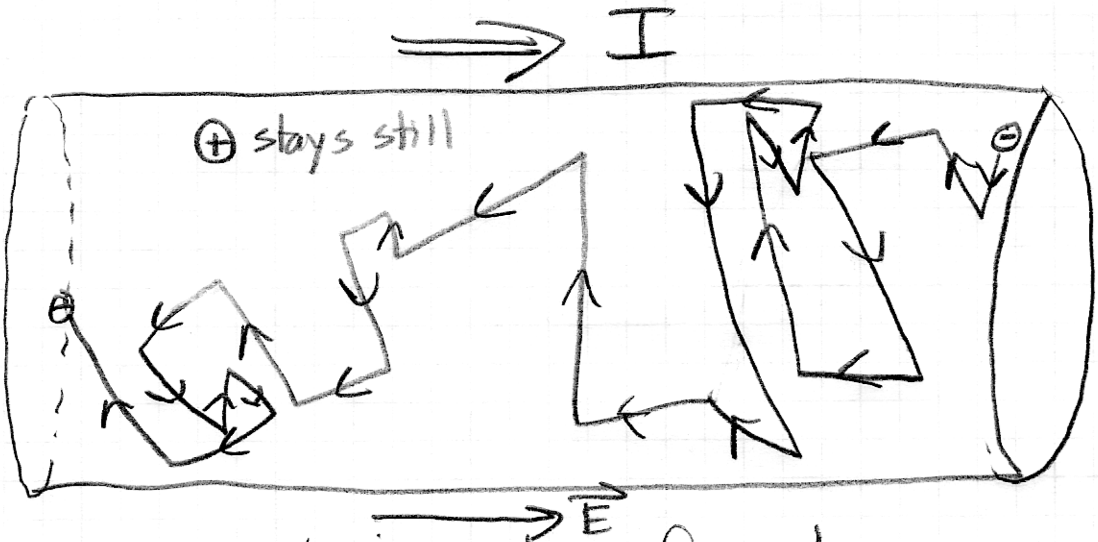



$\rho_q$ = net charge density $= 0$ !
$= \rho_{+q} + \rho_{-q} = 0 \qquad \rho_{+q} = -\rho_{-q}$

net current $\neq 0$ : $\frac{dQ}{dt} = A\left(\rho_{+q} \cdot v + \rho_{-q}(-v)\right) = 2Av\rho_{+q}$

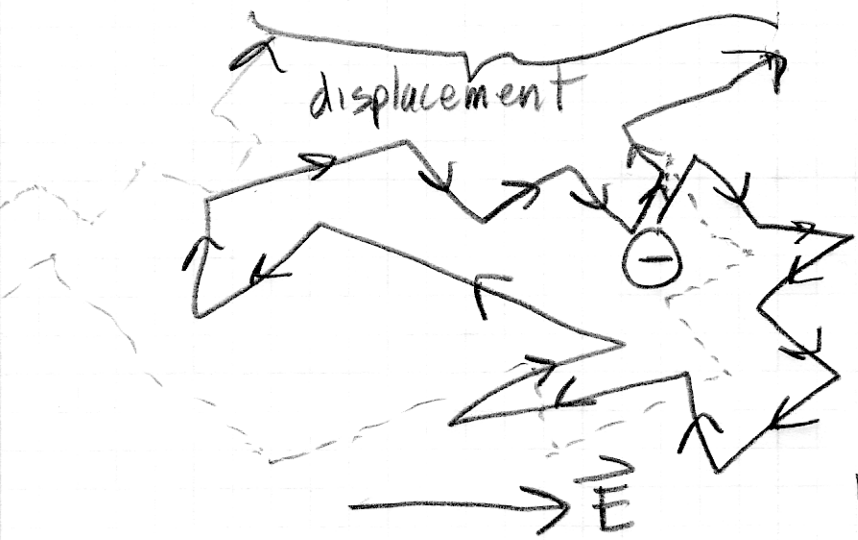
In real materials, charge does not get to go in straight lines (not very often, anyway)

In real materials, often electrons do the moving, atoms stay still, and current looks like this:




actually, this path far less random than real path. Further, the path above is the result of an applied $\vec{E}$ field

in absence of an $\vec{E}$ field, ⊖ wanders without direction




with non-zero $\vec{E}$ applied, each trajectory bent a little... cumulative effect is drift in direction of $q\vec{E}$

$V_{drift} \ll V$ (instantaneous)

letting $\rho_q = n\,q$

$\rho_q$: $\dfrac{\text{charge}}{\text{volume}}$

$n$: $\dfrac{\text{charge carriers}}{\text{volume}}$

$q$: $\underbrace{\text{charge of each carrier}}$ usually $-e$

then $I = \dfrac{dQ}{dt} = n q v_d \cdot A$ depends on size

"extrinsic"

current density

"intrinsic" $\dfrac{I}{A} = n q v_d = J$

$J$: component of $\vec{J}$

$\vec{J} \equiv n q \vec{v}$ vector current density

(note, $I$ never a vector, it is

$I = \int_{\text{surface}} dA\, \hat{n} \cdot \vec{J}$ ⟸ current "flux")

Resistivity & Resistance

$\underbrace{\text{Resistivity}}$: intrinsic characteristic of material

$\underbrace{\text{Resistance}}$: "extrinsic" depends on geometry.

$\rho$ $\vec{J} = \left(\dfrac{1}{\rho}\right) \vec{E}$ in material

$\rho$ large, $J$ small, "high resistivity" units: $\frac{\text{Amps}}{\text{meter}^2} = \left(\frac{1}{\rho}\right)\left(\frac{\text{Volts}}{\text{meter}}\right)$

$$[\rho] = (\text{meter}) \cdot \underbrace{\left(\frac{\text{Volts}}{\text{Amp}}\right)}_{\text{a.k.a. "ohm"}}$$

$$[\rho] = \text{ohm-meters}$$

silver : $\rho \sim 1.5 \cdot 10^{-8}$ ohm-meter

copper : $\rho \sim 1.7 \cdot 10^{-8}$ "

mercury : $\rho \sim 95 \cdot 10^{-8}$ "

(conductor : $\sim (1-100) \cdot 10^{-8}$ ohm-meter)

"free" electrons the charge carrier.

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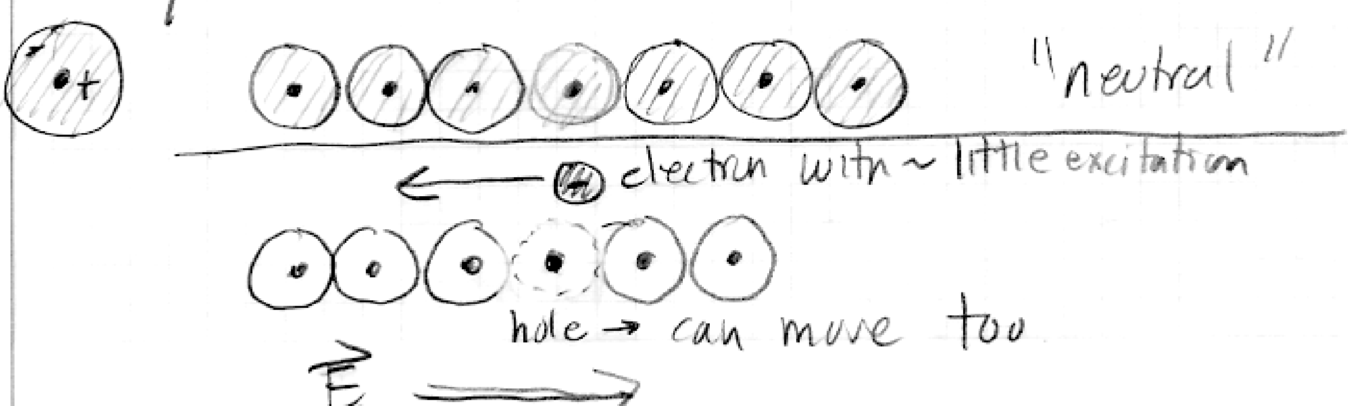
pure : Carbon $\rho \sim 4 \cdot 10^{-5}$ $\Omega$-m

"semiconductors" Silicon $\sim 2 \cdot 10^{3}$ $\Omega$-m

Germanium $\sim 0.6$ $\Omega$-m

electrons can "get" free, but are usually bound.. when they get free they leave a divot or "hole"

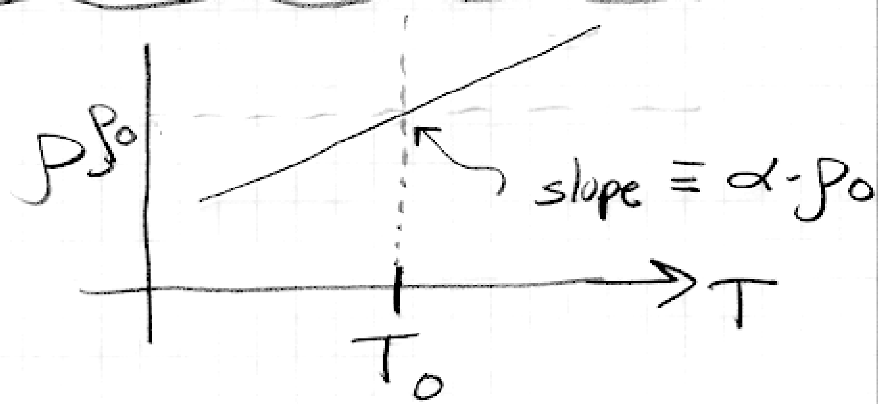


insulators $\rho : 10^{8} \to 10^{18}\ \Omega \cdot m$

(26 orders of magnitude separate best conductors from best insulators)

$\rho$ is dependent on temperature

conductors :

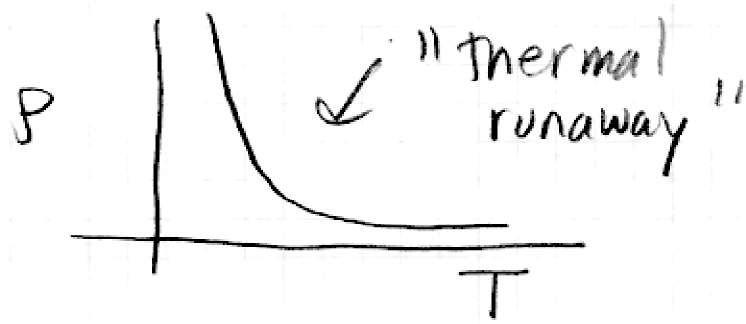



so $\rho(T) \simeq \rho_0 + (T - T_0) \times \alpha \rho_0$

$$\boxed{\rho(T) = \rho_0 (1 + \alpha (T - T_0))}$$

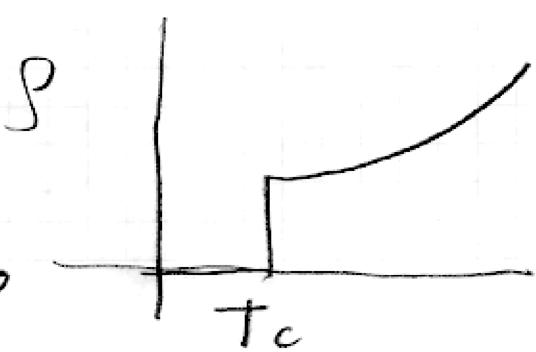
$\alpha : -5 \cdot 10^{-4}$ up to $5 \cdot 10^{-3}\ \frac{1}{{}^{\circ}C}$

semiconductors :



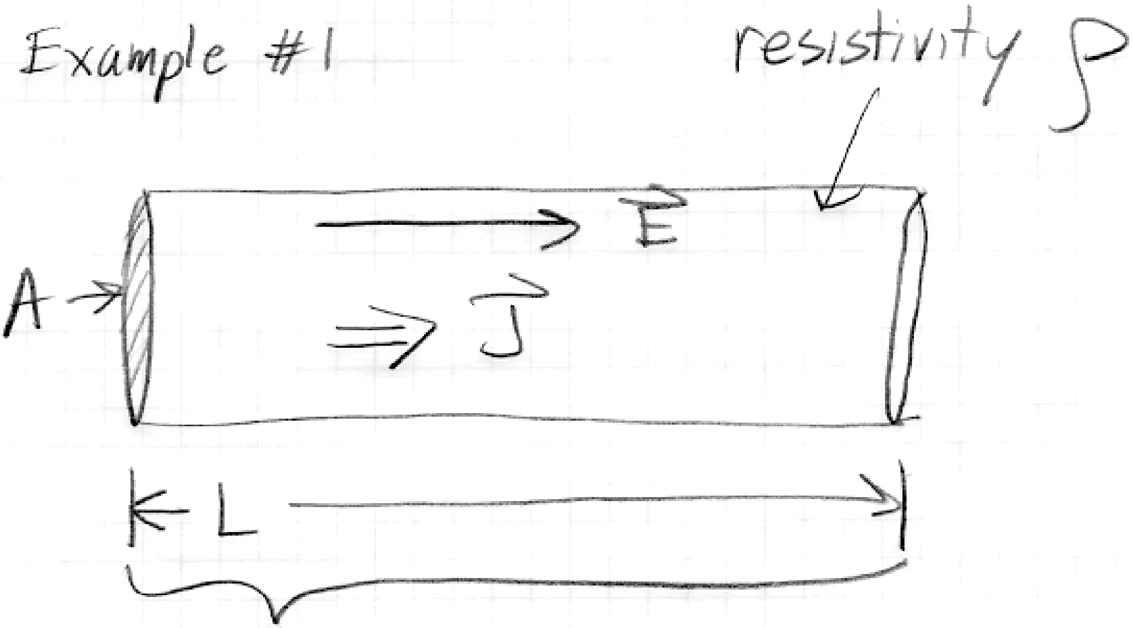

superconductors

really 0! →



# Resistivity + Geometry = Resistance

Example #1




$$V = EL$$

$$J = \frac{1}{\rho} E$$

$$I = J \cdot A = \frac{A}{\rho} E = \frac{A}{\rho L} V$$

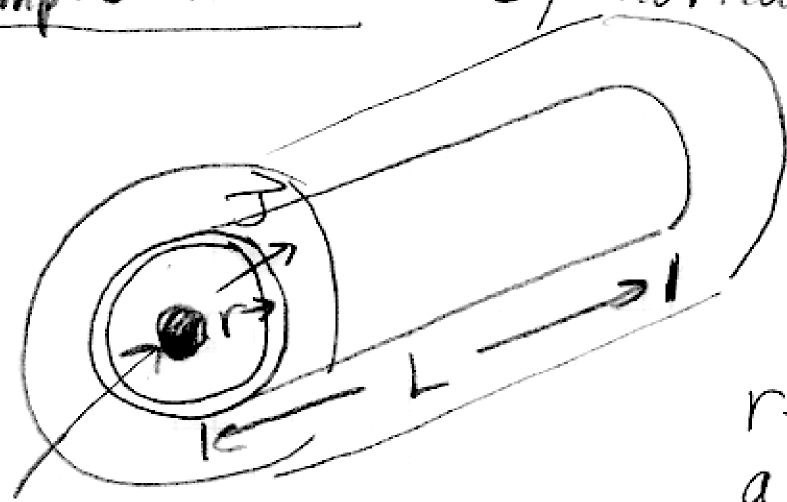
$$V = \left[\rho \cdot \frac{L}{A}\right] \cdot I = I \times \left[\rho \cdot \frac{L}{A}\right]$$

$$V = I \underbrace{R}_{\text{resistance}} \qquad \boxed{\underbrace{R}_{\text{resistance}} = \underbrace{\rho}_{\text{resistivity}} \cdot \underbrace{\frac{L}{A}}_{\text{geometry}}}$$

dimensions

$$[R] = \left[\frac{V}{I}\right] = \frac{\text{volts}}{\text{amperes}} = \text{ohms} = \left[\underbrace{\rho}_{\Omega \cdot m} \cdot \underbrace{\frac{L}{A}}_{1/m}\right]$$

## Example #2 : Cylindrical




outer conductor radius b

center conductor radius a

region between a + b filled with resistive material

slice thickness dr shown contributes resistance:

$$dR = \rho \cdot \frac{(\text{thickness})}{\text{Area} \perp \text{ to } \vec{J}}$$

$$dR = \rho \cdot \frac{dr}{L \times 2\pi r}$$

$$R_{total} = \int dR = \frac{\rho}{2\pi L} \int_a^b \frac{dr}{r} = \frac{\rho \ln(b/a)}{2\pi L}$$

$$R = \rho \times \frac{\ln(b/a)}{2\pi L}$$

resistance     resistivity     geometry.