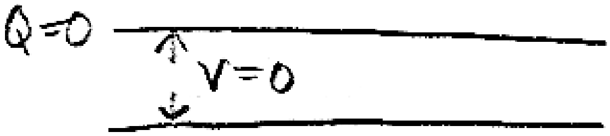
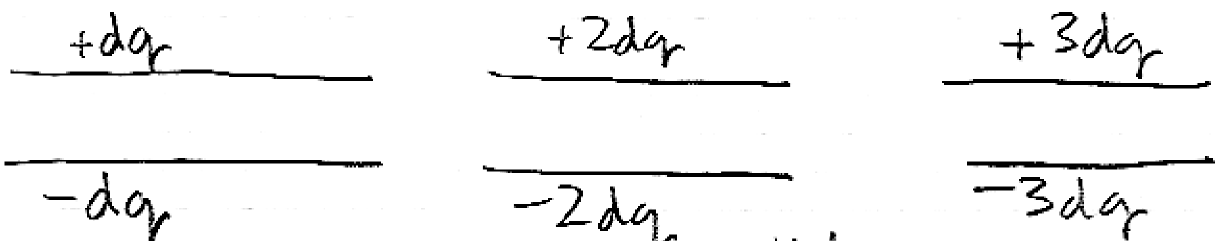
Energy in Capacitors



build up Q out of moving many small parcels of charge



use old voltage

$\Delta W = V dq$

$= 0 \cdot dq$

$= 0$

now some added work.

$\Delta W = \left(\frac{dq}{C}\right) dq$

$\Delta W = \left(\frac{2dq}{C}\right) dq$

or $\quad dW = V dq = \frac{1}{C} q\, dq$

$$W = \int dW = \frac{1}{C}\int_0^Q q\, dq = \frac{1}{2}\frac{Q^2}{C}$$

$$\boxed{U_{cap} = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2}$$

quadratic!

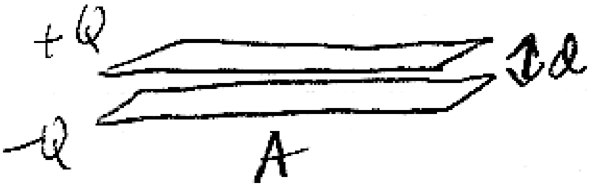
Parallel Plates: $\quad C = \varepsilon_0 \frac{A}{d} \qquad V = Ed$

$$U_{cap} = \frac{1}{2}\varepsilon_0 \frac{A}{d} E^2 d^2 = \frac{1}{2}\varepsilon_0 E^2 \times \underbrace{(Ad)}_{\text{volume}}$$

$$u = \frac{U_{cap}}{\text{Volume}} = \frac{U_{cap}}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$

field energy

Extra: look at forces; direction of force depends on whether plates held at constant charge or voltage.



constant charge:

$$U_{cap} = \frac{1}{2}\frac{Q^2}{C}$$

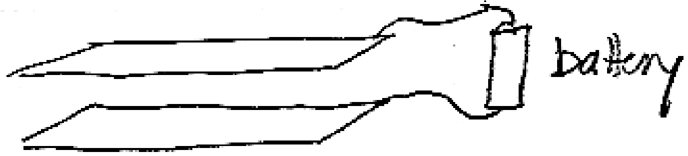
$$= \frac{1}{2\varepsilon_0}\frac{d}{A}Q^2$$

$$= -\frac{U_{cap}}{d}$$

$$F = -\frac{\partial U_{cap}}{\partial d} = -\frac{Q^2}{2\varepsilon_0 A} = -\underbrace{\frac{\sigma}{2\varepsilon_0}}_{\text{field of other}} \times \underbrace{\sigma \cdot A}_{\text{total charge}}$$

$\boxed{\text{attraction}}$

Constant VOLTAGE



$$U_{cap} = \frac{1}{2}CV^2$$

$$= \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2$$

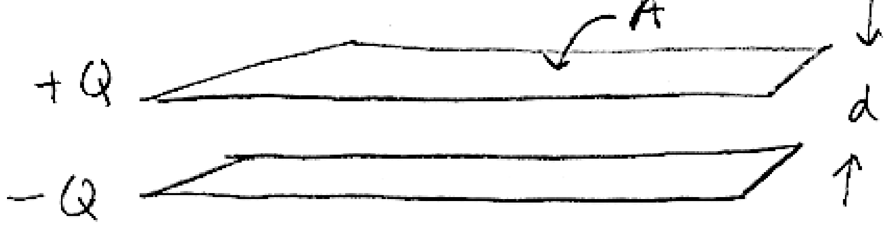
$$F = -\frac{\partial U_{cap}}{\partial d} = +\frac{1}{2}\frac{\varepsilon_0 A}{d^2}\cdot V^2$$

$$F = +\frac{1}{2}\frac{1}{d}CV^2 = +\frac{U_{cap}}{d}$$

$\boxed{\text{repulsive, same magnitude.}}$

Dielectrics

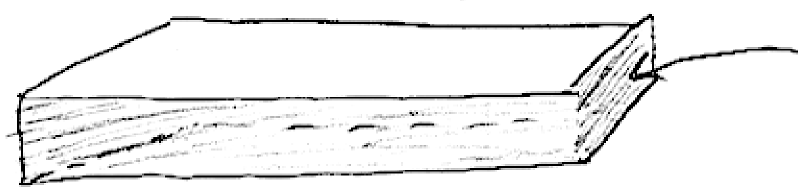
<u>Idea</u>: given a pair of conductors that comprise a capacitor, insert stuff into the "gap" and (greatly) increase the capacitance!



$$C_0 = \varepsilon_0 \frac{A}{d}$$

no stuff

insert insolating "stuff"....



dielectric insolator

$C > C_0$

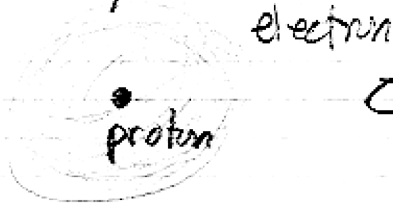
$\frac{C}{C_0} \equiv K$

K:
- 1.00059 air
- 2.1 Teflon
- 80 $\underbrace{\text{water}}$ slightly conductive, purity ↑ resistance ↑
- 310 Strontium Titanate.
- $> 10^4$ similar.

The physics is <u>polarization</u>

Some molecules → "symmetric"

Hydrogen Atom
(also noble gases)



← no "prefered" direction.

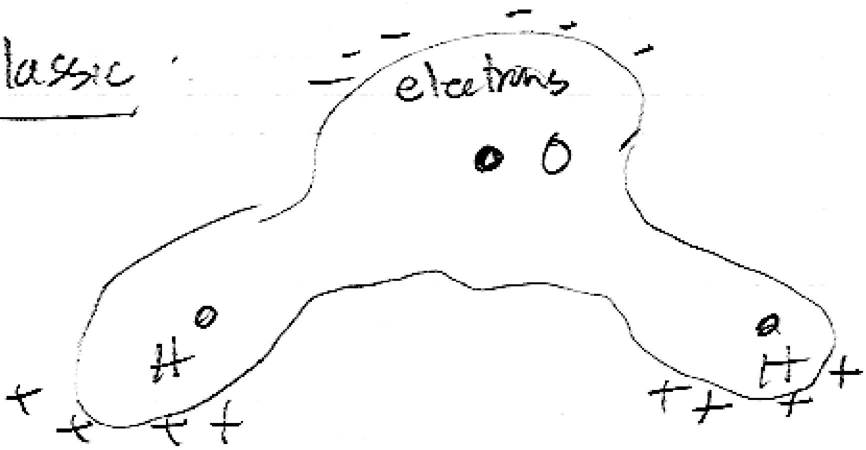
<u>Diatomics</u>:

← "natural" axes
still lots of <u>symmetry</u>

(≈ non "polar," although a bit)

<u>Polar Molecules</u>: Assymmetry.

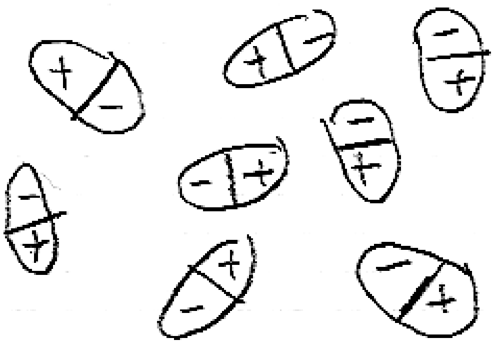
<u>H_2O Classic</u>.



<u>abstraction</u>:

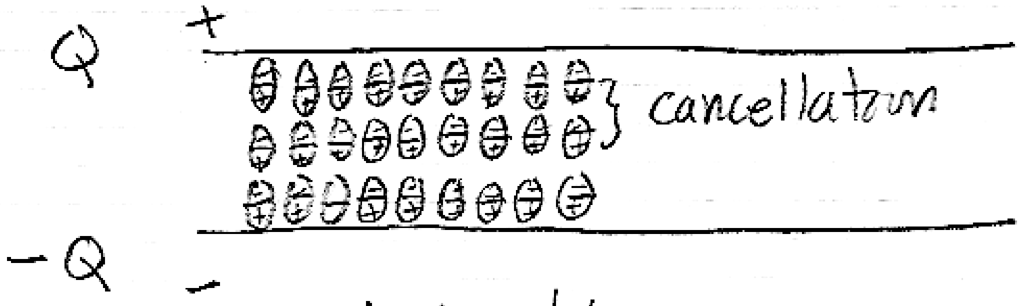
(≈ all non-symmetric molecules)

<u>Usually</u>

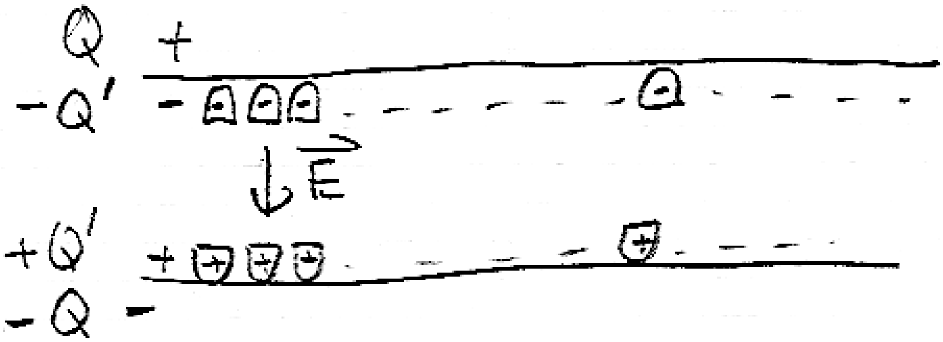


← <u>random orientation</u>

pot in capacitor



looks like ...



Qualitatively $\quad |Q - Q'| < |Q|$

$$|\vec{E}| \propto |Q - Q'|$$

now link to what you know

$$V = Ed = \frac{(Q - Q')}{\varepsilon_0}\frac{d}{A} \quad \left(= \frac{\sigma_{eff}\, d}{\varepsilon_0 A}\right)$$

recall, $\quad Q = CV = KC_0V$

$$V = \frac{Q}{KC_0} = \frac{1}{K}Q \cdot \frac{\cancel{d}}{\varepsilon_0 \cancel{A}} = \frac{Q - Q'}{\cancel{\varepsilon_0}}\frac{\cancel{d}}{\cancel{A}}$$

$$\boxed{\frac{1}{K}Q = Q - Q'}$$

K big, $Q - Q'$ small.

<u>so</u>: $V = Ed = \frac{Q}{K}\frac{d}{\varepsilon_0 A}$ $\left(\text{so, } C = K\frac{\varepsilon_0 A}{d}\right)$

but $KE = \frac{Q}{\varepsilon_0 A} = \left\{\frac{\sigma}{\varepsilon_0} \leftarrow \text{gauss's law}\right\}$.

Generalizes ...

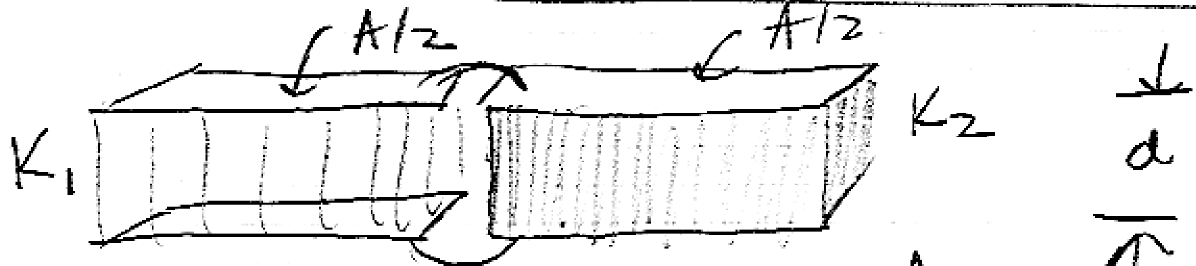
$$\boxed{\begin{array}{l} \text{so} \\ K \cdot \left(\overset{\text{Electric}}{\text{flux}}\right) = \frac{Q_{\text{enclosed, free}}}{\varepsilon_0} \\ \text{or} \quad K \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed, free}}}{\varepsilon_0} \end{array}}$$

"dielectric" Gauss's law.

$$\vec{D} = K\vec{E}$$

"Electric Displacement

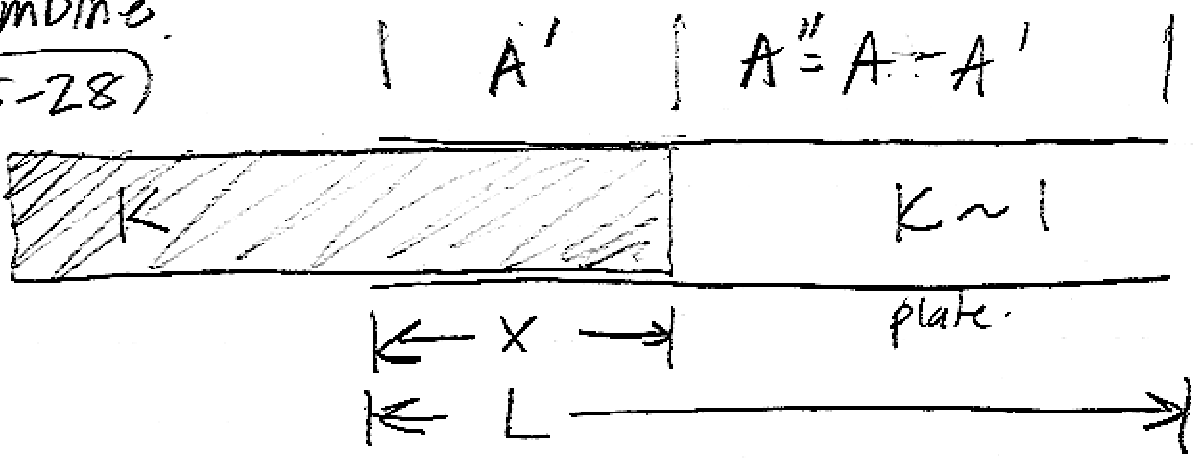
field.



$C_1 = K_1 \frac{A}{\varepsilon_0 2d}$ $C_2 = K_2 \frac{A}{\varepsilon_0 2d}$

$C = C_1 + C_2 = \frac{A}{\varepsilon_0 2d}(K_1 + K_2)$.

Combine
(25-28)

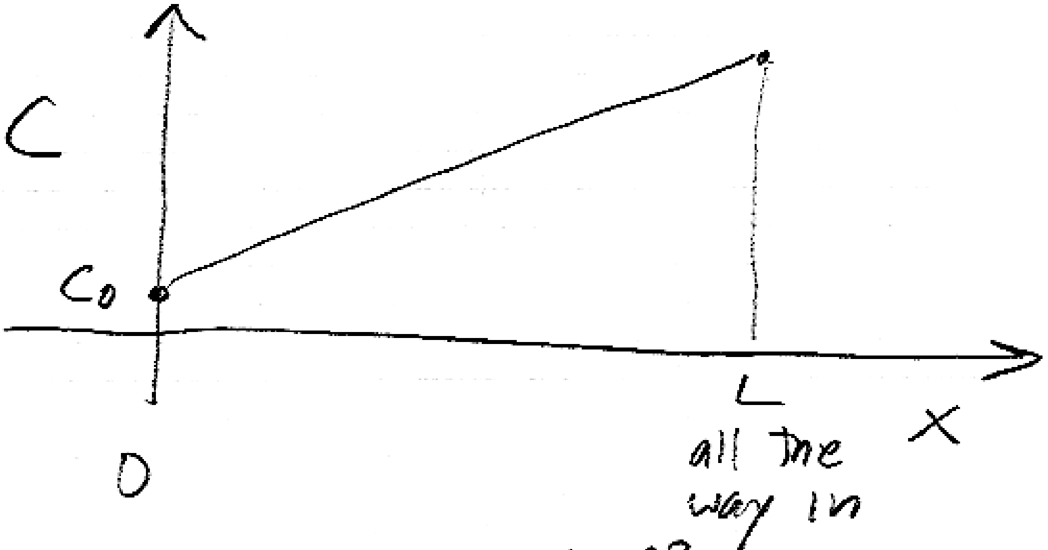


$$A' \propto x = xL$$

$$C' = K \frac{xL}{\varepsilon_0 d} \qquad C'' = 1 \cdot \frac{(L^2 - xL)}{\varepsilon_0 d}$$

$$C = C' + C'' = \frac{L^2}{\varepsilon_0 d}\left[K \cdot \frac{x}{L} + \left(1 - \frac{x}{L}\right)\right]$$

$$C = C_0\left(1 + \frac{x}{L}(K-1)\right)$$



(1) Q constant: $U = \frac{1}{2}\frac{Q^2}{C}$ ← minimized $x = L$
dielectric sucked in.

(2) V constant: $U = \frac{1}{2}CV^2$ ← minimized $x = 0$