Answer to Chapter Opening Question

From Eq. (18.19), the root-mean-square speed of a gas molecule is proportional to the square root of the absolute temperature T. The temperature range we’re considering is from (−40 + 273.15) K = 233 K to (40 + 273.15) K = 313 K. Hence the speeds increase by a factor of $\sqrt{(313 \text{ K})/(233 \text{ K})} = 1.16$; that is, there is a 16% increase. While 40°C feels far warmer than −40°C, the difference in molecular speeds is relatively minor.

Answers to Test Your Understanding Questions

Section 18.1 If there were no holes in the crust, the steam emanating from the filling as it bakes would be confined to a fixed volume V. The amount of steam n and the temperature T both increase as the pie bakes, so the pressure $p = nRT/V$ increases as well, until the crust bursts. Cutting holes in the crust allows steam to escape and the pressure to remain within limits.

Section 18.2 The value of $v_0$ determines the equilibrium separation of the molecules in the solid phase, so doubling $v_0$ means that the separation doubles as well. Hence a solid cube of this compound might grow from 1 cm on a side to 2 cm on a side. The volume would then be $2^3 = 8$ times larger, and the density (mass divided by volume) would be $\frac{1}{8}$ as great.

Section 18.3 The temperature of the corona is 2,000,000°C, or to two significant figures $2 \times 10^6 \text{ K}$ (the difference between Celsius and Kelvin temperatures is irrelevant). Hydrogen has a molar mass (from Appendix D) of $M = 1.008 \text{ g/mol} = 1.008 \times 10^{-3} \text{ kg/mol}$, so

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.314 \text{ J/mol} \cdot \text{K})(2.0 \times 10^6 \text{ K})}{1.008 \times 10^{-3} \text{ kg/mol}}} = 1.8 \times 10^3 \text{ m/s}$$

This is less than the escape speed, so not all of the hydrogen in the corona can escape into space. But remember that not all the atoms move at the rms speed: some are moving fast enough to escape, and they help make up a continuous flow of particles called the solar wind.

Section 18.4 Adding a small amount of heat $dQ$ to the gas changes the temperature by $dT$, where $dQ = nC_vdT$ from Eq. (18.24). Figure 18.16 shows that $C_v$ for H₂ varies with temperature between 25 K and 500 K, so a given amount of heat gives rise to different amounts of temperature change during the process. Hence the temperature will not increase at a constant rate. The temperature change $dT = dQ/mC_v$ is inversely proportional to $C_v$, so the temperature increases most rapidly at the beginning of the process when the temperature is lowest and $C_v$ is smallest (see Fig. 18.16).

Section 18.5 If the curve were symmetrical like a bell curve, the average value would be at the peak of the curve and $v_{in}$ would be equal to $v_{rms}$ (which, by definition, is at the peak of the curve). The curve is not symmetrical, however, and its long tail shifts the average value to the right of the peak. Hence $v_{in} > v_{rms}$.

Section 18.6 The triple-point pressure of water from Table 18.3 is $6.10 \times 10^2 \text{ Pa}$. The present-day pressure on Mars is just less than this value, corresponding to the line labeled $p_0$ in Fig. 18.21. Hence liquid water cannot exist on the present-day Martian surface, and there are no rivers or lakes. Planetary scientists conclude that liquid water could have and almost certainly did exist on Mars in the past, when the atmosphere was thicker.

Discussion Questions

Q18.1 Section 18.1 states that ordinarily, pressure, volume, and temperature cannot change individually without one affecting the others. Yet when a liquid evaporates, its volume changes, even though its pressure and temperature are constant. Is this inconsistent? Why or why not?

Q18.2 In the ideal-gas equation, could an equivalent Celsius temperature be used instead of the Kelvin one if an appropriate numerical value of the constant $R$ is used? Why or why not?

Q18.3 On a chilly morning you can “see your breath.” Can you really? What are you actually seeing? Does this phenomenon depend on the temperature of the air, the humidity, or both? Explain.

Q18.4 When a car is driven some distance, the air pressure in the tires increases. Why? Should you let out some air to reduce the pressure? Why or why not?

Q18.5 The coolant in an automobile radiator is kept at a pressure higher than atmospheric pressure. Why is this desirable? The radiator cap will release coolant when the gauge pressure of the coolant reaches a certain value, typically 15 lb/in.² or so. Why not just seal the system completely?

Q18.6 Unwrapped food placed in a freezer experiences dehydration, known as “freezer burn.” Why?

Q18.7 “Freeze-drying” food involves the same process as “freezer burn,” referred to in the previous question. For freeze-drying, the food is usually frozen first, then placed in a vacuum chamber and irradiated with infrared radiation. What is the purpose of the vacuum? The radiation? What advantages might freeze-drying have in comparison to ordinary drying?

Q18.8 A group of students drove from their university (near sea level) up into the mountains for a skiing weekend. Upon arriving at the slopes, they discovered that the bags of potato chips they had brought for snacks had all burst open. What caused this to happen?

Q18.9 How does evaporation of perspiration from your skin cool your body?

Q18.10 Helium is a mixture of two isotopes, one having molar mass 3 g/mol and the other 4 g/mol. In a sample of helium gas which atoms move faster, on the average? Explain why.

Q18.11 Which has more atoms, a kilogram of hydrogen or a kilogram of lead? Which has more mass? Explain.

Q18.12 Use the concepts of the kinetic-molecular model to explain: a) why the pressure of a gas in a rigid container increases as heat is added to the gas; b) why the pressure of a gas increases as we compress it, even if we do not change its temperature.

Q18.13 The proportion of various gases in the earth’s atmosphere changes somewhat with altitude. Would you expect the proportion of oxygen at high altitude to be greater or less than at sea level compared to the proportion of nitrogen? Why?
Q18.14 Comment on the following statement: When two gases are mixed, if they are to be in thermal equilibrium they must have the same average molecular speed. Is the statement correct? Why or why not?

Q18.15 The kinetic-molecular model contains a hidden assumption about the temperature of the container walls. What is this assumption? What would happen if this assumption were not valid?

Q18.16 The temperature of an ideal gas is directly proportional to the average kinetic energy of its molecules. If a container of ideal gas is moving past you at 2000 m/s, is the temperature of the gas higher than if the container was at rest? Defend your answer.

Q18.17 If the pressure of an ideal monatomic gas is increased while the number of moles is kept constant, what happens to the average translational kinetic energy of one atom of the gas? Is it possible to change both the volume and pressure of an ideal gas and keep the average translational kinetic energy of the atoms constant? Explain.

Q18.18 In deriving the ideal-gas equation from the kinetic-molecular model, we ignored potential energy due to the earth's gravity. Is this omission justified? Why or why not?

Q18.19 The derivation of the ideal-gas equation included the assumption that the number of molecules is very large, so that we could compute the average force due to many collisions. However, the ideal-gas equation holds accurately only at low pressures, where the molecules are few and far between. Is this inconsistent? Why or why not?

Q18.20 A gas storage tank has a small leak. The pressure in the tank drops more quickly if the gas is hydrogen or helium than if it is oxygen. Why?

Q18.21 Consider two specimens of ideal gas at the same temperature. Specimen A has the same total mass as specimen B, but the molecules in specimen A have greater molar mass than they do in specimen B. In which specimen is the total kinetic energy of the gas greater? Does your answer depend on the molecular structure of the gases? Why or why not?

Q18.22 The temperature of an ideal monatomic gas is increased from 25°C to 50°C. Does the average translational kinetic energy of each gas atom double? Explain. If your answer is no, what would the final temperature be if the average translational kinetic energy is doubled?

Q18.23 If the root-mean-square speed of the atoms of an ideal gas is to be doubled, by what factor must the Kelvin temperature of the gas be increased? Explain.

Q18.24 Some elements that form solid crystals have molar heat capacities greater than 3R. What could account for this?

*Q18.25 In a gas that contains N molecules, would it be accurate to say that the number of molecules with speed v is equal to f(v)? Or would it be accurate to say that this number is given by Nf(v)? Explain your answers.

*Q18.26 Fig. 18.19 shows a velocity selector. The two disks are a distance x apart, the slits in the two disks are at an angle θ to each other, and the disks rotate together with angular speed ω. Explain why a molecule that passes through the first slit of the selector will only be able to pass through the second slit if its speed is v = ox/θ.

*Q18.27 Imagine a special air filter placed in a window of a house. The tiny holes in the filter only allow air molecules moving faster than a certain speed to exit the house, and only allow air molecules moving slower than that speed to enter the house from outside. What effect would this filter have on the temperature inside the house? (It turns out that the second law of thermodynamics—which we will discuss in Chapter 20—tells us that such a wonderful air filter would be quite impossible to make.)

Q18.28 A beaker of water at room temperature is placed in an enclosure, and the air pressure in the enclosure is slowly reduced. When the air pressure is reduced sufficiently, the water begins to boil. The temperature of the water does not rise when it boils; in fact, the temperature drops slightly. Explain these phenomena.

Q18.29 Ice is slippery to walk on, and especially slippery if you wear ice skates. What does this tell you about how the melting temperature of ice depends on pressure? Explain.

Q18.30 Hydrothermal vents are openings in the ocean floor that discharge very hot water. The water emerging from one such vent off the Oregon coast, 2400 m below the surface, has a temperature of 270°C. Despite its high temperature, the water doesn’t boil. Why not?

Q18.31 The dark areas on the moon’s surface are called maria. Latin for “seas,” and were once thought to be bodies of water. In fact, the maria are not “seas” at all, but plains of solidified lava. Given that there is no atmosphere on the moon, how can you explain the absence of liquid water on the moon’s surface?

Q18.32 In addition to the normal cooking directions printed on the back of a box of rice, there are also “high altitude directions.” The only difference is that the “high altitude directions” suggest increasing the cooking time and using a greater volume of boiling water in which to cook the rice. Why should the directions depend on the altitude in this way?

Exercises

Section 18.1 Equations of State

18.1 A 20.0-L tank contains 0.225 kg of helium at 18.0°C. The molar mass of helium is 4.00 g/mol. a) How many moles of helium are in the tank? b) What is the pressure in the tank, in pascals and in atmospheres?

18.2 Helium gas with a volume of 2.60 L, under a pressure of 41.0°C, is warmed until both pressure and volume are doubled. a) What is the final temperature? b) How many grams of helium are there? The molar mass of helium is 4.00 g/mol.

18.3 A cylindrical tank has a tight-fitting piston that allows the volume of the tank to be changed. The tank originally contains 0.110 m³ of air at a pressure of 3.40 atm. The piston is slowly pulled out until the volume of the gas is increased to 0.390 m³. If the temperature remains constant, what is the final value of the pressure?

18.4 A 3.00-L tank contains air at 3.00 atm and 20.0°C. The tank is sealed and cooled until the pressure is 1.00 atm. a) What is the temperature in degrees Celsius? Assume that the volume of the tank is constant. b) If the temperature is kept at the value found in
part (a) and the gas is compressed, what is the volume when the pressure again becomes 3.00 atm?

18.5 a) Use the ideal gas law to estimate the number of air molecules in your physics lab room, assuming all the air is N₂. b) Calculate the particle density in the lab (that is, the number of molecules per cubic centimeter).

18.6 You have several identical balloons. You experimentally determine that a balloon will break if its volume exceeds 0.900 L. The pressure of the gas inside the balloon equals air pressure (1.00 atm). a) If the air inside the balloon is at a constant temperature of 22.0°C and behaves as an ideal gas, what mass of air can you blow into one of the balloons before it bursts? b) Repeat part (a) if the gas is helium rather than air.

18.7 A Jaguar XK8 convertible has an eight-cylinder engine. At the beginning of its compression stroke, one of the cylinders contains 499 cm³ of air at atmospheric pressure (1.01 × 10⁵ Pa) and a temperature of 27.0°C. At the end of the stroke the air has been compressed to a volume of 46.2 cm³ and the gauge pressure has increased to 2.72 × 10⁵ Pa. Compute the final temperature.

18.8 A welder using a tank of volume 0.0750 m³ fills it with oxygen (molar mass = 32.0 g/mol) at a gauge pressure of 3.00 × 10⁵ Pa and temperature of 37.0°C. The tank has a small leak, and in time some of the oxygen leaks out. On a day when the temperature is 22.0°C, the gauge pressure of the oxygen in the tank is 1.80 × 10⁵ Pa. Find a) the initial mass of oxygen; b) the mass of oxygen that has leaked out.

18.9 A large cylindrical tank contains 0.750 m³ of nitrogen gas at 27°C and 1.50 × 10⁵ Pa (absolute pressure). The tank has a tight-fitting piston that allows the volume to be changed. What will be the pressure if the volume is decreased to 0.480 m³ and the temperature is increased to 157°C?

18.10 A room with dimensions 7.00 m × 8.00 m × 2.50 m is filled with pure oxygen at 22.0°C and 1.00 atm. The molar mass of oxygen is 32.0 g/mol. a) How many moles of oxygen are required? b) What is the mass of this oxygen, in kilograms?

18.11 The gas inside a balloon will always have a pressure nearly equal to atmospheric pressure, since that is the pressure applied to the outside of the balloon. You fill a balloon with helium (a nearly ideal gas) to a volume of 0.600 L at a temperature of 19.0°C. What is the volume of the balloon if you cool it to the boiling point of liquid nitrogen (77.3 K)?

18.12 Deviations from the Ideal-Gas Equation. For carbon dioxide gas (CO₂), the constants in the van der Waals equation are \( a = 0.364 \text{ J·m}^3/\text{mol}^2 \) and \( b = 4.27 \times 10^{-5} \text{ m}^3/\text{mol} \). a) If 1.00 mol of CO₂ gas at 350 K is confined to a volume of 400 cm³, find the pressure of the gas using the ideal-gas equation and the van der Waals equation. b) Which equation gives a lower pressure? Why? What is the percentage difference of the van der Waals result from the ideal-gas equation result? c) The gas is kept at the same temperature as it expands to a volume of 4000 cm³. Repeat the calculations of parts (a) and (b). d) Explain how your calculations show that the van der Waals equation is equivalent to the ideal-gas equation if \( n/V \) is small.

18.13 The total lung volume for a typical physics student is 6.00 L. A physics student fills her lungs with air at an absolute pressure of 1.00 atm. Then, holding her breath, she compresses her chest cavity, decreasing her lung volume to 5.70 L. What is the pressure of the air in her lungs then? Assume that the temperature of the air remains constant.

18.14 A diver observes a bubble of air rising from the bottom of a lake (where the absolute pressure is 3.50 atm) to the surface (where the pressure is 1.00 atm). The temperature at the bottom is 4.0°C, and the temperature at the surface is 23.0°C. a) What is the ratio of the volume of the bubble as it reaches the surface to its volume at the bottom? b) Would it be safe for the diver to hold his breath while ascending from the bottom of the lake to the surface? Why or why not?

18.15 A metal tank with volume 3.10 L will burst if the absolute pressure of the gas it contains exceeds 100 atm. a) If 1.10 mol of an ideal gas is put into the tank at a temperature of 23.0°C, to what temperature can the gas be warmed before the tank ruptures? You can ignore the thermal expansion of the tank. b) Based on your answer to part (a), is it reasonable to ignore the thermal expansion of the tank? Explain.

18.16 Three moles of ideal gas are in a rigid cubical box with sides of length 0.200 m. a) What is the force that the gas exerts on each of the six sides of the box when the gas temperature is 20.0°C? b) What is the force when the temperature of the gas is increased to 100.0°C?

18.17 With the assumptions of Example 18.4 (Section 18.1), at what altitude above sea level is air pressure 90% of the pressure at sea level?

18.18 Make the same assumptions as in Example 18.4 (Section 18.1). How does the percent decrease in air pressure in going from sea level to an altitude of 100 m compare to that when going from sea level to an altitude of 1000 m? If your second answer is not ten times your first answer, explain why.

18.19 With the assumptions of Example 18.4 (Section 18.1), how does the density of air at sea level compare to that at an altitude of 100 m above sea level?

18.20 With the assumption (not very realistic) that the air temperature is a uniform 0°C, what is the atmospheric pressure at an altitude of 5000 m? This is roughly the maximum altitude usually attained by aircraft without pressurized cabins.

18.21 At an altitude of 11,000 m (a typical cruising altitude for a jet airliner), the air temperature is −56.5°C and the air density is 0.364 kg/m³. What is the pressure of the atmosphere at that altitude? (Note that the temperature at this altitude is not the same as at the surface of the earth, so that the calculation of Example 18.4 (Section 18.1) doesn’t apply.)

Section 18.2 Molecular Properties of Matter

18.22 A large organic molecule has a mass of 1.41 × 10⁻²¹ kg. What is the molar mass of this compound?

18.23 What is the volume of 3.00 moles of copper?

18.24 Modern vacuum pumps make it easy to attain pressures of the order of 10⁻¹⁰ atm in the laboratory. At a pressure of 9.00 × 10⁻¹⁰ atm and an ordinary temperature (say \( T = 300 \text{ K} \)), how many molecules are present in a volume of 1.00 cm³?

18.25 The Lagoon Nebula (Fig. 18.25) is a cloud of hydrogen gas located 3900 light years from the earth. The cloud is about 45 light years in diameter, and glows because of its high temperature of 7500 K. (The gas is raised to this temperature by the stars that lie...
within the nebula). The cloud is also very thin; there are only 80 molecules per cubic centimeter. a) Find the gas pressure (in atmospheres) in the Lagoon Nebula. Compare to the laboratory pressure referred to in Exercise 18.24. b) Science fiction films sometimes show starships being buffeted by turbulence as they fly through gas clouds such as the Lagoon Nebula. Does this seem realistic? Why or why not?

Figure 18.25 Exercise 18.25.

18.26 In a gas at standard conditions, what is the length of the side of a cube that contains a number of molecules equal to the population of the earth (about $6 \times 10^9$)?

18.27 How many moles are there in a 1.00-kg bottle of water? How many molecules? The molar mass of water is 18.0 g/mol.

18.28 Consider an ideal gas at $27^\circ$C and 1.00 atm pressure. Imagine the molecules to be, on average, uniformly spaced, with each molecule at the center of a small cube. a) What is the length of an edge of each small cube if adjacent cubes touch but don’t overlap? b) How does this distance compare with the diameter of a molecule?

18.29 Consider 5.00 mol of liquid water. a) What volume is occupied by this amount of water? The molar mass of water is 18.0 g/mol. b) Imagine the molecules to be, on average, uniformly spaced, with each molecule at the center of a small cube. What is the length of an edge of each small cube if adjacent cubes touch but don’t overlap? c) How does this distance compare with the diameter of a molecule?

18.30 A flask contains a mixture of neon (Ne), krypton (Kr), and radon (Rn) gases. Compare a) the average kinetic energies of the three types of atoms; b) the root-mean-square speeds. (Hint: the periodic table in Appendix D shows the molar mass (in g/mol) of each element under the chemical symbol for that element.)

18.31 Gaseous Diffusion of Uranium. a) A process called gaseous diffusion is often used to separate isotopes of uranium, that is, atoms of the elements that have different masses, such as $^{235}\text{U}$ and $^{238}\text{U}$. The only gaseous compound of uranium at ordinary temperatures is uranium hexafluoride, UF$_6$. Speculate on how $^{235}\text{UF}_6$ and $^{238}\text{UF}_6$ molecules might be separated by diffusion. b) The molar masses for $^{235}\text{UF}_6$ and $^{238}\text{UF}_6$ molecules are 0.349 kg/mol and 0.352 kg/mol, respectively. If uranium hexafluoride acts as an ideal gas, what is the ratio of the root-mean-square speed of $^{235}\text{UF}_6$ molecules to that of $^{238}\text{UF}_6$ molecules if the temperature is uniform?

18.32 The ideas of average and root-mean-square value can be applied to any distribution. A class of 150 students had the following scores on a 100-point quiz:

<table>
<thead>
<tr>
<th>Score</th>
<th>Number of Students</th>
</tr>
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<tbody>
<tr>
<td>10</td>
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<td>20</td>
<td>12</td>
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<td>70</td>
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<tr>
<td>80</td>
<td>20</td>
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<tr>
<td>90</td>
<td>17</td>
</tr>
<tr>
<td>100</td>
<td>10</td>
</tr>
</tbody>
</table>

a) Find the average score for the class. b) Find the root-mean-square score for the class.

18.33 We have two equal size boxes, A and B. Each box contains gas that behaves as an ideal gas. We insert a thermometer into each box and find that the gas in box A is at a temperature of 50°C while that in B is at 10°C. This is all we know about the gas in the boxes. Which of the following statements must be true? Which could be true? a) The pressure in A is higher than in B. b) There are more molecules in A than in B. c) A and B cannot contain the same type of gas. d) The molecules in A have more average kinetic energy per molecule than those in B. e) The molecules in A are moving faster than those in B. Explain the reasoning behind your answers.

18.34 We have two equal size boxes, A and B. Each box contains gas that behaves as an ideal gas. We put a pressure gauge into each box and find that the gauge reads 0.200 atm in box A, but only 0.040 atm in box B. This is all we know about the gas in the boxes. Which of the following statements must be true? Which could be true? a) There are more molecules in A than in B. b) The molecules in A are moving faster than those in B. c) The temperature in A is higher than in B. d) The molecules in A are heavier than those in B. e) The molecules in A have more average kinetic energy per molecule than those in B. Explain the reasoning behind your answers.

18.35 a) A deuteron, $^1\text{H}$, is the nucleus of a hydrogen isotope and consists of one proton and one neutron. The plasma of deuterons in a nuclear fusion reactor must be heated to about 300 million K. What is the rms speed of the deuterons? Is this a significant fraction of the speed of light ($c = 3.0 \times 10^8$ m/s)? b) What would the temperature of the plasma be if the deuterons had an rms speed equal to 0.10$c$?

18.36 Initially, the translational rms speed of an atom of a monatomic ideal gas is 250 m/s. The pressure and volume of the gas are each doubled while the number of moles of the gas is kept constant. What is the final translational rms speed of the atoms?

18.37 a) Oxygen (O$_2$) has a molar mass of 32.0 g/mol. What is the average translational kinetic energy of an oxygen molecule at a temperature of 300 K? b) What is the average value of the square of its speed? c) What is the root-mean-square speed? d) What is the momentum of an oxygen molecule traveling at this speed? e) Suppose an oxygen molecule traveling at this speed bounces back and forth between opposite sides of a cubical vessel 0.10 m on a side.
What is the average force it exerts on one of the walls of the container? (Assume that the molecule’s velocity is perpendicular to the two sides that it strikes.) f) What is the average force per unit area? 
g) How many oxygen molecules traveling at this speed are necessary to produce an average pressure of 1 atm? h) Compute the number of oxygen molecules that are actually contained in a vessel of this size at 300 K and atmospheric pressure. i) Your answer for part (h) should be three times as large as the answer for part (g). Where does this discrepancy arise?

18.38 Calculate the mean free path of air molecules at a pressure of $3.50 \times 10^{-13}$ atm and a temperature of 300 K. (This pressure is readily attainable in the laboratory; see Exercise 18.24.) As in Example 18.9, model the air molecules as spheres of radius 2.0 \times 10^{-10} m.

18.39 At what temperature is the root-mean-square speed of nitrogen molecules equal to the root-mean-square speed of hydrogen molecules at 20.0°C? (Hint: The periodic table in Appendix D shows the molar mass (in g/mol) of each element under the chemical symbol for that element. The molar mass of H$_2$ is twice the molar mass of hydrogen atoms, and similarly for N$_2$.)

18.40 Smoke particles in the air typically have masses of the order of 10^{-16} kg. The Brownian motion (rapid, irregular movement) of these particles, resulting from collisions with air molecules, can be observed with a microscope. a) Find the root-mean-square speed of Brownian motion for a particle with a mass of 3.00 \times 10^{-16} kg in air at 300 K. b) Would the root-mean-square speed be different if the particle were in hydrogen gas at the same temperature? Explain.

Section 18.4 Heat Capacities

18.41 a) Calculate the specific heat capacity at constant volume of water vapor, assuming the nonlinear triatomic molecule has three translational and three rotational degrees of freedom and that vibrational motion does not contribute. The molar mass of water is 18.0 g/mol. b) The actual specific heat capacity of water vapor at low pressures is about 2000 J/kg·K. Compare this with your calculation and comment on the actual role of vibrational motion.

18.42 a) The specific heat of ice at constant volume is 833 J/kg·K at −180°C, 1640 J/kg·K at −60°C, and 2060 J/kg·K at −5.0°C. Calculate $C_v$, the molar heat capacity at constant volume for ice, at each of these temperatures. The molar mass of H$_2$O is 18.0 g/mol. b) Why do you suppose the value of $C_v$ increases with increasing temperature? c) Do the values that you calculate approach the value of 3R (given in the rule of Dulong and Petit) as the temperature increases? Speculate about why this should be so.

18.43 a) How much heat does it take to increase the temperature of 2.50 mol of a diatomic ideal gas by 30.0 K near room temperature if the gas is held at constant volume? b) What is the answer to the question in part (a) if the gas is monatomic rather than diatomic?

18.44 a) Compute the specific heat capacity at constant volume of nitrogen (N$_2$) gas, and compare with the specific heat capacity of liquid water. The molar mass of N$_2$ is 28.0 g/mol. b) You warm 1.00 kg of water at a constant volume of 1.00 L from 20.0°C to 30.0°C in a kettle. For the same amount of heat, how many kilograms of 20.0°C air would you be able to warm to 30.0°C? What volume (in liters) would this air occupy at 20.0°C and a pressure of 1.00 atm? Make the simplifying assumption that air is 100% N$_2$.

Section 18.5 Molecular Speeds

18.45 For a gas of nitrogen molecules (N$_2$), what must the temperature be if 94.7% of all the molecules have speeds less than a) 1500 m/s; b) 1000 m/s; c) 500 m/s? Use Table 18.2. The molar mass of N$_2$ is 28.0 g/mol.

18.46 Derive Eq. (18.33) from Eq. (18.32).

18.47 Prove that $f(v)$ as given by Eq. (18.33) is maximum for $v = kT$. Use this result to obtain Eq. (18.34).

18.48 For diatomic carbon dioxide gas (CO$_2$, molar mass = 44.0 g/mol) at $T = 300$ K, calculate a) the most probable speed $v_{mp}$; b) the average speed $v_{av}$; c) the root-mean-square speed $v_{rms}$.

Section 18.6 Phases of Matter

18.49 Puffy cumulus clouds, which are made of water droplets, occur at lower altitudes in the atmosphere. Wispy cirrus clouds, which are made of ice crystals, occur only at higher altitudes. Find the altitude $y$ (measured from sea level) above which only cirrus clouds can occur. On a typical day and at altitudes less than 11 km, the temperature at an altitude $y$ is given by $T = T_0 - ay$, where $T_0 = 15.0^\circ C$ and $a = 6.0^\circ C/1000$ m.

18.50 Solid water (ice) is slowly warmed from a very low temperature. a) What minimum external pressure $p_t$ must be applied to the solid if a melting phase transition is to be observed? Describe the sequence of phase transitions that occur if the applied pressure $p$ is such that $p < p_t$. b) Above a certain maximum pressure $p_m$, no boiling transition is observed. What is this pressure? Describe the sequence of phase transitions that occur if $p < p < p_m$.

18.51 A physicist places a piece of ice at 0.00°C and a beaker of water at 0.00°C inside a glass box and closes the lid of the box. All the air is then removed from the box. If the ice, water, and beaker are all maintained at a temperature of 0.00°C, describe the final equilibrium state inside the box.

18.52 The atmosphere of the planet Mars is 95.3% carbon dioxide (CO$_2$) and about 0.03% water vapor. The atmospheric pressure is only 600 Pa, and the surface temperature varies from −30°C to +100°C. The polar ice caps contain both CO$_2$ ice and water ice. Could there be liquid CO$_2$ on the surface of Mars? Could there be liquid water? Why or why not?

Problems

18.53 a) Use Eq. (18.1) to estimate the change in the volume of a solid steel sphere of volume 11 L when the temperature and pressure increase from 21°C and 1,013 \times 10^5 Pa to 42°C and 2.10 \times 10^5 Pa. (Hint: Consult Chapters 11 and 17 to determine the values of $\beta$ and $k$.) b) In Example 18.3 the change in volume of an 11-L steel scuba tank was ignored. Was this a good approximation? Explain.

18.54 At an absolute pressure of 2.00 \times 10^{-13} atm, a partial vacuum easily obtained in laboratories (see Exercise 18.24), calculate the mass of nitrogen present in a volume of 3000 cm$^3$ if the temperature of the gas is 22.0°C. The molar mass of nitrogen (N$_2$) is 28.0 g/mol.

18.55 A cylinder 1.00 m tall with inside diameter 0.120 m is used to hold propane gas (molar mass 44.1 g/mol) for use in a bar-
becue. It is initially filled with gas until the gauge pressure is 1.30 \times 10^5 \text{ Pa} and the temperature is 22.0°C. The temperature of the gas remains constant as it is partially emptied out of the tank, until the gauge pressure is 2.50 \times 10^4 \text{ Pa}. Calculate the mass of propane that has been used.

**18.56** During a test dive in 1939, prior to being accepted by the U.S. Navy, the submarine *Squalus* sank at a point where the depth of water was 73.0 m. The temperature at the surface was 27.0°C and at the bottom it was 7.0°C. The density of seawater is 1030 kg/m³. a) A diving bell was used to rescue 33 trapped crewmen from the *Squalus*. The diving bell was in the form of a circular cylinder 2.30 m high, open at the bottom and closed at the top. When the diving bell was lowered to the bottom of the sea, to what height did water rise within the diving bell? (Hint: You may ignore the relatively small variation in water pressure between the bottom of the bell and the surface of the water within the bell.) b) At what gauge pressure must compressed air have been supplied to the bell while on the bottom to expel all the water from it?

**18.57** A glassblower makes a barometer using a tube 0.900 m long and with a cross-sectional area of 0.620 cm². Mercury stands in this tube to a height of 0.750 m. The room temperature is 20.0°C. A small amount of air is accidentally introduced into the evacuated space above the mercury, and the column drops to a height of 0.690 m. How many grams of air were introduced? The molar mass of air is 28.8 g/mol.

**18.58** A hot-air balloon stays aloft because hot air at atmospheric pressure is less dense than cooler air at the same pressure; the calculation of the buoyant force is discussed in Chapter 14. If the volume of the balloon is 500 m³ and the surrounding air is at 15.0°C, what must the temperature of the air in the balloon be for it to lift a total load of 290 kg (in addition to the mass of the hot air)? The density of air at 15.0°C and atmospheric pressure is 1.23 kg/m³.

**18.59** An automobile tire has a volume of 0.0150 m³ on a cold day when the temperature of the air in the tire is 5.0°C and atmospheric pressure is 1.02 atm. Under these conditions the gauge pressure is measured to be 1.70 atm (about 25 lb/in.²). After the car is driven on the highway for 30 min, the temperature of the air in the tires has risen to 45.0°C and the volume to 0.0159 m³. What then is the gauge pressure?

**18.60** A flask with a volume of 1.50 L, provided with a stopcock, contains ethane gas (C₂H₆) at 300 K and atmospheric pressure (1.013 \times 10^5 \text{ Pa}). The molar mass of ethane is 30.1 g/mol. The system is warmed to a temperature of 380 K, with the stopcock open to the atmosphere. The stopcock is then closed, and the flask cooled to its original temperature. a) What is the final pressure of the ethane in the flask? b) How many grams of ethane remain in the flask?

**18.61** A balloon whose volume is 750 m³ is to be filled with hydrogen at atmospheric pressure (1.01 \times 10^5 \text{ Pa}). a) If the hydrogen is stored in cylinders with volumes of 1.90 m³ at a gauge pressure of 1.20 \times 10^7 \text{ Pa}, how many cylinders are required? Assume that the temperature of the hydrogen remains constant. b) What is the total weight (in addition to the weight of the gas) that can be supported by the balloon if the gas in the balloon and the surrounding air are both at 15.0°C? The molar mass of hydrogen (H₂) is 2.02 g/mol. The density of air at 15.0°C and atmospheric pressure is 1.23 kg/m³. See Chapter 14 for a discussion of buoyancy.

**18.62** A vertical cylindrical tank 0.900 m high has its top end closed by a tightly fitting frictionless piston of negligible weight. The air inside the cylinder is at an absolute pressure of 1.00 atm. The piston is depressed by pouring mercury on it slowly (Fig. 18.26). How far will the piston descend before mercury spills over the top of the cylinder? The temperature of the air is kept constant.

**18.63** A large tank of water has a hose connected to it, as shown in Fig. 18.27. The tank is sealed at the top and has compressed air between the water surface and the top. When the water height h has the value 3.50 m, the absolute pressure p of the compressed air is 4.20 \times 10^5 \text{ Pa}. Assume that the air above the water expands at constant temperature, and take the atmospheric pressure to be 1.00 \times 10^5 \text{ Pa}. a) What is the speed with which water flows out of the hose when h = 3.50 m? b) As water flows out of the tank, h decreases. Calculate the speed of flow for h = 3.00 m and for h = 2.00 m. c) At what value of h does the flow stop?

**18.64** In one hour, an average person at rest consumes 14.5 L of oxygen at a pressure of 1.00 atm and a temperature of 20.0°C. a) Express this rate of oxygen consumption in terms of the number of molecules per second. b) A person at rest inhales and exhales 0.5 L of air with each breath. The inhaled air is 21.0% oxygen, and the exhaled air is 16.3% oxygen. How many breaths per minute will satisfy this person’s oxygen requirements? c) Repeat part (b) for a resting person at an elevation of 3000 m, where the pressure is 0.72 atm and the temperature is 0.0°C. Assume that the oxygen percentages and volume per inhalation are the same as stated in part (b). (To maintain its functions, the body still requires the same number of oxygen molecules per second as at sea level.) Explain why some people report “shortness of breath” at such elevations.

**18.65** How Many Atoms Are You? Estimate the number of atoms in the body of a 50-kg physics student. Note that the human body is mostly water, which has molar mass 18.0 g/mol, and that each water molecule contains three atoms.

**18.66** The size of an oxygen molecule is about 2.0 \times 10^{-10} \text{ m}. Make a rough estimate of the pressure at which the finite volume of the molecules should cause noticeable deviations from ideal-gas behavior at ordinary temperatures (T = 300 K).

**18.67** Successive Approximations and the van der Waals Equation. In the ideal-gas equation, the number of moles per vol-
ume $n/V$ is simply equal to $p/RT$. In the van der Waals equation, solving for $n/V$ in terms of the pressure $p$ and temperature $T$ is somewhat more involved. a) Show that the van der Waals equation can be written as

$$n/V = \left( \frac{p + a n^2/V^2}{RT} \right) \left( 1 - \frac{bn}{V} \right)$$

b) The van der Waals parameters for hydrogen sulfide gas ($\text{H}_2\text{S}$) are $a = 0.448 \text{ J m}^{-3} \text{ mol}^{-2}$ and $b = 4.29 \times 10^{-5} \text{ m}^3 \text{ mol}^{-1}$. Determine the number of moles per volume of $\text{H}_2\text{S}$ gas at 127°C and an absolute pressure of $9.80 \times 10^3 \text{ Pa}$ as follows: i) Calculate a first approximation using the ideal-gas equation, $n/V = p/RT$. ii) Substitute this approximation for $n/V$ into the right-hand side of the equation in part (a). The result is a new, improved approximation for $n/V$. iii) Substitute the new approximation for $n/V$ into the right-hand side of the equation in (a). The result is a further-improved approximation for $n/V$. iv) Repeat step (iii) until successive approximations agree to the desired level of accuracy (in this case, to three significant figures). c) Compare your final result in part (b) to the result $p/RT$ obtained using the ideal-gas equation. Which result gives a larger value of $n/V$? Why?

18.68 a) Compute the increase in gravitational potential energy for a nitrogen molecule (molar mass 28.0 glmol) for an increase in elevation of 400 m near the earth’s surface. b) At what temperature is this equal to the average kinetic energy of a nitrogen molecule? c) Is it possible that a nitrogen molecule near sea level where $T = 15.0^\circ\text{C}$ could rise to an altitude of 400 m? Is it likely that it could do so without hitting any other molecules along the way? Explain.

18.69 The Lennard-Jones Potential. A commonly used potential-energy function for the interaction of two molecules (Fig. 18.6) is the Lennard-Jones 6-12 potential

$$U(r) = U_0 \left[ \left( \frac{R_o}{r} \right)^{12} - 2 \left( \frac{R_o}{r} \right)^6 \right]$$

where $r$ is the distance between the centers of the molecules and $U_0$ and $R_o$ are positive constants. The corresponding force $F(r)$ is given in Eq. (13.26). a) Graph $U(r)$ and $F(r)$ versus $r$. b) Let $r_1$ be the value of $r$ at which $U(r) = 0$, and let $r_2$ be the value of $r$ at which $F(r) = 0$. Show the locations of $r_1$ and $r_2$ on your graphs of $U(r)$ and $F(r)$. Which of these values represents the equilibrium separation between the molecules? c) Find the values of $r_1$ and $r_2$ in terms of $R_o$ and find the ratio $r_1/r_2$. d) If the molecules are located a distance $r_3$ apart (as calculated in part (c)), how much work must be done to pull them apart so that $r \rightarrow \infty$?

18.70 a) What is the total random translational kinetic energy of 5.00 L of hydrogen gas, with pressure 1.01 \times 10^5 \text{ Pa} and temperature 300 K? b) If the tank containing the gas is moved with a speed of 30.0 m/s, by what percentage is the total kinetic energy of the gas increased? The molar mass of hydrogen (H$_2$) is 2.016 glmol.

18.71 The speed of propagation of a sound wave in air at 27°C is about 350 m/s. Calculate, for comparison, a) $v_{rms}$ for nitrogen molecules; b) the rms value of $v_x$ at this temperature. The molar mass of nitrogen (N$_2$) is 28.0 glmol.

18.72 The surface of the sun has a temperature of about 5800 K and consists largely of hydrogen atoms. a) Find the rms speed of a hydrogen atom at this temperature. (The mass of a single hydrogen atom is 1.67 \times 10^{-27} \text{ kg}.) b) The escape speed for a particle to leave the gravitational influence of the sun is given by $(2GM/R)^{1/2}$, where $M$ is the sun’s mass, $R$ its radius, and $G$ the gravitational constant (Example 12.5 of Section 12.3). Use the data in Appendix F to calculate this escape speed. c) Can appreciable quantities of hydrogen escape from the sun? Can any hydrogen escape? Explain.

18.73 a) Show that a projectile with mass $m$ can “escape” from the surface of a planet if it is launched vertically upward with a kinetic energy greater than $mgR_p$, where $g$ is the acceleration due to gravity at the planet’s surface and $R_p$ is the planet’s radius. Ignore air resistance. (See Problem 18.72.) b) If the planet in question is the earth, at what temperature does the average translational kinetic energy of a nitrogen molecule (molar mass 28.0 glmol) equal that required to escape? What about a hydrogen molecule (molar mass 2.02 glmol)? c) Repeat part (b) for the moon, for which $g = 1.63 \text{ m/s}^2$ and $R_p = 1740 \text{ km}$. d) While the earth and the moon have similar average surface temperatures, the moon has essentially no atmosphere. Use your results from parts (b) and (c) to explain why.

18.74 Planetary Atmospheres. a) The temperature near the top of Jupiter’s multicolored cloud layer is about 140 K. The temperature at the top of the earth’s troposphere, at an altitude of about 20 km, is about 220 K. Calculate the rms speed of hydrogen molecules in both these environments. Give your answers in m/s and as a fraction of the escape speed from the respective planet (see Problem 18.72). b) Hydrogen gas (H$_2$) is a rare element in the earth’s atmosphere. In the atmosphere of Jupiter, by contrast, 89% of all molecules are H$_2$. Explain why, using your results from part (a). c) Suppose an astronomer claims to have discovered an oxygen (O$_2$) atmosphere on the asteroid Ceres. How likely is this? Ceres has a mass equal to 0.014 times the mass of the moon, a density of 2400 kg/m$^3$, and a surface temperature of about 200 K.

18.75 a) For what mass of molecule or particle is $v_{rms}$ equal to 1.00 mm/s at 300 K? b) If the particle is an ice crystal, how many molecules does it contain? The molar mass of water is 18.0 glmol.

c) Calculate the diameter of the particle if it is a spherical piece of ice. Would it be visible to the naked eye?

18.76 In describing the heat capacities of solids in Section 18.4, we stated that the potential energy $U = 1/2kx^2$ of a harmonic oscillator averaged over one period of the motion is equal to the kinetic energy $K = 1/2mv^2$ averaged over one period. Prove this result using Eqs. (13.13) and (13.15) for the position and velocity of a simple harmonic oscillator. For simplicity, assume that the initial position and velocity make the phase angle $\phi$ equal to zero. (Hint: Use the trigonometric identities $\cos^2(\theta) = [1 + \cos(2\theta)]/2$ and $\sin^2(\theta) = [1 - \cos(2\theta)]/2$. What is the average value of $\cos(2\omega t)$ averaged over one period?)

18.77 It is possible to make crystalline solids that are only one layer of atoms thick. Such “two-dimensional” crystals can be created by depositing atoms on a very flat surface. a) If the atoms in such a two-dimensional crystal can only move within the plane of the crystal, what will be its molar heat capacity near room temperature? Give your answer as a multiple of $R$ and in J/mol·K. b) At very low temperatures, will the molar heat capacity of a two-
dimensional crystal be greater than, less, or equal to the result you found in part (a)? Explain why.

18.78 a) Calculate the total rotational kinetic energy of the molecules in 1.00 mol of a diatomic gas at 300 K. b) Calculate the moment of inertia of an oxygen molecule \( \text{O}_2 \) for rotation about either the y- or z-axis shown in Fig. 18.15. Treat the molecule as two massive points (representing the oxygen atoms) separated by a distance of \( 1.21 \times 10^{-10} \) m. The molar mass of oxygen atoms is 16.0 g/mol. c) Find the rms angular velocity of rotation of an oxygen molecule about either the y- or z-axis shown in Fig. 18.15. How does your answer compare to the angular velocity of a typical piece of rapidly rotating machinery (10,000 rev/min)?

18.79 For each polyatomic gas in Table 18.1, compute the value of the molar heat capacity at constant volume, \( C_v \), on the assumption that there is no vibrational energy. Compare with the measured values in the table, and compute the fraction of the total heat capacity that there is no vibrational energy. Compare with the measured values in the table, and compute the fraction of the total heat capacity that there is no vibrational energy. Compare with the measured values in the table, and compute the fraction of the total heat capacity that there is no vibrational energy.

\[ C_v = \frac{\text{total heat capacity}}{\text{molar mass of gas}} \]

18.80 a) Show that \( \int_0^\infty f(v) \, dv = 1 \), where \( f(v) \) is the Maxwell-Boltzmann distribution of Eq. (18.32). b) In terms of the physical definition of \( f(v) \), explain why the integral in part (a) must have this value.

18.81 Calculate the integral in Eq. (18.31), \( \int_0^\infty v^2 f(v) \, dv \), and compare this result to \( \langle v^2 \rangle_m \) as given by Eq. (18.16). (Hint: You may use the tabulated integral

\[ \int_0^\infty x^n e^{-ax} \, dx = \frac{1}{a^n} \left( \frac{\pi}{2} \right) \sqrt{n} \]

where \( n \) is a positive integer and \( a \) is a positive constant.)

18.82 Calculate the integral in Eq. (18.30), \( \int_0^\infty v f(v) \, dv \), and compare this result to \( \langle v \rangle_m \) as given by Eq. (18.35). (Hint: Make the change of variable \( v^2 = x \) and use the tabulated integral

\[ \int_0^\infty x^n e^{-ax} \, dx = \frac{n!}{a^{n+1}} \]

where \( n \) is a positive integer and \( a \) is a positive constant.)

18.83 a) Explain why in a gas of \( N \) molecules, the number of molecules having speeds in the \( \text{finite} \) interval \( v \) to \( v + \Delta v \) is \( \Delta N = N \int_v^{v+\Delta v} f(v) \, dv \), if \( \Delta v \) is small, then \( f(v) \) is approximately constant over the interval and \( \Delta N \approx NF(v) \Delta v \). For oxygen gas (\( \text{O}_2 \), molar mass = 32.0 g/mol) at \( T = 300 \) K, use this approximation to calculate the number of molecules with speeds within \( \Delta v = 20 \) m/s of \( v \). Express your answer as a multiple of \( N \). c) Repeat part (b) for speeds within \( \Delta v = 20 \) m/s of \( 7v \). d) Repeat parts (b) and (c) for a temperature of 600 K. e) Repeat parts (b) and (c) for a temperature of 150 K. f) What do your results tell you about the shape of the distribution as a function of temperature? Do your conclusions agree with what is shown in Fig. 18.20?

18.84 The vapor pressure is the pressure of the vapor phase of a substance when it is in equilibrium with the solid or liquid phase of the substance. The relative humidity is the partial pressure of water vapor in the air divided by the vapor pressure of water at that same temperature, expressed as a percentage. The air is saturated when the humidity is 100%. a) The vapor pressure of water at 20.0°C is 2.34 \times 10^3 \text{ Pa}. If the air temperature is 20.0°C and the relative humidity is 60%, what is the partial pressure of water vapor in the atmosphere (that is, the pressure due to water vapor alone)? b) Under the conditions of part (a), what is the mass of water in 1.00 m³ of air? (The molar mass of water is 18.0 g/mol. Assume that water vapor can be treated as an ideal gas.)

18.85 The Dew Point. The vapor pressure of water (Problem 18.84) decreases as the temperature decreases. If the amount of water vapor in the air is kept constant as the air is cooled, a temperature is reached, called the dew point, at which the partial pressure and vapor pressure coincide and the vapor is saturated. If the air is cooled further, vapor condenses to liquid until the partial pressure again equals the vapor pressure at that temperature. The temperature in a room is 30.0°C. A meteorologist cools a metal can by gradually adding cold water. When the can temperature reaches 16.0°C, water droplets form on its outside surface. What is the relative humidity of the 30.0°C air in the room? The table below lists the vapor pressure of water at various temperatures:

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Vapor Pressure (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>1.23 \times 10^3</td>
</tr>
<tr>
<td>12.0</td>
<td>1.40 \times 10^3</td>
</tr>
<tr>
<td>14.0</td>
<td>1.60 \times 10^3</td>
</tr>
<tr>
<td>16.0</td>
<td>1.81 \times 10^3</td>
</tr>
<tr>
<td>18.0</td>
<td>2.06 \times 10^3</td>
</tr>
<tr>
<td>20.0</td>
<td>2.34 \times 10^3</td>
</tr>
<tr>
<td>22.0</td>
<td>2.65 \times 10^3</td>
</tr>
<tr>
<td>24.0</td>
<td>2.99 \times 10^3</td>
</tr>
<tr>
<td>26.0</td>
<td>3.36 \times 10^3</td>
</tr>
<tr>
<td>28.0</td>
<td>3.78 \times 10^3</td>
</tr>
<tr>
<td>30.0</td>
<td>4.25 \times 10^3</td>
</tr>
</tbody>
</table>

18.86 Altitude at Which Clouds Form. On a spring day in the midwestern United States, the air temperature at the surface is 28.0°C. Puffy cumulus clouds form at an altitude where the air temperature equals the dew point (Problem 18.85). If the air temperature decreases with altitude at a rate of 0.6°C/100 m, at approximately what height above the ground will clouds form? Assume that the relative humidity at the surface is a) 35% b) 80%? (Hint: Use the table in Problem 18.85.)

Challenge Problems

18.87 Dark Nebulae and the Interstellar Medium. The dark area in Fig. 18.28 that appears devoid of stars is a dark nebula, a cold gas cloud in interstellar space that contains enough material to block out light from the stars behind it. A typical dark nebula is about 20 light years in diameter and contains about 50 hydrogen atoms per cubic centimeter (monatomic hydrogen, \( \text{H}_2 \)) at a temperature of about 20 K. (A light year is the distance light travels in vacuum in one year and is equal to \( 9.46 \times 10^{16} \text{ m} \).) a) Estimate the mean free path for a hydrogen atom in a dark nebula. The radius of a hydrogen atom is \( 5.0 \times 10^{-11} \) m. b) Estimate the rms speed of a hydrogen
atom and the mean free time (the average time between collisions for a given atom). Based on this result, do you think that atomic collisions, such as those leading to H₂ molecule formation, are very important in determining the composition of the nebula? c) Estimate the pressure inside a dark nebula. d) Compare the rms speed of a hydrogen atom to the escape speed at the surface of the nebula (assumed spherical). If the space around the nebula were a vacuum, would such a cloud be stable or would it tend to evaporate? e) The stability of dark nebulae is explained by the presence of the interstellar medium (ISM), an even thinner gas that permeates space and into which the dark nebulae are embedded. Show that for dark nebulae to be in equilibrium with the ISM, the numbers of atoms per volume \( \frac{N}{V} \) and the temperatures \( T \) of dark nebulae and the ISM must be related by

\[
\frac{\left( \frac{N}{V} \right)_{\text{nebula}}}{\left( \frac{N}{V} \right)_{\text{ISM}}} = \frac{T_{\text{ISM}}}{T_{\text{nebula}}}
\]

f) In the vicinity of the sun, the ISM contains about 1 hydrogen atom per 200 cm³. Estimate the temperature of the ISM in the vicinity of the sun. Compare to the temperature of the sun’s surface, about 5800 K. Would a spacecraft coasting through interstellar space burn up? Why or why not?

18.88 In the troposphere, the part of the atmosphere that extends from the surface to an altitude of about 11 km, the temperature is not uniform but decreases with increasing elevation. a) Show that if the temperature variation is approximated by the linear relation

\[ T = T_0 - \alpha y \]

where \( T_0 \) is the temperature at the earth’s surface and \( T \) is the temperature at height \( y \), the pressure \( p \) at height \( y \) is given by

\[ \ln \left( \frac{p}{p_0} \right) = \frac{M}{R \alpha} \ln \left( \frac{T_0 - \alpha y}{T_0} \right) \]

where \( p_0 \) is the pressure at the earth’s surface and \( M \) is the molar mass of air. The coefficient \( \alpha \) is called the lapse rate of temperature. It varies with atmospheric conditions, but an average value is about 0.6 °C/100 m. b) Show that the above result reduces to the result of Example 18.4 (Section 18.1) in the limit that \( \alpha \to 0 \).

c) With \( \alpha = 0.6 \, \text{°C}/100 \, \text{m} \), calculate \( p \) for \( y = 8863 \, \text{m} \) and compare your answer to the result of Example 18.4. Take \( T_0 = 288 \, \text{K} \) and \( p_0 = 1.00 \, \text{atm} \).

18.89 Van der Waals Equation and Critical Points. a) In \( pV \) diagrams the slope \( \partial p/\partial V \) along an isotherm is never positive. Explain why. b) Regions where \( \partial p/\partial V = 0 \) represent equilibrium between two phases; volume can change with no change in pressure, as when water boils at atmospheric pressure. We can use this to determine the temperature, pressure, and volume per mole at the critical point using the equation of state \( p = p(V, T, n) \). If \( T > T_c \), then \( p(V) \) has no maximum along an isotherm, but if \( T < T_c \), then \( p(V) \) has a maximum. Show how this leads to the following condition for determining the critical point:

\[ \frac{\partial p}{\partial V} = 0 \quad \text{and} \quad \frac{\partial^2 p}{\partial V^2} = 0 \]

at the critical point.

c) Solve the van der Waals equation (Eq. 18.7) for \( p \). That is, find \( p(V, T, n) \). Find \( \partial p/\partial V \) and \( \partial^2 p/\partial V^2 \). Set these equal to zero to obtain two equations for \( V, T, \) and \( n \). d) Simultaneous solution of the two equations obtained in part (c) gives the temperature and volume per mole at the critical point, \( T_c \) and \( (V/n)_c \). Find these constants in terms of \( a \) and \( b \). (Hint: Divide one equation by the other to eliminate \( T \).) e) Substitute these values into the equation of state to find \( p_c \), the pressure at the critical point. f) Use the results from parts (d) and (e) to find the ratio \( \nu_c / p_c (V/n)_c \). This should not contain either \( a \) or \( b \), and so should have the same value for all gases. g) Compute the ratio \( \nu_c / p_c (V/n)_c \) for the gases \( \text{H}_2, \text{N}_2, \) and \( \text{H}_2\text{O} \) using the critical point data given below.

<table>
<thead>
<tr>
<th>Gas</th>
<th>( T_c ) (K)</th>
<th>( p_c ) (Pa)</th>
<th>( (V/n)_c ) (m³/mol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{H}_2 )</td>
<td>33.3</td>
<td>( 13.0 \times 10^5 )</td>
<td>( 65.0 \times 10^{-6} )</td>
</tr>
<tr>
<td>( \text{N}_2 )</td>
<td>126.2</td>
<td>( 33.9 \times 10^5 )</td>
<td>( 90.1 \times 10^{-6} )</td>
</tr>
<tr>
<td>( \text{H}_2\text{O} )</td>
<td>647.4</td>
<td>( 221.2 \times 10^5 )</td>
<td>( 56.0 \times 10^{-6} )</td>
</tr>
</tbody>
</table>

h) Discuss how well the results of part (g) compare to the prediction of part (f) based on the van der Waals equation. What do you conclude about the accuracy of the van der Waals equation as a description of the behavior of gases near the critical point?

18.90 In Example 18.7 (Section 18.3) we saw that \( \nu_{\text{rms}} > \nu_{\text{av}} \). It is not difficult to show that this is always the case. (The only exception is when the particles have the same speed, in which case \( \nu_{\text{rms}} = \nu_{\text{av}} \).) a) For two particles with speeds \( v_1 \) and \( v_2 \), show that \( \nu_{\text{rms}} \geq \nu_{\text{av}} \), regardless of the numerical values of \( v_1 \) and \( v_2 \). Then show that \( \nu_{\text{rms}} > \nu_{\text{av}} \) if \( v_1 \neq v_2 \). Suppose that for a collection of \( N \) particles you know that \( \nu_{\text{rms}} > \nu_{\text{av}} \). Another particle, with speed \( u \), is added to the collection of particles. If the new rms and average speeds are denoted as \( \nu'_{\text{rms}} \) and \( \nu'_{\text{av}} \), show that

\[ \nu'_{\text{rms}} = \sqrt{\frac{N\nu_{\text{rms}}^2 + u^2}{N + 1}} \quad \text{and} \quad \nu'_{\text{av}} = \frac{N\nu_{\text{av}} + u}{N + 1} \]

c) Use the expressions in part (b) to show that \( \nu'_{\text{rms}} > \nu'_{\text{av}} \) regardless of the numerical value of \( u \). d) Explain why your results for (a) and (c) together show that \( \nu_{\text{rms}} > \nu_{\text{av}} \) for any collection of particles if the particles do not all have the same speed.