 Perhaps the highest-temperature material you will ever see is the sun's outer atmosphere, or corona. At a temperature of about 2,000,000°C (3,600,000°F), the corona glows with a light that is literally unearthly. But because the corona is also very thin, its light is rather faint. You can only see the corona during a total solar eclipse when the sun's disk is covered by the moon, as shown here.

Is it accurate to say that the corona contains heat?

Whether it's a sweltering summer day or a frozen midwinter night, your body needs to be kept at a nearly constant temperature. It has effective temperature-control mechanisms, but sometimes it needs help. On a hot day you wear less clothing to improve heat transfer from your body to the air and for better cooling by evaporation of perspiration. You probably drink cold beverages possibly with ice in them, and sit near a fan or in an air-conditioned room. On a cold day you wear more clothes or stay indoors where it's warm. When you're outside, you keep active and drink hot liquids to stay warm. The concepts in this chapter will help you understand the basic physics of keeping warm or cool.

The terms temperature and heat are often used interchangeably in everyday language. In physics, however, these two terms have very different meanings. In this chapter we'll define temperature in terms of how it's measured and see how temperature changes affect the dimensions of objects. We'll see that heat refers to energy transfer caused by temperature differences and learn how to calculate and control such energy transfers.

Our emphasis in this chapter is on the concepts of temperature and heat as they relate to macroscopic objects such as cylinders of gas, ice cubes, and the human body. In Chapter 18 we'll look at these same concepts from a microscopic viewpoint in terms of the behavior of individual atoms and molecules. These two chapters lay the groundwork for the subject of thermodynamics, the study of energy.
transformations involving heat, mechanical work, and other aspects of energy and how these transformations relate to the properties of matter. Thermodynamics forms an indispensable part of the foundation of physics, chemistry, and the life sciences, and its applications turn up in such places as car engines, refrigerators, biochemical processes, and the structure of stars. We’ll explore the key ideas of thermodynamics in Chapters 19 and 20.

17.1 | Temperature and Thermal Equilibrium

The concept of temperature is rooted in qualitative ideas of “hot” and “cold” based on our sense of touch. A body that feels hot usually has a higher temperature than a similar body that feels cold. That’s pretty vague, and the senses can be deceived. But many properties of matter that we can measure depend on temperature. The length of a metal rod, steam pressure in a boiler, the ability of a wire to conduct an electric current, and the color of a very hot glowing object—all these depend on temperature.

Temperature is also related to the kinetic energies of the molecules of a material. In general this relationship is fairly complex, so it’s not a good place to start defining temperature. In Chapter 18 we will look at the relationship between temperature and the energy of molecular motion for an ideal gas. It is important to understand, however, that temperature and heat can be defined independently of any detailed molecular picture. In this section we’ll develop a macroscopic definition of temperature.

To use temperature as a measure of hotness or coldness, we need to construct a temperature scale. To do this, we can use any measurable property of a system that varies with its “hotness” or “coldness.” Figure 17.1a shows a familiar system that is used to measure temperature. When the system becomes hotter, the colored liquid (usually mercury or ethanol) expands and rises in the tube, and the value of \( L \) increases. Another simple system is a quantity of gas in a constant-volume container (Fig. 17.1b). The pressure \( p \), measured by the gauge, increases or decreases as the gas becomes hotter or colder. A third example is the electrical resistance \( R \) of a conducting wire, which also varies when the wire becomes hotter or colder. Each of these properties gives us a number \((L, p, R)\) that varies with hotness and coldness, so each property can be used to make a thermometer.

To measure the temperature of a body, you place the thermometer in contact with the body. If you want to know the temperature of a cup of hot coffee, you stick the thermometer in the coffee; as the two interact, the thermometer becomes hotter and the coffee cools off a little. After the thermometer settles down to a steady value, you read the temperature. The system has reached an equilibrium condition, in which the interaction between the thermometer and the coffee causes no further change in the system. We call this a state of thermal equilibrium.

If two systems are separated by an insulating material or insulator such as wood, plastic foam, or fiberglass, they influence each other more slowly. Camping coolers are made with insulating materials to delay the ice and cold food inside from warming up and attaining thermal equilibrium with the hot summer air outside. An ideal insulator is a material that permits no interaction at all between the two systems. It prevents the systems from attaining thermal equilibrium if they aren’t in thermal equilibrium at the start. An ideal insulator is just that, an idealization; real insulators, like those in camping coolers, aren’t ideal, so the contents of the cooler will warm up eventually.
We can discover an important property of thermal equilibrium by considering three systems, A, B, and C, that initially are not in thermal equilibrium (Fig. 17.2). We surround them with an ideal insulating box so that they cannot interact with anything except each other. We separate systems A and B with an ideal insulating wall (the blue slab in Fig. 17.2a), but we let system C interact with both systems A and B. This interaction is shown in the figure by an orange slab representing a thermal conductor, a material that permits thermal interactions through it. We wait until thermal equilibrium is attained; then A and B are each in thermal equilibrium with C. But are they in thermal equilibrium with each other?

To find out, we separate system C from systems A and B with an ideal insulating wall (Fig. 17.2b), and then we replace the insulating wall between A and B with a conducting wall that lets A and B interact. What happens? Experiment shows that nothing happens; there are no additional changes to A or B. We conclude that if C is initially in thermal equilibrium with both A and B, then A and B are also in thermal equilibrium with each other. This result is called the zeroth law of thermodynamics. (The importance of this law was recognized only after the first, second, and third laws of thermodynamics had been named. Since it is fundamental to all of them, the name "zeroth" seemed appropriate.)

Now suppose system C is a thermometer, such as the tube-and-liquid system of Fig. 17.1a. In Fig. 17.2a the thermometer C is in contact with both A and B. In thermal equilibrium, when the thermometer reading reaches a stable value, the thermometer measures the temperature of both A and B; hence A and B both have the same temperature. Experiment shows that thermal equilibrium isn’t affected by adding or removing insulators, so the reading of thermometer C wouldn’t change if it were in contact only with A or only with B. We conclude that two systems are in thermal equilibrium if and only if they have the same temperature. This is what makes a thermometer useful; a thermometer actually measures its own temperature, but when a thermometer is in thermal equilibrium with another body, the temperatures must be equal. When the temperatures of two systems are different, they cannot be in thermal equilibrium.

Test Your Understanding

Why does a nurse taking your temperature wait for the thermometer reading to stop changing? What object is it whose temperature the nurse is reading?

17.2 | Thermometers and Temperature Scales

To make the liquid-in-tube device shown in Fig. 17.1a into a useful thermometer, we need to mark a scale on the tube wall with numbers on it. These numbers are arbitrary, and historically many different schemes have been used. Suppose we label the thermometer’s liquid level at the freezing temperature of pure water “zero” and the level at the boiling temperature “100,” and divide the distance between these two points into 100 equal intervals called degrees. The result is the Celsius temperature scale (formerly called the centigrade scale in English-speaking countries). The Celsius temperature for a state colder than freezing water is a negative number. The Celsius scale is used, both in everyday life and in science and industry, almost everywhere in the world.

Another common type of thermometer uses a bimetallic strip, made by bonding strips of two different metals together (Fig. 17.3a). When the temperature of
the composite strip increases, one metal expands more than the other and the strip bends. This strip is usually formed into a spiral, with the outer end anchored to the thermometer case and the inner end attached to a pointer (Fig. 17.3c). The pointer rotates in response to temperature changes.

In a resistance thermometer the changing electrical resistance of a coil of fine wire, a carbon cylinder, or a germanium crystal is measured. Because resistance can be measured very precisely, resistance thermometers are usually more precise than most other types.

Some thermometers don’t require physical contact with the object they’re measuring. An example is an ear thermometer (Fig. 17.4). Such a thermometer uses a device called a thermopile to measure the amount of infrared radiation emitted by the eardrum, which indicates the eardrum’s temperature. (We’ll see in Section 17.7 that all objects emit electromagnetic radiation as a consequence of their temperature.) The advantage of this technique is that it doesn’t require touching the fragile and easily damaged eardrum.

In the Fahrenheit temperature scale, still used in everyday life in the United States, the freezing temperature of water is 32°F (thirty-two degrees Fahrenheit) and the boiling temperature is 212°F, both at standard atmospheric pressure. There are 180 degrees between freezing and boiling, compared to 100 on the Celsius scale, so one Fahrenheit degree represents only \( \frac{100}{180} \) or \( \frac{5}{9} \), as great a temperature change as one Celsius degree.

To convert temperatures from Celsius to Fahrenheit, note that a Celsius temperature \( T_c \) is the number of Celsius degrees above freezing; the number of Fahrenheit degrees above freezing is \( \frac{9}{5} \) of this. But freezing on the Fahrenheit scale is at 32°F, so to obtain the actual Fahrenheit temperature \( T_f \), multiply the Celsius value by \( \frac{9}{5} \) and then add 32°F. Symbolically,

\[
T_f = \frac{9}{5} T_c + 32^\circ \quad (17.1)
\]

To convert Fahrenheit to Celsius, solve this equation for \( T_c \):

\[
T_c = \frac{5}{9} (T_f - 32^\circ) \quad (17.2)
\]

In words, subtract 32°F to get the number of Fahrenheit degrees above freezing, and then multiply by \( \frac{5}{9} \) to obtain the number of Celsius degrees above freezing, that is, the Celsius temperature.

We don’t recommend memorizing Eqs. (17.1) and (17.2). Instead, try to understand the reasoning that led to them so that you can derive them on the spot when you need them, checking your reasoning with the relation 100°C = 212°F.

It is useful to distinguish between an actual temperature and a temperature interval (a difference or change in temperature). An actual temperature of 20°C is stated as 20°C (twenty degrees Celsius), and a temperature interval of 10°C is 10°C (ten Celsius degrees). A beaker of water heated from 20°C to 30°C has a temperature change of 10°C.

**Test Your Understanding**

Find the average Fahrenheit temperature on the planet Venus (average Celsius temperature 460°C), and find the temperature at which the Fahrenheit and Celsius scales agree.
17.3 | Gas Thermometers and the Kelvin Scale

When we calibrate two thermometers, such as a liquid-in-tube system and a resistance thermometer, so that they agree at 0°C and 100°C, they may not agree exactly at intermediate temperatures. Any temperature scale defined in this way always depends somewhat on the specific properties of the material used. Ideally, we would like to define a temperature scale that doesn’t depend on the properties of a particular material. To establish a truly material-independent scale, we first need to develop some principles of thermodynamics. We’ll return to this fundamental problem in Chapter 20. Here we’ll discuss a thermometer that comes close to the ideal, the gas thermometer.

The principle of a gas thermometer is that the pressure of a gas at constant volume increases with temperature. A quantity of gas is placed in a constant-volume container (Fig. 17.5a), and its pressure is measured by one of the devices described in Section 14.2. To calibrate a constant-volume gas thermometer, we measure the pressure at two temperatures, say 0°C and 100°C, plot these points on a graph, and draw a straight line between them. Then we can read from the graph the temperature corresponding to any other pressure. Figure 17.5b shows the results of three such experiments, each using a different type and quantity of gas.

By extrapolating this graph, we see that there is a hypothetical temperature, $-273.15^\circ C$, at which the absolute pressure of the gas would become zero. We might expect that this temperature would be different for different gases, but it turns out to be the same for many different gases (at least in the limit of very low gas density). We can’t actually observe this zero-pressure condition. Gases liquefy and solidify at very low temperatures, and the proportionality of pressure to temperature no longer holds.

We use this extrapolated zero-pressure temperature as the basis for a temperature scale with its zero at this temperature. This is the Kelvin temperature scale,
Gas Thermometers and the Kelvin Scale

The units are the same size as those on the Celsius scale, but the zero is shifted so that 0 K = -273.15°C and 273.15 K = 0°C; that is,

\[ T_k = T_C + 273.15 \]  

This scale is shown in Fig. 17.5b. A common room temperature, 20°C (= 68°F), is 20 + 273.15, or about 293 K.

**CAUTION** In SI nomenclature, “degree” is not used with the Kelvin scale; the temperature mentioned above is read “293 kelvins,” not “degrees Kelvin” (Fig. 17.6). We capitalize Kelvin when it refers to the temperature scale; however, the unit of temperature is the kelvin, which is not capitalized (but is nonetheless abbreviated as a capital K).

**Example 17.1** Body temperature

You place a small piece of melting ice in your mouth. Eventually, the water all converts from ice at \( T_f = 32.00°F \) to body temperature, \( T_B = 98.60°F \). Express these temperatures as °C and K, and find \( \Delta T = T_B - T_f \) in both cases.

**SOLUTION**

**IDENTIFY and SET UP:** We convert Fahrenheit to Celsius temperatures using Eq. (17.2), and Celsius to Kelvin temperatures using Eq. (17.3).

**EXECUTE:** First we find the Celsius temperatures. We know that \( T_f = 32.00°F = 0.00°C \), and 98.60°F is 98.60 - 32.00 = 66.60°F above freezing; multiply this by \( \frac{5°C}{9°F} \) to find 37.00°C above freezing, or \( T_B = 37.00°C \).

To get the Kelvin temperatures, we just add 273.15 to each Celsius temperature: \( T_f = 273.15 K \) and \( T_B = 310.15 K \). “Normal” body temperature is 37.0°C, but if your doctor says that your temperature is 310 K, don’t be alarmed.

The temperature difference \( \Delta T = T_B - T_f \) is 37.00°C = 37.00 K.

**EVALUATE:** The Celsius and Kelvin scales have different zero points but the same size degrees. Therefore any temperature difference is the same on the Celsius and Kelvin scales but not the same on the Fahrenheit scale.

The Celsius scale has two fixed points, the normal freezing and boiling temperatures of water. But we can define the Kelvin scale using a gas thermometer with only a single reference temperature. We define the ratio of any two temperatures \( T_1 \) and \( T_2 \) on the Kelvin scale as the ratio of the corresponding gas-thermometer pressures \( p_1 \) and \( p_2 \):

\[ \frac{T_2}{T_1} = \frac{p_2}{p_1} \quad \text{(constant-volume gas thermometer, } T \text{ in kelvins)} \]  

The pressure \( p \) is directly proportional to the Kelvin temperature, as shown in Fig. 17.5b. To complete the definition of \( T \), we need only specify the Kelvin temperature of a single specific state. For reasons of precision and reproducibility the state
chosen is the *triple point* of water. This is the unique combination of temperature and pressure at which solid water (ice), liquid water, and water vapor can all coexist. It occurs at a temperature of 0.01°C and a water-vapor pressure of 610 Pa (about 0.006 atm). (This is the pressure of the *water*: it has nothing to do directly with the gas pressure in the *thermometer*.) The triple-point temperature \( T_{\text{triple}} \) of water is *defined* to have the value \( T_{\text{triple}} = 273.16 \text{ K} \), corresponding to 0.01°C. From Eq. (17.4), if \( p_{\text{triple}} \) is the pressure in a gas thermometer at temperature \( T_{\text{triple}} \) and \( p \) is the pressure at some other temperature \( T \), then \( T \) is given on the Kelvin scale by

\[
T = T_{\text{triple}} \frac{p}{p_{\text{triple}}} = (273.16 \text{ K}) \frac{p}{p_{\text{triple}}} \tag{17.5}
\]

Low-pressure gas thermometers using various gases are found to agree very closely, but they are large, bulky, and very slow to come to thermal equilibrium. They are used principally to establish high-precision standards and to calibrate other thermometers.

The relationships among the three temperature scales we have discussed are shown graphically in Fig. 17.7. The Kelvin scale is called an *absolute temperature scale*, and its zero point (\( T = 0 \text{ K} = -273.15\text{°C} \), the temperature at which \( p = 0 \) in Eq. (17.5)) is called *absolute zero*. At absolute zero a system of molecules (such as a quantity of a gas, a liquid, or a solid) has its *minimum* possible total energy (kinetic plus potential); because of quantum effects, however, it is *not* correct to say that all molecular motion ceases at absolute zero. To define more completely what we mean by absolute zero, we need to use the thermodynamic principles developed in the next several chapters. We will return to this concept in Chapter 20.

**Test Your Understanding**

The temperature of the solar corona (see the photograph that opens this chapter) is \( 2.0 \times 10^7 \text{°C} \), and the temperature at which helium becomes a liquid at standard pressure is \(-268.93\text{°C}\). Express these temperatures in kelvins, and explain why the Kelvin scale is usually employed for very high and very low temperatures.

### 17.4 | Thermal Expansion

Most materials expand when their temperatures increase. Rising temperatures make the liquid expand in a liquid-in-tube thermometer (Fig. 17.1a) and bend
bimetallic strips (Fig. 17.3). The decks of bridges need special joints and supports to allow for expansion. A completely filled and tightly capped bottle of water cracks when it is heated, but you can loosen a metal jar lid by running hot water over it. These are all examples of thermal expansion.

**Linear Expansion**

Suppose a rod of material has a length \( L_0 \) at some initial temperature \( T_0 \). When the temperature changes by \( \Delta T \), the length changes by \( \Delta L \). Experiments show that if \( \Delta T \) is not too large (say, less than 100°C or so), \( \Delta L \) is directly proportional to \( \Delta T \). If two rods made of the same material have the same temperature change, but one is twice as long as the other, then the change in its length is also twice as great. Therefore \( \Delta L \) must also be proportional to \( L_0 \). Introducing a proportionality constant \( \alpha \) (which is different for different materials), we may express these relations in an equation:

\[
\Delta L = \alpha L_0 \Delta T \quad \text{(linear thermal expansion)} \quad (17.6)
\]

If a body has length \( L_0 \) at temperature \( T_0 \), then its length \( L \) at a temperature \( T = T_0 + \Delta T \) is

\[
L = L_0 + \Delta L = L_0 + \alpha L_0 \Delta T = L_0 (1 + \alpha \Delta T) \quad (17.7)
\]

The constant \( \alpha \), which describes the thermal expansion properties of a particular material, is called the **coefficient of linear expansion**. The units of \( \alpha \) are K\(^{-1}\) or (°C\(^{-1}\)). (Remember that a temperature interval is the same in the Kelvin and Celsius scales.) For many materials, every linear dimension changes according to Eq. (17.6) or (17.7). Thus \( L \) could be the thickness of a rod, the side length of a square sheet, or the diameter of a hole. Some materials, such as wood or single crystals, expand differently in different directions. We won’t consider this complication.

We can understand thermal expansion qualitatively on a molecular basis. Picture the interatomic forces in a solid as springs, as in Fig. 17.8. (We explored the analogy between spring forces and interatomic forces in Section 13.4.) Each atom vibrates about its equilibrium position. When the temperature increases, the energy and amplitude of the vibration also increase. The interatomic spring forces are not symmetrical about the equilibrium position; they usually behave like a spring that is easier to stretch than to compress. As a result, when the amplitude of vibration increases, the average distance between atoms also increases. As the atoms get farther apart, every dimension increases.

**CAUTION** If a solid object has a hole in it, what happens to the size of the hole when the temperature of the object increases? A common misconception is that if the object expands, the hole will shrink because material expands into the hole. But the truth of the matter is that if the object expands, the hole will expand too (Fig. 17.9); as we stated above, every linear dimension of an object changes in the same way when the temperature changes. If you’re not convinced, think of the atoms in Fig. 17.8a as outlining a cubical hole. When the object expands, the atoms move apart and the hole
increases in size. The only situation in which a “hole” will fill in due to thermal expansion is when two separate objects expand and close the gap between them (Fig. 17.10).

The direct proportionality expressed by Eq. (17.6) is not exact; it is approximately correct only for sufficiently small temperature changes. For a given material, $\alpha$ varies somewhat with the initial temperature $T_0$ and the size of the temperature interval. We’ll ignore this complication here, however. Average values of $\alpha$ for several materials are listed in Table 17.1. Within the precision of these values we don’t need to worry whether $T_0$ is 0°C or 20°C or some other temperature. Note that typical values of $\alpha$ are very small; even for a temperature change of 100°C, the fractional length change $\Delta L/L_0$ is only of the order of 1/1000 for the metals in the table.

### Volume Expansion

Increasing temperature usually causes increases in volume for both solid and liquid materials. Just as with linear expansion, experiments show that if the temperature change $\Delta T$ is not too great (less than 100°C or so), the increase in volume $\Delta V$ is approximately proportional to both the temperature change $\Delta T$ and the initial volume $V_0$:

$$\Delta V = \beta V_0 \Delta T \quad \text{(volume thermal expansion)} \quad (17.8)$$

The constant $\beta$ characterizes the volume expansion properties of a particular material; it is called the coefficient of volume expansion. The units of $\beta$ are K$^{-1}$ or $(^\circ C)^{-1}$. As with linear expansion, $\beta$ varies somewhat with temperature, and Eq. (17.8) is an approximate relation that is valid only for small temperature changes. For many substances, $\beta$ decreases at low temperatures. Several values of $\beta$ in the neighborhood of room temperature are listed in Table 17.2. Note that the values for liquids are generally much larger than those for solids.

For solid materials there is a simple relation between the volume expansion coefficient $\beta$ and the linear expansion coefficient $\alpha$. To derive this relation, we consider a cube of material with side length $L$ and volume $V = L^3$. At the initial temperature the values are $L_0$ and $V_0$. When the temperature increases by $dT$, the side length increases by $dL$ and the volume increases by an amount $dV$ given by

$$dV = \frac{dV}{dL} dL = 3L^2 dL$$

Now we replace $L$ and $V$ by the initial values $L_0$ and $V_0$. From Eq. (17.6), $dL$ is

$$dL = \alpha L_0 dT$$

since $V_0 = L_0^3$, this means that $dV$ can also be expressed as

$$dV = 3L_0^2 \alpha L_0 dT = 3\alpha V_0 dT$$

This is consistent with the infinitesimal form of Eq. (17.8), $dV = \beta V_0 dT$, only if

$$\beta = 3\alpha \quad (17.9)$$

You should check this relation for some of the materials listed in Tables 17.1 and 17.2.
### Thermal Expansion

**Identify the relevant concepts:** Decide whether the problem involves changes in length (linear thermal expansion) or in volume (volume thermal expansion).

**Set up the problem using the following steps:**
1. Choose Eq. (17.6) for linear expansion and Eq. (17.8) for volume expansion.
2. Identify which quantities in Eq. (17.6) or (17.8) are known and which are the unknown target variables.

**Execute the solution as follows:**
1. Solve for the target variables. Often you will be given two temperatures and asked to compute $\Delta T$. Or you may be given an initial temperature $T_0$ and asked to find a final temperature corresponding to a given length or volume change. In this case, plan to find $\Delta T$ first; then the final temperature is $T_0 + \Delta T$.
2. Unit consistency is crucial, as always. $L_0$ and $\Delta L$ (or $V_0$ and $\Delta V$) must have the same units, and if you use a value of $a$ or $b$ in $K^{-1}$ or $(C^\circ)^{-1}$, then $\Delta T$ must be in kelvins or Celsius degrees ($C^\circ$). But you can use $K$ and $C^\circ$ interchangeably.

**Evaluate your answer:** Check whether your results make sense. Remember that the sizes of holes in a material expand with temperature just the same way as any other linear dimension, and the volume of a hole (such as the volume of a container) expands the same way as the corresponding solid shape.

---

### Example 17.2

**Length change due to temperature change I**

A surveyor uses a steel measuring tape that is exactly 50.000 m long at a temperature of 20°C. What is its length on a hot summer day when the temperature is 35°C?

**Solution**

**Identify and set up:** This problem concerns linear expansion, so we use Eq. (17.6). We are given $L = 50.000$ m, $T_0 = 20^\circ C$, and $T = 35^\circ C$, and the value of $a$ is found from Table 17.1. The target variable is the new length $L$.

**Execute:** The temperature change is $\Delta T = T - T_0 = 15^\circ C$, so from Eq. (17.6) the change in length $\Delta L$ and the final length $L = L_0 + \Delta L$ are

$$\Delta L = aL_0 \Delta T = (1.2 \times 10^{-5} \text{ K}^{-1})(50 \text{ m})(15 \text{ K}) = 9.0 \times 10^{-3} \text{ m} = 9.0 \text{ mm}$$

$$L = L_0 + \Delta L = 50.000 \text{ m} + 0.009 \text{ m} = 50.009 \text{ m}$$

Thus the length at 35°C is 50.009 m.
**EVALUATE:** Note that $L_0$ is given to five significant figures but that we need only two of them to compute $\Delta L$. Note also that $\Delta L$ is proportional to the initial length $L_0$: a 50-m tape would expand by 0.90 mm, and a 0.50-m (50-cm) tape would expand by a mere 0.090 mm.

This example shows that metals expand very little under moderate temperature changes. Even a metal baking pan in a 200°C (392°F) oven is only slightly larger than it is at room temperature.

**Example 17.3**

**Length change due to temperature change II**

In Example 17.2 the surveyor uses the measuring tape to measure a distance when the temperature is 35°C; the value that she reads off the tape is 35.794 m. What is the actual distance? Assume that the tape is calibrated for use at 20°C.

**EXECUTE:** The ratio $L/L_0$ is $(50.009\, \text{m})/(50.000\, \text{m})$, so the true distance is

$$\frac{50.009\, \text{m}}{50.000\, \text{m}} = 35.800\, \text{m}$$

**EVALUATE:** Although the difference of 0.008 m = 8 mm between the scale reading and the actual distance seems small, it can be important in precision work.

**Example 17.4**

**Volume change due to temperature change**

A glass flask with volume 200 cm$^3$ is filled to the brim with mercury at 20°C. How much mercury overflows when the temperature of the system is raised to 100°C? The coefficient of linear expansion of the glass is $0.40 \times 10^{-5} \, \text{K}^{-1}$.

**SOLUTION**

**IDENTIFY** and **SET UP:** As we saw in Example 17.2, at 35°C the tape has expanded slightly. The distance between two successive meter marks is slightly more than one meter, so the scale underestimates the actual distance. Hence the actual distance is larger than the distance read off the tape by a factor equal to the ratio of the tape's length $L$ at 35°C to its length $L_0$ at 20°C.

**EXECUTE:** The coefficient of volume expansion for the glass is

$$\Delta V_{\text{glass}} = \beta_{\text{glass}} V_0 \Delta T$$

where

$$\beta_{\text{glass}} = 3\alpha_{\text{glass}} = 3(0.40 \times 10^{-5} \, \text{K}^{-1}) = 1.2 \times 10^{-5} \, \text{K}^{-1}$$

The increase in volume of the glass flask is

$$\Delta V_{\text{glass}} = (1.2 \times 10^{-5} \, \text{K}^{-1})(200\, \text{cm}^3)(100°C - 20°C) = 0.19\, \text{cm}^3$$

The increase in volume of the mercury is

$$\Delta V_{\text{mercury}} = \beta_{\text{mercury}} V_0 \Delta T$$

where

$$\beta_{\text{mercury}} = 18 \times 10^{-5} \, \text{K}^{-1}$$

The volume of mercury that overflows is

$$\Delta V_{\text{mercury}} - \Delta V_{\text{glass}} = 2.9\, \text{cm}^3 - 0.19\, \text{cm}^3 = 2.7\, \text{cm}^3$$

**EVALUATE:** This is basically how a mercury-in-glass thermometer works, except that instead of letting the mercury overflow and run all over the place, the thermometer has it rise inside a sealed tube as $T$ increases.

As Tables 17.1 and 17.2 show, glass has smaller coefficients of expansion $\alpha$ and $\beta$ than do most metals. This is why you can use hot water to loosen a metal lid on a glass jar; the metal expands more than the glass does.

**Thermal Expansion of Water**

Water, in the temperature range from 0°C to 4°C, decreases in volume with increasing temperature. In this range its coefficient of volume expansion is negative. Above 4°C, water expands when heated (Fig. 17.11). Hence water has its
greatest density at 4°C. Water also expands when it freezes, which is why ice humps up in the middle of the compartments in an ice cube tray. By contrast, most materials contract when they freeze.

This anomalous behavior of water has an important effect on plant and animal life in lakes. A lake cools from the surface down; above 4°C, the cooled water at the surface flows to the bottom because of its greater density. But when the surface temperature drops below 4°C, the water near the surface is less dense than the warmer water below. Hence the downward flow ceases, and the water near the surface remains colder than that at the bottom. As the surface freezes, the ice floats because it is less dense than water. The water at the bottom remains at 4°C until nearly the entire lake is frozen. If water behaved like most substances, contracting continuously on cooling and freezing, lakes would freeze from the bottom up. Circulation due to density differences would continuously carry warmer water to the surface for efficient cooling, and lakes would freeze solid much more easily. This would destroy all plant and animal life that cannot withstand freezing. If water did not have this special property, the evolution of life would have taken a very different course.

**Thermal Stress**

If we clamp the ends of a rod rigidly to prevent expansion or contraction and then change the temperature, tensile or compressive stresses called thermal stresses develop. The rod would like to expand or contract, but the clamps won’t let it. The resulting stresses may become large enough to strain the rod irreversibly or even break it. (You may want to review the discussion of stress and strain in Section 11.4).

Engineers must account for thermal stress when designing structures. Concrete highways and bridge decks usually have gaps between sections, filled with a flexible material or bridged by interlocking teeth (Fig. 17.12), to permit expansion and contraction of the concrete. Long steam pipes have expansion joints or U-shaped sections to prevent buckling or stretching with temperature changes. If one end of a steel bridge is rigidly fastened to its abutment, the other end usually rests on rollers.

To calculate the thermal stress in a clamped rod, we compute the amount the rod would expand (or contract) if not held and then find the stress needed to compress (or stretch) it back to its original length. Suppose that a rod with length $L_0$ and cross-sectional area $A$ is held at constant length while the temperature is reduced (negative $\Delta T$), causing a tensile stress. The fractional change in length if the rod were free to contract would be

$$\left(\frac{\Delta L}{L_0}\right)_{\text{thermal}} = \alpha \Delta T$$  \hspace{1cm} (17.10)

Both $\Delta L$ and $\Delta T$ are negative. The tension must increase by an amount $F$ that is just enough to produce an equal and opposite fractional change in length $(\Delta L/L_0)_{\text{tension}}$. From the definition of Young’s modulus, Eq. (11.10),

$$Y = \frac{F/A}{\Delta L/L_0} \quad \text{so} \quad \left(\frac{\Delta L}{L_0}\right)_{\text{tension}} = \frac{F}{AY}$$

If the length is to be constant, the total fractional change in length must be zero. From Eqs. (17.10) and (17.11), this means that

$$\left(\frac{\Delta L}{L_0}\right)_{\text{thermal}} + \left(\frac{\Delta L}{L_0}\right)_{\text{tension}} = \alpha \Delta T + \frac{F}{AY} = 0$$

17.11 The volume of one gram of water in the temperature range from 0°C to 10°C. By 100°C the volume has increased to 1.034 cm³. If the coefficient of volume expansion were constant, the curve would be a straight line.

17.12 The interlocking teeth of an expansion joint on a bridge. These joints are needed to accommodate changes in length that result from thermal expansion.
Solving for the tensile stress $F/A$ required to keep the rod's length constant, we find

$$\frac{F}{A} = -Y\alpha \Delta T \quad \text{(thermal stress)} \quad (17.12)$$

For a decrease in temperature, $\Delta T$ is negative, so $F$ and $F/A$ are positive; this means that a tensile force and stress are needed to maintain the length. If $\Delta T$ is positive, $F$ and $F/A$ are negative, and the required force and stress are compressive.

If there are temperature differences within a body, non-uniform expansion or contraction will result and thermal stresses can be induced. You can break a glass bowl by pouring very hot water into it; the thermal stress between the hot and cold parts of the bowl exceeds the breaking stress of the glass, causing cracks. The same phenomenon makes ice cubes crack when dropped into warm water. Heat-resistant glasses such as PyrexTM have exceptionally low expansion coefficients and high strength.

**Example 17.5**  
**Thermal stress**

An aluminum cylinder 10 cm long, with a cross-sectional area of 20 cm², is to be used as a spacer between two steel walls. At 17.2°C it just slips in between the walls. When it warms to 22.3°C, calculate the stress in the cylinder and the total force it exerts on each wall, assuming that the walls are perfectly rigid and a constant distance apart.

**SOLUTION**

**IDENTIFY and SET UP:** We use Eq. (17.12) to relate the stress (our target variable) to the temperature change. The relevant values of Young’s modulus $Y$ and the coefficient of linear expansion $\alpha$ are those for aluminum, the material of which the cylinder is made; we find these values from Tables 11.1 and 17.1, respectively.

**EXECUTE:** For aluminum, $Y = 7.0 \times 10^{10}$ Pa and $\alpha = 2.4 \times 10^{-5}$ K⁻¹. The temperature change is $\Delta T = 22.3^\circ C - 17.2^\circ C = 5.1$ K. The stress is $F/A$; from Eq. (17.12),

$$\frac{F}{A} = -Y\alpha \Delta T = - (0.70 \times 10^{11} \text{ Pa}) (2.4 \times 10^{-5} \text{ K}^{-1}) (5.1 \text{ K})$$

$$= -8.6 \times 10^6 \text{ Pa} \quad \text{(or -1200 lb/in.}^2\text{)}$$

The negative sign indicates that compressive rather than tensile stress is needed to keep the cylinder’s length constant. This stress is independent of the length and cross-sectional area of the cylinder. The total force $F$ is the cross-sectional area times the stress:

$$F = A \frac{F}{A} = (20 \times 10^{-4} \text{ m}^2) (-8.6 \times 10^6 \text{ Pa})$$

$$= -1.7 \times 10^4 \text{ N}$$

or nearly two tons. The negative sign indicates compression.

**EVALUATE:** The stress on the cylinder and the force it exerts on each wall are immense. This points out the importance of accounting for such thermal stresses in engineering.

**Test Your Understanding**

In the bimetallic strip shown in Fig. 17.3, metal 1 is copper. What are two different materials that could be used for metal 2?

**17.5 | Quantity of Heat**

When you put a cold spoon into a cup of hot coffee, the spoon warms up and the coffee cools down as they approach thermal equilibrium. The interaction that causes these temperature changes is fundamentally a transfer of energy from one substance to another. Energy transfer that takes place solely because of a temper-
An understanding of the relation between heat and other forms of energy emerged gradually during the 18th and 19th centuries. Sir James Joule (1818–1889) studied how water can be warmed by vigorous stirring with a paddle wheel (Fig. 17.13a). The paddle wheel adds energy to the water by doing work on it, and Joule found that the temperature rise is directly proportional to the amount of work done. The same temperature change can also be caused by putting the water in contact with some hotter body (Fig. 17.13b); hence this interaction must also involve an energy exchange. We will explore the relation between heat and mechanical energy in greater detail in Chapters 19 and 20.

**CAUTION** It is absolutely essential for you to keep clearly in mind the distinction between temperature and heat. Temperature depends on the physical state of a material and is a quantitative description of its hotness or coldness. In physics the term “heat” always refers to energy in transit from one body or system to another because of a temperature difference, never to the amount of energy contained within a particular system. We can change the temperature of a body by adding heat to it or taking heat away, or by adding or subtracting energy in other ways, such as mechanical work (Fig. 17.13a). If we cut a body in half, each half has the same temperature as the whole; but to raise the temperature of each half by a given interval, we add half as much heat as for the whole.

We can define a unit of quantity of heat based on temperature changes of some specific material. The calorie (abbreviated cal) is defined as the amount of heat required to raise the temperature of one gram of water from 14.5°C to 15.5°C. The kilocalorie (kcal), equal to 1000 cal, is also used; a food-value calorie is actually a kilocalorie. A corresponding unit of heat using Fahrenheit degrees and British units is the British thermal unit, or Btu. One Btu is the quantity of heat required to raise the temperature of one pound (weight) of water 1°F from 63°F to 64°F.

Because heat is energy in transit, there must be a definite relation between these units and the familiar mechanical energy units such as the joule. Experiments similar in concept to Joule’s have shown that

\[
1 \text{ cal} = 4.186 \text{ J} \\
1 \text{kcal} = 1000 \text{ cal} = 4186 \text{ J} \\
1 \text{ Btu} = 778 \text{ ft-lb} = 252 \text{ cal} = 1055 \text{ J}
\]

The calorie is not a fundamental SI unit. The International Committee on Weights and Measures recommends using the joule as the basic unit of energy in all forms, including heat. We will follow that recommendation in this book.

**Specific Heat**

We use the symbol \( Q \) for quantity of heat. When it is associated with an infinitesimal temperature change \( dT \), we call it \( dQ \). The quantity of heat \( Q \) required to increase the temperature of a mass \( m \) of a certain material from \( T_1 \) to \( T_2 \) is found to be approximately proportional to the temperature change \( \Delta T = T_2 - T_1 \). It is also proportional to the mass \( m \) of material. When you’re heating water to make
CHAPTER 17 | Temperature and Heat

Tea, you need twice as much heat for two cups as for one if the temperature interval is the same. The quantity of heat needed also depends on the nature of the material; raising the temperature of one kilogram of water by 1°C requires 4190 J of heat, but only 910 J is needed to raise the temperature of one kilogram of aluminum by 1°C.

Putting all these relationships together, we have

\[ Q = mc \Delta T \quad \text{(heat required for temperature change of mass } m) \] (17.13)

where \( c \) is a quantity, different for different materials, called the specific heat of the material. For an infinitesimal temperature change \( dT \) and corresponding quantity of heat \( dQ \),

\[ dQ = mc \, dT \] (17.14)

\[ c = \frac{1}{m} \frac{dQ}{dT} \quad \text{(specific heat)} \] (17.15)

In Eqs. (17.13), (17.14), and (17.15), \( Q \) (or \( dQ \)) and \( \Delta T \) (or \( dT \)) can be either positive or negative. When they are positive, heat enters the body and its temperature increases; when they are negative, heat leaves the body and its temperature decreases.

**CAUTION** Remember that \( dQ \) does not represent a change in the amount of heat contained in a body; this is a meaningless concept. Heat is always energy in transit as a result of a temperature difference. There is no such thing as “the amount of heat in a body.”

The specific heat of water is approximately

\[ 4190 \text{ J/kg·K} \quad \frac{1}{3} \text{ cal/g·C}^\circ \quad \text{or } 1 \text{ Btu/lb·F}^\circ \]

The specific heat of a material always depends somewhat on the initial temperature and the temperature interval. Figure 17.14 shows this variation for water. In the problems and examples in this chapter we will usually ignore this small variation.

---

**Example 17.6** Feed a cold, starve a fever

During a bout with the flu an 80-kg man ran a fever of 39.0°C (102.2°F) instead of the normal body temperature of 37.0°C (98.6°F). Assuming that the human body is mostly water, how much heat is required to raise his temperature by that amount?

**SOLUTION**

**IDENTIFY:** This problem uses the relationship between heat (the target variable), mass, specific heat, and temperature change.

**SET UP:** We are given the values of \( m = 80 \text{ kg} \), \( c = 4190 \text{ J/kg·K} \) (for water), and \( \Delta T = 39.0°C - 37.0°C = 2.0 \text{ C}^\circ = 2.0 \text{ K} \). We use Eq. (17.13) to determine the required heat.

**EXECUTE:** From Eq. (17.13),

\[ Q = mc \Delta T = (80 \text{ kg})(4190 \text{ J/kg·K})(2.0 \text{ K}) = 6.7 \times 10^5 \text{ J} \]

**EVALUATE:** This corresponds to 160 kcal, or 160 food-value calories. In fact, the specific heat of the human body is more nearly equal to 3480 J/kg·K, about 83% that of water. The difference is due to the presence of protein, fat, and minerals, which have lower specific heats. With this value of \( c \), the required heat is \( 5.6 \times 10^5 \text{ J} = 133 \text{ kcal} \). Either result shows us that were it not for the body’s temperature-regulating systems, taking in energy in the form of food would produce measurable changes in body temperature. (In the case of a person with the flu, the elevated temperature results from the body’s extra activity in fighting infection.)
You are designing an electronic circuit element made of 23 mg of silicon. The electric current through it adds energy at the rate of 7.4 mW = 7.4 \times 10^{-3} J/s. If your design doesn’t allow any heat transfer out of the element, at what rate does its temperature increase? The specific heat of silicon is 705 J/kg\cdot K.

**SOLUTION**

**IDENTIFY and SET UP:** The target variable is the rate of temperature change. From Eq. (17.14), the temperature change \( \Delta T \) in kelvins is proportional to the heat transferred in joules, so the rate of temperature change in K/s is proportional to the rate of heat transfer in J/s.

**EXECUTE:** In one second, \( Q = (7.4 \times 10^{-3} \text{ J/s}) (1 \text{ s}) = 7.4 \times 10^{-3} \text{ J} \). From Eq. (17.13), \( Q = mc \Delta T \), the temperature change in one second is

\[
\Delta T = \frac{Q}{mc} = \frac{7.4 \times 10^{-3} \text{ J}}{(23 \times 10^{-6} \text{ kg})(705 \text{ J/kg} \cdot \text{K})} = 0.46 \text{ K/s}
\]

Alternatively, we can divide both sides of Eq. (17.14) by \( dt \) and rearrange:

\[
\frac{dT}{dt} = \frac{dQ/dt}{mc} = \frac{7.4 \times 10^{-3} \text{ J/s}}{(23 \times 10^{-6} \text{ kg})(705 \text{ J/kg} \cdot \text{K})} = 0.46 \text{ K/s}
\]

**EVALUATE:** At this rate of temperature rise (27 K every minute), the circuit element would soon self-destruct. Heat transfer is an important design consideration in electronic circuit elements.

---

**Molar Heat Capacity**

Sometimes it’s more convenient to describe a quantity of substance in terms of the number of moles \( n \) rather than the mass \( m \) of material. Recall from your study of chemistry that a mole of any pure substance always contains the same number of molecules. (We will discuss this point in more detail in Chapter 18.) The molar mass \( M \) of any substance, denoted by \( M \), is the mass per mole. (The quantity \( M \) is sometimes called molecular weight, but molar mass is preferable; the quantity depends on the mass of a molecule, not its weight.) For example, the molar mass of water is 18.0 g/mol = 18.0 \times 10^{-3} kg/mol; one mole of water has a mass of 18.0 g = 0.0180 kg. The total mass \( m \) of material is equal to the mass per mole \( M \) times the number of moles \( n \):

\[
m = nM
\]

Replacing the mass \( m \) in Eq. (17.13) by the product \( nM \), we find

\[
Q = nM \Delta T
\]

The product \( nM \) is called the molar heat capacity (or molar specific heat) and is denoted by \( C \) (capitalized). With this notation we rewrite Eq. (17.17) as

\[
Q = nC \Delta T \quad \text{(heat required for temperature change of } n \text{ moles)} \quad (17.18)
\]

Comparing to Eq. (17.15), we can express the molar heat capacity \( C \) (heat per mole per temperature change) in terms of the specific heat \( c \) (heat per mass per temperature change) and the molar mass \( M \) (mass per mole):

\[
C = \frac{1}{n} \frac{dQ}{dT} = \frac{M}{n} \quad \text{(molar heat capacity)} \quad (17.19)
\]

For example, the molar heat capacity of water is

\[
C = Mc = \left(0.0180 \text{ kg/mol}\right) \left(4190 \text{ J/kg} \cdot \text{K}\right) = 75.4 \text{ J/mol} \cdot \text{K}
\]
Table 17.3 Approximate Specific Heats and Molar Heat Capacities (Constant Pressure)

<table>
<thead>
<tr>
<th>Substance</th>
<th>Specific Heat, $c$ (J/kg·K)</th>
<th>M (kg/mol)</th>
<th>Molar Heat Capacity, $C$ (J/mol·K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>910</td>
<td>0.0270</td>
<td>24.6</td>
</tr>
<tr>
<td>Beryllium</td>
<td>1970</td>
<td>0.00901</td>
<td>17.7</td>
</tr>
<tr>
<td>Copper</td>
<td>390</td>
<td>0.0635</td>
<td>24.8</td>
</tr>
<tr>
<td>Ethanol</td>
<td>2428</td>
<td>0.0461</td>
<td>111.9</td>
</tr>
<tr>
<td>Ethylene glycol</td>
<td>2386</td>
<td>0.0620</td>
<td>148.0</td>
</tr>
<tr>
<td>Ice (near 0°C)</td>
<td>2100</td>
<td>0.0180</td>
<td>37.8</td>
</tr>
<tr>
<td>Iron</td>
<td>470</td>
<td>0.0559</td>
<td>26.3</td>
</tr>
<tr>
<td>Lead</td>
<td>130</td>
<td>0.207</td>
<td>26.9</td>
</tr>
<tr>
<td>Marble (CaCO$_3$)</td>
<td>879</td>
<td>0.100</td>
<td>87.9</td>
</tr>
<tr>
<td>Mercury</td>
<td>138</td>
<td>0.201</td>
<td>27.7</td>
</tr>
<tr>
<td>Salt (NaCl)</td>
<td>879</td>
<td>0.0585</td>
<td>51.4</td>
</tr>
<tr>
<td>Silver</td>
<td>234</td>
<td>0.108</td>
<td>25.3</td>
</tr>
<tr>
<td>Water (liquid)</td>
<td>4190</td>
<td>0.0180</td>
<td>75.4</td>
</tr>
</tbody>
</table>

Values of specific heat and molar heat capacity for several substances are given in Table 17.3. Note the remarkably large specific heat for water (Fig. 17.15).

**CAUTION** The term heat capacity is unfortunate because it gives the erroneous impression that a body contains a certain amount of heat. Remember, heat is energy in transit to or from a body, not the energy residing in the body.

Precise measurements of specific heats and molar heat capacities require great experimental skill. Usually, a measured quantity of energy is supplied by an electric current in a heater wire wound around the specimen. The temperature change $\Delta T$ is measured with a resistance thermometer or thermocouple embedded in the specimen. This sounds simple, but great care is needed to avoid or compensate for unwanted heat transfer between the sample and its surroundings. Measurements for solid materials are usually made at constant atmospheric pressure; the corresponding values are called the specific heat and molar heat capacity at constant pressure, denoted by $c_p$ and $C_p$. For a gas it is usually easier to keep the substance in a container with constant volume; the corresponding values are called the specific heat and molar heat capacity at constant volume, denoted by $c_v$ and $C_v$. For a given substance, $C_v$ and $C_p$ are different. If the system can expand while heat is added, there is additional energy exchange through the performance of work by the system on its surroundings. If the volume is constant, the system does no work. For gases the difference between $C_p$ and $C_v$ is substantial. We will study heat capacities of gases in detail in Section 19.7.

The last column of Table 17.3 shows something interesting. The molar heat capacities for most elemental solids are about the same, about 25 J/mol·K. This correlation, named the rule of Dulong and Petit (for its discoverers), forms the basis for a very important idea. The number of atoms in one mole is the same for all elemental substances. This means that on a per atom basis, about the same amount of heat is required to raise the temperature of each of these elements by a given amount, even though the masses of the atoms are very different. The heat required for a given temperature increase depends only on how many atoms the sample contains, not on the mass of an individual atom. We will see the reason...
17.6 | Calorimetry and Phase Changes

Calorimetry means “measuring heat.” We have discussed the energy transfer (heat) involved in temperature changes. Heat is also involved in phase changes, such as the melting of ice or boiling of water. Once we understand these additional heat relationships, we can analyze a variety of problems involving quantity of heat.

Phase Changes

We use the term phase to describe a specific state of matter, such as a solid, liquid, or gas. The compound H₂O exists in the solid phase as ice, in the liquid phase as water, and in the gaseous phase as steam. (These are also referred to as states of matter: the solid state, the liquid state, and the gaseous state.) A transition from one phase to another is called a phase change or phase transition. For any given pressure a phase change takes place at a definite temperature, usually accompanied by absorption or emission of heat and a change of volume and density.

A familiar example of a phase change is the melting of ice. When we add heat to ice at 0°C and normal atmospheric pressure, the temperature of the ice does not increase. Instead, some of it melts to form liquid water. If we add the heat slowly, to maintain the system very close to thermal equilibrium, the temperature remains at 0°C until all the ice is melted (Fig. 17.16). The effect of adding heat to this system is not to raise its temperature but to change its phase from solid to liquid.

To change 1 kg of ice at 0°C to 1 kg of liquid water at 0°C and normal atmospheric pressure requires \(3.34 \times 10^5\) J of heat. The heat required per unit mass is called the heat of fusion (or sometimes latent heat of fusion), denoted by \(L_f\). For water at normal atmospheric pressure the heat of fusion is

\[ L_f = 3.34 \times 10^5 \text{ J/kg} = 79.6 \text{ cal/g} = 143 \text{ Btu/lb} \]

More generally, to melt a mass \(m\) of material that has a heat of fusion \(L_f\) requires a quantity of heat \(Q\) given by

\[ Q = mL_f \]

This process is reversible. To freeze liquid water to ice at 0°C, we have to remove heat; the magnitude is the same, but in this case, \(Q\) is negative because heat is removed rather than added. To cover both possibilities and to include other kinds of phase changes, we write

\[ Q = \pm mL \quad \text{(heat transfer in a phase change)} \quad (17.20) \]
The plus sign (heat entering) is used when the material melts; the minus sign (heat leaving) is used when it freezes. The heat of fusion is different for different materials, and it also varies somewhat with pressure.

For any given material at any given pressure, the freezing temperature is the same as the melting temperature. At this unique temperature the liquid and solid phases (liquid water and ice, for example) can coexist in a condition called phase equilibrium.

We can go through this whole story again for boiling or evaporation, a phase transition between liquid and gaseous phases. The corresponding heat (per unit mass) is called the heat of vaporization $L_v$. At normal atmospheric pressure the heat of vaporization for water is

$$L_v = 2.256 \times 10^6 \text{J/kg} = 539 \text{cal/g} = 970 \text{Btu/lb}$$

That is, it takes $2.256 \times 10^6$ J to change 1 kg of liquid water at 100°C to 1 kg of water vapor at 100°C. By comparison, to raise the temperature of a kilogram of water from 0°C to 100°C requires $Q = mc \Delta T = (1.00 \text{ kg})(4190 \text{ J/kg} \cdot \text{C}^\circ) \times (100 \text{ C}^\circ) = 4.19 \times 10^5 \text{ J}$, less than one-fifth as much heat as is required for vaporization at 100°C. This agrees with everyday kitchen experience; a pot of water may reach boiling temperature in a few minutes, but it takes a much longer time to completely evaporate all the water away.

Like melting, boiling is a reversible transition. When heat is removed from a gas at the boiling temperature, the gas returns to the liquid phase, or condenses, giving up to its surroundings the same quantity of heat (heat of vaporization) that was needed to vaporize it. At a given pressure the boiling and condensation temperatures are always the same; at this temperature the liquid and gaseous phases can coexist in phase equilibrium.

Both $L_v$ and the boiling temperature of a material depend on pressure. Water boils at a lower temperature (about 95°C) in Denver than in Pittsburgh because Denver is at higher elevation and the average atmospheric pressure is less. The heat of vaporization is somewhat greater at this lower pressure, about $2.27 \times 10^6$ J/kg.

Table 17.4 lists heats of fusion and vaporization for several materials and their melting and boiling temperatures at normal atmospheric pressure. Very few ele-

<table>
<thead>
<tr>
<th>Substance</th>
<th>Normal Melting Point</th>
<th>Normal Boiling Point</th>
<th>Heat of Fusion, $L_f$</th>
<th>Heat of Vaporization, $L_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>°C</td>
<td>(J/kg)</td>
<td>K</td>
<td>°C</td>
</tr>
<tr>
<td>Helium</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>4.216</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>13.84</td>
<td>-259.31</td>
<td>58.6 $\times 10^3$</td>
<td>20.26</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>63.18</td>
<td>-209.97</td>
<td>25.5 $\times 10^3$</td>
<td>77.34</td>
</tr>
<tr>
<td>Oxygen</td>
<td>54.36</td>
<td>-218.79</td>
<td>13.8 $\times 10^3$</td>
<td>90.18</td>
</tr>
<tr>
<td>Ethanol</td>
<td>159</td>
<td>-114</td>
<td>104.2 $\times 10^3$</td>
<td>351</td>
</tr>
<tr>
<td>Mercury</td>
<td>234</td>
<td>-39</td>
<td>11.8 $\times 10^3$</td>
<td>630</td>
</tr>
<tr>
<td>Water</td>
<td>273.15</td>
<td>0.00</td>
<td>334 $\times 10^3$</td>
<td>373.15</td>
</tr>
<tr>
<td>Sulfur</td>
<td>392</td>
<td>119</td>
<td>38.1 $\times 10^3$</td>
<td>717.75</td>
</tr>
<tr>
<td>Lead</td>
<td>600.5</td>
<td>327.3</td>
<td>24.5 $\times 10^3$</td>
<td>2023</td>
</tr>
<tr>
<td>Antimony</td>
<td>903.65</td>
<td>630.50</td>
<td>165 $\times 10^3$</td>
<td>1713</td>
</tr>
<tr>
<td>Silver</td>
<td>1233.95</td>
<td>960.80</td>
<td>88.3 $\times 10^3$</td>
<td>2466</td>
</tr>
<tr>
<td>Gold</td>
<td>1336.15</td>
<td>1063.00</td>
<td>64.5 $\times 10^3$</td>
<td>2933</td>
</tr>
<tr>
<td>Copper</td>
<td>1356</td>
<td>1083</td>
<td>134 $\times 10^3$</td>
<td>1460</td>
</tr>
</tbody>
</table>

*A pressure in excess of 25 atmospheres is required to make helium solidify. At 1 atmosphere pressure, helium remains a liquid down to absolute zero.
ments have melting temperatures in the vicinity of ordinary room temperatures; one of the few is the metal gallium, shown in Fig. 17.17.

Figure 17.18 shows how the temperature varies when we add heat continuously to a specimen of ice with an initial temperature below 0°C (point a). The temperature rises until we reach the melting point (point b). As more heat is added, the temperature remains constant until all the ice has melted (point c). Then the temperature starts to rise again until the boiling temperature is reached (point d). At that point the temperature again is constant until all the water is transformed into the vapor phase (point e). If the rate of heat input is constant, the slope of the line for the solid phase (ice) has a steeper slope than does the line for the liquid phase (water). Do you see why? (See Table 17.3.)

A substance can sometimes change directly from the solid to the gaseous phase. This process is called sublimation, and the solid is said to sublime. The corresponding heat is called the heat of sublimation, \( L_s \). Liquid carbon dioxide cannot exist at a pressure lower than about \( 5 \times 10^5 \) Pa (about 5 atm), and “dry ice” (solid carbon dioxide) sublimes at atmospheric pressure. Sublimation of water from frozen food causes freezer burn. The reverse process, a phase change from gas to solid, occurs when frost forms on cold bodies such as refrigerator cooling coils.

Very pure water can be cooled several degrees below the freezing temperature without freezing; the resulting unstable state is described as supercooled. When a small ice crystal is dropped in or the water is agitated, it crystallizes within a second or less. Supercooled water vapor condenses quickly into fog droplets when a disturbance, such as dust particles or ionizing radiation, is introduced. This principle is used in “seeding” clouds, which often contain supercooled water vapor, to cause condensation and rain.

A liquid can sometimes be superheated above its normal boiling temperature. Any small disturbance such as agitation causes local boiling with bubble formation.

Steam heating systems for buildings use a boiling–condensing process to transfer heat from the furnace to the radiators. Each kilogram of water that is turned to steam in the boiler absorbs over \( 2 \times 10^6 \) J (the heat of vaporization \( L_v \) of water) from the boiler and gives it up when it condenses in the radiators. Boiling–condensing processes are also used in refrigerators, air conditioners, and heat pumps. We will discuss these systems in Chapter 20.

The temperature-control mechanisms of many warm-blooded animals make use of heat of vaporization, removing heat from the body by using it to evaporate
The water may be warm and it may be a hot day, but these children will feel cold when they first step out of the swimming pool. That’s because as water evaporates from their skin, it removes the heat of vaporization from their bodies. To stay warm, they will need to dry off immediately.

Evaporative cooling is also used to cool buildings in hot, dry climates and to condense and recirculate “used” steam in coal-fired or nuclear-powered electric-generating plants. That’s what goes on in the large, tapered concrete towers that you see at such plants.

Chemical reactions such as combustion are analogous to phase changes in that they involve definite quantities of heat. Complete combustion of one gram of gasoline produces about 46,000 J or about 11,000 cal, so the heat of combustion \( L_c \) of gasoline is

\[
L_c = 46,000 \text{ J/g} = 4.6 \times 10^7 \text{ J/kg}
\]

Energy values of foods are defined similarly: the U.S. unit of food energy, although called a calorie, is a kilocalorie, equal to 1000 cal or 4186 J. When we say that a gram of peanut butter “contains 6 calories,” we mean that 6 kcal of heat (6,000 cal or 25,000 J) is released when the carbon and hydrogen atoms in the peanut butter react with oxygen (with the help of enzymes) and are completely converted to \( \text{CO}_2 \) and \( \text{H}_2\text{O} \). Not all of this energy is directly useful for mechanical work. We will study the efficiency of energy utilization in Chapter 20.

Heat Calculations

Let’s look at some examples of calorimetry calculations (calculations with heat). The basic principle is very simple: when heat flow occurs between two bodies that are isolated from their surroundings, the amount of heat lost by one body must equal the amount gained by the other. Heat is energy in transit, so this principle is really just conservation of energy. Calorimetry, dealing entirely with one conserved quantity, is in many ways the simplest of all physical theories!

**Problem-Solving Strategy**

**Calorimetry Problems**

**IDENTIFY the relevant concepts:** The basic principle underlying calorimetry calculations (calculations with heat) is very simple: when heat flow occurs between two bodies that are isolated from their surroundings, the amount of heat lost by one body must equal the amount gained by the other. Heat is energy in transit, so this principle is really just conservation of energy. Calorimetry, dealing entirely with one conserved quantity, is in many ways the simplest of all physical theories!

**SET UP the problem using the following steps:**

1. Identify which objects exchange heat. To avoid confusion with algebraic signs, take each quantity of heat added to a body as positive and each quantity leaving a body as negative. When several bodies interact, the algebraic sum of the quantities of heat transferred to all the bodies must be zero.
2. Each object will undergo a temperature change with no phase change, a phase change at constant temperature, or both. Use Eq. (17.13) to describe temperature changes and Eq. (17.20) to describe phase changes.
3. Consult Table 17.3 for values of the specific heat or molar heat capacity and Table 17.4 for heats of fusion or vaporization.
4. Be certain to identify which quantities are known and which are the unknown target variables.
EXECUTE the solution as follows:

1. Solve Eq. (17.13) and/or Eq. (17.20) for the target variables. Often you will need to find an unknown temperature. Represent it by an algebraic symbol such as $T$. Then if a body has an initial temperature of 20°C and an unknown final temperature $T$, the temperature change for the body is

$$
\Delta T = T_{\text{final}} - T_{\text{initial}} = T - 20°C \text{ (not } 20°C - T).$

2. In problems where a phase change takes place, as when ice melts, you may not know in advance whether all the material undergoes a phase change or only part of it. You can always assume one or the other, and if the resulting calculation gives an absurd result (such as a final temperature higher or lower than any of the initial temperatures), you know the initial assumption was wrong. Back up and try again!

EVALUATE your answer: A common error is to use the wrong algebraic sign for either a $Q$ or $\Delta T$ term. Double-check your calculations, and make sure that the final results are physically sensible.

Example 17.8

A temperature change with no phase change

A geologist working in the field drinks her morning coffee out of an aluminum cup. The cup has a mass of 0.120 kg and is initially at 20.0°C when she pours in 0.300 kg of coffee initially at 70.0°C. What is the final temperature after the coffee and the cup attain thermal equilibrium? (Assume that coffee has the same specific heat as water and that there is no heat exchange with the surroundings.)

IDENTIFY and SET UP: No phase changes occur in this situation, so we need use only Eq. (17.13). The two objects we must consider are the cup and the coffee, and the target variable is their common final temperature.

EXECUTE: By using Table 17.3, the (negative) heat gained by the coffee is

$$
Q_{\text{coffee}} = m_{\text{coffee}} c_{\text{water}} \Delta T_{\text{coffee}} = (0.300 \text{ kg}) (4190 \text{ J/kg} \cdot \text{K}) (T - 70.0°C)
$$

The (positive) heat gained by the aluminum cup is

$$
Q_{\text{aluminum}} = m_{\text{aluminum}} c_{\text{aluminum}} \Delta T_{\text{aluminum}} = (0.120 \text{ kg}) (910 \text{ J/kg} \cdot \text{K}) (T - 20.0°C)
$$

We equate the sum of these two quantities of heat to zero, obtaining an algebraic equation for $T$:

$$
Q_{\text{coffee}} + Q_{\text{aluminum}} = 0 \quad \text{or}
$$

$$
(0.300 \text{ kg}) (4190 \text{ J/kg} \cdot \text{K}) (T - 70.0°C) + (0.120 \text{ kg}) (910 \text{ J/kg} \cdot \text{K}) (T - 20.0°C) = 0
$$

Solution of this equation gives $T = 66.0°C$.

EVALUATE: The final temperature is much closer to the initial temperature of the coffee than to that of the cup; water has a much larger specific heat than aluminum, and we have more than twice as much mass of water. We can also find the quantities of heat by substituting this value for $T$ back into the original equations. We find that $Q_{\text{coffee}} = -5.0 \times 10^3 \text{ J}$ and $Q_{\text{aluminum}} = +5.0 \times 10^3 \text{ J}; Q_{\text{coffee}}$ is negative, which means that the coffee loses heat.

Example 17.9

Changes in both temperature and phase

A physics student wants to cool 0.25 kg of Diet Omni-Cola (mostly water), initially at 25°C, by adding ice initially at −20°C. How much ice should she add so that the final temperature will be 0°C with all the ice melted if the heat capacity of the container may be neglected?

IDENTIFY and SET UP: The ice and the Omni-Cola are the objects that exchange heat. The Omni-Cola undergoes a temperature change only, while the ice undergoes both a temperature change and a phase change from solid to liquid. The target variable is the mass of ice, $m_{\text{ice}}$.

EXECUTE: The Omni-Cola loses heat, so the heat added to it is negative:

$$
Q_{\text{Omni}} = m_{\text{Omni}} c_{\text{water}} \Delta T_{\text{Omni}} = (0.25 \text{ kg}) (4190 \text{ J/kg} \cdot \text{K}) (0°C - 25°C) = -26,000 \text{ J}
$$

From Table 17.3, the specific heat of ice (not the same as for liquid water) is $2.1 \times 10^3 \text{ J/kg} \cdot \text{K}$. Let the mass of ice be $m_{\text{ice}}$; then the heat $Q$ needed to warm it from −20°C to 0°C is
\[ Q_1 = m_{\text{ice}} c_{\text{ice}} \Delta T_{\text{ice}} \]
\[ = m_{\text{ice}} (2.1 \times 10^3 \text{ J/kg} \cdot \text{K}) \left[ 0^\circ \text{C} - (-20^\circ \text{C}) \right] \]
\[ = m_{\text{ice}} (4.2 \times 10^3 \text{ J/kg}) \]

From Eq. (17.20) the additional heat \( Q_2 \) needed to melt this mass of ice is the mass times the heat of fusion. Using Table 17.4, we find
\[ Q_2 = m_{\text{ice}} L_f \]
\[ = m_{\text{ice}} (3.34 \times 10^5 \text{ J/kg}) \]

The sum of these three quantities must equal zero:
\[ Q_{\text{burn}} + Q_1 + Q_2 = -26,000 \text{ J} + m_{\text{ice}} (42,000 \text{ J/kg}) + m_{\text{ice}} (334,000 \text{ J/kg}) = 0 \]
Solving this for \( m_{\text{ice}} \) we get \( m_{\text{ice}} = 0.069 \text{ kg} = 69 \text{ g} \).

**EVALUATE:** This mass of ice corresponds to three or four medium-size ice cubes, which seems reasonable for the quantity of Omni-Cola in this problem.

---

**Example 17.10**

**What's cooking?**

A heavy copper pot of mass 2.0 kg (including the copper lid) is at a temperature of 150°C. You pour 0.10 kg of water at 25°C into the pot, then quickly close the lid of the pot so that no steam can escape. Find the final temperature of the pot and its contents, and determine the phase (liquid or gas) of the water. Assume that no heat is lost to the surroundings.

**SOLUTION**

**IDENTIFY and SET UP:** The two objects that exchange heat are the water and the pot. Note that there are three conceivable outcomes in this situation. One, none of the water boils, and the final temperature is less than 100°C; two, a portion of the water boils, giving a mixture of water and steam at 100°C; or three, all the water boils, giving 0.10 kg of steam at a temperature of 100°C or greater.

**EXECUTE:** The simplest case to calculate is the first possibility, so let's try that first. Let the common final temperature of the liquid water and the copper pot be \( T \). Since we are assuming that no phase changes take place, the sum of the quantities of heat added to the two materials is
\[ Q_{\text{water}} + Q_{\text{copper}} = m_{\text{water}} c_{\text{water}} (T - 25^\circ \text{C}) + m_{\text{copper}} c_{\text{copper}} (T - 150^\circ \text{C}) \]
\[ = (0.10 \text{ kg}) (4190 \text{ J/kg} \cdot \text{K}) (T - 25^\circ \text{C}) + (2.0 \text{ kg}) (390 \text{ J/kg} \cdot \text{K}) (T - 150^\circ \text{C}) \]
\[ = 0 \]
Solving this for \( T \), we find \( T = 106^\circ \text{C} \). But this is above the boiling point of water, which contradicts our assumption that none of the water boils! So this assumption can't be correct; at least some of the water undergoes a phase change.

If we try the second possibility, in which the final temperature is 100°C, we have to find the fraction of water that changes to the gaseous phase. Let this fraction be \( x \); the (positive) amount of heat needed to vaporize this water is \( (xm_{\text{water}}) L_v \). Setting the final temperature \( T \) equal to 100°C, we have
\[ Q_{\text{water}} = m_{\text{water}} c_{\text{water}} (100^\circ \text{C} - 25^\circ \text{C}) + x m_{\text{water}} L_v \]
\[ = (0.10 \text{ kg}) (4190 \text{ J/kg} \cdot \text{K}) (75 \text{ K}) + x (0.10 \text{ kg}) (2.256 \times 10^6 \text{ J/kg}) \]
\[ = 3.14 \times 10^4 \text{ J} + x (2.256 \times 10^5 \text{ J}) \]
\[ Q_{\text{copper}} = m_{\text{copper}} c_{\text{copper}} (100^\circ \text{C} - 150^\circ \text{C}) \]
\[ = (2.0 \text{ kg}) (390 \text{ J/kg} \cdot \text{K}) (-50 \text{ K}) = -3.90 \times 10^4 \text{ J} \]
The requirement that the sum of all the quantities of heat be zero then gives
\[ Q_{\text{water}} + Q_{\text{copper}} = 3.14 \times 10^4 \text{ J} + x (2.256 \times 10^5 \text{ J}) - 3.90 \times 10^4 \text{ J} = 0 \]
\[ x = \frac{3.90 \times 10^4 \text{ J} - 3.14 \times 10^4 \text{ J}}{2.256 \times 10^5 \text{ J}} = 0.034 \]
This makes sense, and we conclude that the final temperature of the water and copper is 100°C. Of the original 0.10 kg of water, 0.034(0.10 kg) = 0.0034 kg = 3.4 g has been converted to steam at 100°C.

**EVALUATE:** Had \( x \) turned out to be greater than 1, we would have again had a contradiction (the fraction of water that vaporized can't be greater than 1). In this case the third possibility would have been the correct description, all the water would have vaporized, and the final temperature would have been greater than 100°C. Can you show that this would have been the case if we had originally poured less than 15 g of 25°C water into the pot?
Example 17.11

**Combustion, temperature change, and phase change**

In a particular gasoline camp stove, 30% of the energy released in burning the fuel actually goes to heating the water in the pot on the stove. If we heat 1.00 L (1.00 kg) of water from 20°C to 100°C and boil 0.25 kg of it away, how much gasoline do we burn in the process?

**SOLUTION**

**IDENTIFY and SET UP:** In this problem, Eqs. (17.13) and (17.20) apply to the water, all of which undergoes a temperature change and part of which also undergoes a phase change from liquid to gas. This requires a certain amount of heat, which we use to determine the amount of gasoline that must be burned (the target variable).

**EXECUTE:** The heat required to raise the temperature of the water from 20°C to 100°C is

\[ Q_1 = mc \Delta T = (1.00 \text{ kg}) (4190 \text{ J/kg·K}) (80 \text{ K}) \]

\[ = 3.35 \times 10^5 \text{ J} \]

To boil 0.25 kg of water at 100°C requires

\[ Q_2 = mL_v = (0.25 \text{ kg}) (2.256 \times 10^6 \text{ J/kg}) = 5.64 \times 10^6 \text{ J} \]

The total energy needed is the sum of these, or 8.99 \( \times \) \( 10^6 \) J. This is only 0.30 of the total heat of combustion, so that energy is \( (8.99 \times 10^6 \text{ J})/0.30 = 3.00 \times 10^7 \text{ J} \). As we mentioned earlier, one gram of gasoline releases 46,000 J, so the mass of gasoline required is

\[ \frac{3.00 \times 10^7 \text{ J}}{46,000 \text{ J/g}} = 65 \text{ g} \]

or a volume of about 0.09 L of gasoline.

**EVALUATE:** This result is a testament to the tremendous amount of energy that can be released by burning even a small quantity of gasoline. Note that most of the heat delivered was used to boil away 0.25 L of water. Can you show that another 123 g of gasoline would be required to boil away the remaining water?

Test Your Understanding

You take a block of ice at 0°C and add heat to it at a steady rate. After a time \( t \), the block of ice has been converted completely to steam at 100°C. What is the temperature of the (ice? water? steam?) at time \( t/2 \), and what is its phase?

17.7 | Mechanisms of Heat Transfer

We have talked about conductors and insulators, materials that permit or prevent heat transfer between bodies. Now let’s look in more detail at rates of energy transfer. In the kitchen you use an aluminum pot for good heat transfer from the stove to whatever you’re cooking, but your refrigerator is insulated with a material that prevents heat from flowing into the food inside the refrigerator. How do we describe the difference between these two materials?

The three mechanisms of heat transfer are conduction, convection, and radiation. Conduction occurs within a body or between two bodies in contact. Convection depends on motion of mass from one region of space to another. Radiation is heat transfer by electromagnetic radiation, such as sunshine, with no need for matter to be present in the space between bodies.

**Conduction**

If you hold one end of a copper rod and place the other end in a flame, the end you are holding gets hotter and hotter, even though it is not in direct contact with the flame. Heat reaches the cooler end by conduction through the material. On the atomic level, the atoms in the hotter regions have more kinetic energy, on the average, than
Chapter 17 | Temperature and Heat

Temperature and Heat

Insulators heat their cooler neighbors. They jostle their neighbors, giving them some of their energy. The neighbors jostle their neighbors, and so on through the material. The atoms themselves do not move from one region of material to another, but their energy does.

Most metals also use another, more effective mechanism to conduct heat. Within the metal, some electrons can leave their parent atoms and wander through the crystal lattice. These “free” electrons can rapidly carry energy from the hotter to the cooler regions of the metal, so metals are generally good conductors of heat. A metal rod at 20°C feels colder than a piece of wood at 20°C because heat can flow more easily from your hand into the metal. The presence of “free” electrons also causes most metals to be good electrical conductors.

Heat transfer occurs only between regions that are at different temperatures, and the direction of heat flow is always from higher to lower temperature. Figure 17.20 shows a rod of conducting material with cross-sectional area \( A \) and length \( L \). The left end of the rod is kept at a temperature \( T_H \) and the right end at a lower temperature \( T_C \), so heat flows from left to right. The sides of the rod are covered by an ideal insulator, so no heat transfer occurs at the sides.

When a quantity of heat \( dQ \) is transferred through the rod in a time \( dt \), the rate of heat flow is \( \frac{dQ}{dt} \). We call this rate the heat current, denoted by \( H \). That is, \( H = \frac{dQ}{dt} \). Experiments show that the heat current is proportional to the cross-sectional area \( A \) of the rod and to the temperature difference \( (T_H - T_C) \) and is inversely proportional to the rod length \( L \). Introducing a proportionality constant \( k \) called the thermal conductivity of the material, we have

\[
H = \frac{dQ}{dt} = kA\frac{T_H - T_C}{L} \tag{17.21}
\]

The quantity \( (T_H - T_C)/L \) is the temperature difference per unit length; it is called the magnitude of the temperature gradient. The numerical value of \( k \) depends on the material of the rod. Materials with large \( k \) are good conductors of heat; materials with small \( k \) are poor conductors or insulators. Equation (17.21) also gives the heat current through a slab or through any homogeneous body with uniform cross section \( A \) perpendicular to the direction of flow; \( L \) is the length of the heat-flow path.

The units of heat current \( H \) are units of energy per time, or power; the SI unit of heat current is the watt (1 W = 1 J/s). We can find the units of \( k \) by solving Eq. (17.21) for \( k \); you can show that the SI units are W/m·K. Some numerical values of \( k \) are given in Table 17.5.

The thermal conductivity of “dead” (that is, nonmoving) air is very small. A wool sweater keeps you warm because it traps air between the fibers. In fact, many insulating materials such as Styrofoam and fiberglass are mostly dead air. Figure 17.21 shows a ceramic material with very unusual thermal properties, including very small conductivity.

If the temperature varies in a non-uniform way along the length of the conducting rod, we introduce a coordinate \( x \) along the length and generalize the temperature gradient to be \( dT/dx \). The corresponding generalization of Eq. (17.21) is

\[
H = \frac{dQ}{dt} = -kA\frac{dT}{dx} \tag{17.22}
\]
The negative sign shows that heat always flows in the direction of decreasing temperature.

For thermal insulation in buildings, engineers use the concept of thermal resistance, denoted by $R$. The thermal resistance $R$ of a slab of material with area $A$ is defined so that the heat current $H$ through the slab is

$$H = \frac{A(T_H - T_C)}{R}$$

where $T_H$ and $T_C$ are the temperatures on the two sides of the slab. Comparing this with Eq. (17.21), we see that $R$ is given by

$$R = \frac{L}{k}$$

where $L$ is the thickness of the slab. The SI unit of $R$ is $1 \text{ m}^2 \cdot \text{K}/\text{W}$. In the units used for commercial insulating materials in the United States, $H$ is expressed in Btu/ft$^2 \cdot \text{hr}$, $A$ is in ft$^2$, and $T_H - T_C$ in °F. (1 Btu/ft$^2 \cdot \text{hr} = 0.293$ W.) The units of $R$ are then ft$^2 \cdot $°F·hr/Btu, though values of $R$ are usually quoted without units; a 6-inch-thick layer of fiberglass has an $R$ value of 19 (that is, $R = 19 \text{ ft}^2 \cdot \text{°F} \cdot \text{hr}/\text{Btu}$), a two-inch-thick slab of polyurethane foam has a value of 12, and so on. Doubling the thickness doubles the $R$-value. Common practice in new construction in severe northern climates is to specify $R$ values of around 30 for exterior walls and ceilings. When the insulating material is in layers, such as a plastered wall, fiberglass insulation, and wood exterior siding, the $R$ values are additive. Do you see why? (See Problem 17.110.)

**Problem-Solving Strategy**

**Heat Conduction**

**IDENTIFY the relevant concepts:** The concept of heat conduction comes into play whenever two objects at different temperature are placed in contact.

**SET UP the problem using the following steps:**
1. Identify the direction of heat flow in the problem (from hot to cold). In Eq. (17.21), $L$ is always measured along this direction, and $A$ is always an area perpendicular to this direction. Often when a box or other container has an irregular shape but uniform wall thickness, you can approximate it as a flat slab with the same thickness and total wall area.
2. Identify the target variable.

**EXECUTE the solution as follows:**
1. If heat flows through a single object, use Eq. (17.21) to solve for the target variable.
2. In some problems the heat flows through two different materials in succession. The temperature at the interface between the two materials is then intermediate between $T_H$ and $T_C$; represent it by a symbol such as $T$. The temperature differences for the two materials are then $(T_H - T)$ and $(T - T_C)$. In steady-state heat flow, the same heat has to pass through both materials in succession, so the heat current $H$ must be the same in both materials.
3. If there are two parallel heat-flow paths, so that some heat flows through each, then the total $H$ is the sum of the quantities $H_1$ and $H_2$ for the separate paths. An example is heat flow from inside to outside a house, both through the glass in a window and through the surrounding frame. In this case the temperature difference is the same for the two paths, but $L$, $A$, and $k$ may be different for the two paths.
4. As always, it is essential to use a consistent set of units. If you use a value of $k$ expressed in W/m·°K, don’t use distances in centimeters, heat in calories, or $T$ in degrees Fahrenheit!

**EVALUATE your answer:** As always, ask yourself whether the results are physically reasonable.
**Example 17.12**

**Conduction through a picnic cooler**

A Styrofoam box used to keep drinks cold at a picnic has total wall area (including the lid) of 0.80 m² and wall thickness 2.0 cm. It is filled with ice, water, and cans of Omni-Cola at 0°C. What is the rate of heat flow into the box if the temperature of the outside wall is 30°C? How much ice melts in one day?

**SOLUTION**

**IDENTIFY and SET UP:** The first target variable is the heat current \( H \). The second is the amount of ice melted, which depends on the heat current (heat per unit time), the elapsed time, and the heat of fusion.

**EXECUTE:** We assume that the total heat flow is approximately the same as it would be through a flat slab of area 0.80 m² and thickness 2.0 cm = 0.020 m (Fig. 17.22). We find \( k \) from Table 17.5. From Eq. (17.21) the heat current (rate of heat flow) is

\[
H = \frac{kA}{L} (T_H - T_C) = \frac{(0.010 \text{ W/m} \cdot \text{K})(0.80 \text{ m}^2)}{0.020 \text{ m}} (30^\circ \text{C} - 0^\circ \text{C}) = 12 \text{ W} = 12 \text{ J/s}
\]

The total heat flow \( Q \) in one day (86,400 s) is

\[
Q = Ht = (12 \text{ J/s})(86,400 \text{ s}) = 1.04 \times 10^6 \text{ J}
\]

The heat of fusion of ice is \( 3.34 \times 10^5 \text{ J/kg} \), so the quantity of ice melted by this quantity of heat is

\[
m = \frac{Q}{L_f} = \frac{1.04 \times 10^6 \text{ J}}{3.34 \times 10^5 \text{ J/kg}} = 3.1 \text{ kg}
\]

**EVALUATE:** The low heat current is a result of the low thermal conductivity of Styrofoam. A substantial amount of heat flows in 24 hours, but a relatively small amount of ice melts because the heat of fusion is high.

**Example 17.13**

**Conduction through two bars I**

A steel bar 10.0 cm long is welded end-to-end to a copper bar 20.0 cm long (Fig. 17.23). Both bars are insulated perfectly on their sides. Each bar has a square cross section, 2.00 cm on a side. The free end of the steel bar is maintained at 100°C by placing it in contact with steam, and the free end of the copper bar is maintained at 0°C by placing it in contact with ice. Find the temperature at the junction of the two bars and the total rate of heat flow.

**SOLUTION**

**IDENTIFY and SET UP:** As we discussed in the Problem-Solving Strategy, the heat currents in the two bars must be equal; this idea is the key to the solution. We write Eq. (17.21) twice, once for each bar, and set the heat currents \( H_{steel} \) and \( H_{copper} \) equal to each other. Both expressions for the heat current involve the temperature \( T \) at the junction, which is one of our target variables.
Rearranging and solving for $T$, we find

$$T = 20.7^\circ\text{C}$$

We can find the total heat current by substituting this value for $T$ back into either of the above expressions:

$$H_{\text{steel}} = \frac{(50.2 \text{ W/m} \cdot \text{K})(0.0200 \text{ m})^2(100^\circ\text{C} - 20.7^\circ\text{C})}{0.100 \text{ m}} = 18.9 \text{ W}$$

or

$$H_{\text{copper}} = \frac{(385 \text{ W/m} \cdot \text{K})(0.0200 \text{ m})^2(20.7^\circ\text{C})}{0.200 \text{ m}} = 15.9 \text{ W}$$

**EVALUATE:** Even though the steel bar is shorter, the temperature drop across it is much greater than across the copper bar (from 100°C to 20.7°C in the steel versus from 20.7°C to 0°C in the copper). This difference arises because steel is a much poorer conductor than copper.

---

**Example 17.14 Conduction through two bars II**

In Example 17.13, suppose the two bars are separated. One end of each bar is maintained at 100°C and the other end of each bar is maintained at 0°C (Fig. 17.24). What is the total rate of heat flow in the two bars?

**EXECUTE:** We write the heat currents for the two rods individually, then add them to get the total heat current:

$$H = H_{\text{steel}} + H_{\text{copper}} = \frac{k_{\text{steel}}A(T_{\text{steel}} - T_{\text{copper}})}{L_{\text{steel}}} + \frac{k_{\text{copper}}A(T_{\text{copper}} - T_{\text{copper}})}{L_{\text{copper}}}$$

$$= \frac{(50.2 \text{ W/m} \cdot \text{K})(0.0200 \text{ m})^2(100 \text{ K})}{0.100 \text{ m}} + \frac{(385 \text{ W/m} \cdot \text{K})(0.0200 \text{ m})^2(106 \text{ K})}{0.200 \text{ m}}$$

$$= 20.1 \text{ W} + 77.0 \text{ W} = 97.1 \text{ W}$$

**EVALUATE:** The heat flow in the copper bar is much greater than that in the steel bar, even though it is longer, because the thermal conductivity of copper is much larger. The total heat flow is much greater than in Example 17.13, partly because the total cross section for heat flow is greater and partly because the full 100-K temperature difference appears across each bar.
Convection

Convection is the transfer of heat by mass motion of a fluid from one region of space to another. Familiar examples include hot-air and hot-water home heating systems, the cooling system of an automobile engine, and the flow of blood in the body. If the fluid is circulated by a blower or pump, the process is called forced convection; if the flow is caused by differences in density due to thermal expansion, such as hot air rising, the process is called natural convection or free convection (Fig. 17.25).

Free convection in the atmosphere plays a dominant role in determining the daily weather, and convection in the oceans is an important global heat-transfer mechanism. On a smaller scale, soaring hawks and glider pilots make use of thermal updrafts from the warm earth. The most important mechanism for heat transfer within the human body (needed to maintain nearly constant temperature in various environments) is forced convection of blood, with the heart serving as the pump.

Convective heat transfer is a very complex process, and there is no simple equation to describe it. Here are a few experimental facts:

1. The heat current due to convection is directly proportional to the surface area. This is the reason for the large surface areas of radiators and cooling fins.
2. The viscosity of fluids slows natural convection near a stationary surface, giving a surface film that on a vertical surface typically has about the same insulating value as 1.3 cm of plywood ($R$ value = 0.7). Forced convection decreases the thickness of this film, increasing the rate of heat transfer. This is the reason for the “wind-chill factor”; you get cold faster in a cold wind than in still air with the same temperature.
3. The heat current due to convection is found to be approximately proportional to the $\frac{3}{2}$ power of the temperature difference between the surface and the main body of fluid.

Radiation

Radiation is the transfer of heat by electromagnetic waves such as visible light, infrared, and ultraviolet radiation. Everyone has felt the warmth of the sun’s radiation and the intense heat from a charcoal grill or the glowing coals in a fireplace. Most of the heat from these very hot bodies reaches you not by conduction or convection in the intervening air but by radiation. This heat transfer would occur even if there were nothing but vacuum between you and the source of heat.

Every body, even at ordinary temperatures, emits energy in the form of electromagnetic radiation. At ordinary temperatures, say 20°C, nearly all the energy is carried by infrared waves with wavelengths much longer than those of visible light (see Figs. 17.4 and 17.26). As the temperature rises, the wavelengths shift to shorter values. At 800°C a body emits enough visible radiation to be self-luminous and appears “red-hot,” although even at this temperature most of the energy is carried by infrared waves. At 3000°C, the temperature of an incandescent lamp filament, the radiation contains enough visible light so the body appears “white-hot.”

The rate of energy radiation from a surface is proportional to the surface area $A$. The rate increases very rapidly with temperature, depending on the fourth power of the absolute (Kelvin) temperature. The rate also depends on the nature of the surface; this dependence is described by a quantity called the emissivity. A dimensionless number between 0 and 1, it represents the ratio of the rate of radiation from a particular surface to the rate of radiation from an equal area of an ideal radiating surface at the same temperature. Emissivity also depends some-
what on temperature. Thus the heat current \( H = \frac{dQ}{dt} \) due to radiation from a surface area \( A \) with emissivity \( e \) at absolute temperature \( T \) can be expressed as

\[
H = A\varepsilon\sigma T^4 \quad \text{(heat current in radiation)} \tag{17.25}
\]

where \( \sigma \) is a fundamental physical constant called the Stefan-Boltzmann constant. This relation is called the Stefan-Boltzmann law in honor of its late-19th-century discoverers. The current best numerical value of \( \sigma \) is

\[
\sigma = 5.670400(40) \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4
\]

We invite you to check unit consistency in Eq. (17.25). Emissivity \( (e) \) is often larger for dark surfaces than for light ones. The emissivity of a smooth copper surface is about 0.3, but \( e \) for a dull black surface can be close to unity.

**Example 17.15**

**Heat transfer by radiation**

A thin square steel plate, 10 cm on a side, is heated in a blacksmith's forge to a temperature of 800°C. If the emissivity is 0.60, what is the total rate of radiation of energy?

**SOLUTION**

**IDENTIFY and SET UP:** We use Eq. (17.25). The target variable is \( H \), the rate of emission of energy. All of the other quantities are given.

**EXECUTE:** The total surface area, including both sides, is \( 2(0.10 \text{ m})^2 = 0.020 \text{ m}^2 \). We must convert the temperature to the Kelvin scale; 800°C = 1073 K. Then Eq. (17.25) gives

\[
H = A\varepsilon\sigma T^4 = (0.020 \text{ m}^2)(0.60)(5.67 \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4)(1073 \text{ K})^4 = 900 \text{ W}
\]

While a body at absolute temperature \( T \) is radiating, its surroundings at temperature \( T_s \) are also radiating, and the body absorbs some of this radiation. If it is in thermal equilibrium with its surroundings, \( T = T_s \), and the rates of radiation and absorption must be equal. For this to be true, the rate of absorption must be given in general by \( H = A\varepsilon\sigma T_s^4 \). Then the net rate of radiation from a body at temperature \( T \) with surroundings at temperature \( T_s \) is

\[
H_{\text{net}} = A\varepsilon\sigma T^4 - A\varepsilon\sigma T_s^4 = A\varepsilon\sigma(T^4 - T_s^4) \tag{17.26}
\]

In this equation a positive value of \( H \) means a net heat flow out of the body. Equation (17.26) shows that for radiation, as for conduction and convection, the heat current depends on the temperature difference between two bodies.

**Example 17.16**

**Radiation from the human body**

If the total surface area of the human body is 1.20 m² and the surface temperature is 30°C = 303 K, find the total rate of radiation of energy from the body. If the surroundings are at a temperature of 20°C, what is the net rate of heat loss from the body by radiation? The emissivity of the body is very close to unity, irrespective of skin pigmentation.

**SOLUTION**

**IDENTIFY and SET UP:** The rate of radiation of energy from the body is given by Eq. (17.25), and the net rate of heat loss is given by Eq. (17.26).

**EXECUTE:** Taking \( e = 1 \) in Eq. (17.25), we find that the body radiates at a rate
Heat transfer by radiation is important in some surprising places. A premature baby in an incubator can be cooled dangerously by radiation if the walls of the incubator happen to be cold, even when the air in the incubator is warm. Some incubators regulate the air temperature by measuring the baby’s skin temperature.

A body that is a good absorber must also be a good emitter. An ideal radiator, with an emissivity of unity, is also an ideal absorber, absorbing all of the radiation that strikes it. Such an ideal surface is called an ideal black body or simply a blackbody. Conversely, an ideal reflector, which absorbs no radiation at all, is also a very ineffective radiator.

This is the reason for the silver coatings on vacuum (“Thermos”) bottles, invented by Sir James Dewar (1842–1923). A vacuum bottle has double glass walls. The air is pumped out of the spaces between the walls; this eliminates nearly all heat transfer by conduction and convection. The silver coating on the walls reflects most of the radiation from the contents back into the container, and the wall itself is a very poor emitter. Thus a vacuum bottle can keep coffee or soup hot for several hours. The Dewar flask, used to store very cold liquefied gases, is exactly the same in principle.

**Test Your Understanding**

The ear thermometer in Fig. 17.4 measures the radiation emitted by the eardrum. By what percentage does the radiation rate increase if the eardrum’s temperature increases from 37.00°C to 37.10°C?
A thermometer measures temperature. Two bodies in thermal equilibrium must have the same temperature. A conducting material between two bodies permits them to interact and come to thermal equilibrium; an insulating material prevents or impedes this interaction.

The Celsius and Fahrenheit temperature scales are based on the freezing temperature (0°C = 32°F) and boiling temperature (100°C = 212°F). One Celsius degree equals 5 Fahrenheit degrees. (See Example 17.1)

\[ T_F = \frac{9}{5} T_C + 32 \]  
(17.1)

\[ T_C = \frac{5}{9} (T_F - 32) \]  
(17.2)

The Kelvin scale has its zero at the extrapolated zero-pressure temperature for a gas thermometer, -273.15°C = 0 K. In the gas-thermometer scale, the ratio of two temperatures \( T_2 \) and \( T_1 \) is defined to be equal to the ratio of the two corresponding gas-thermometer pressures \( p_2 \) and \( p_1 \). The triple-point temperature of water (0.01°C) is defined to be 273.16 K.

\[ T_K = T_C + 273.15 \]  
(17.3)

\[ \frac{T_2}{T_1} = \frac{p_2}{p_1} \]  
(17.4)

A temperature change \( \Delta T \) causes a change in any linear dimension \( L_0 \) of a solid body. The change \( \Delta L \) is approximately proportional to \( L_0 \) and \( \Delta T \). Similarly, a temperature change causes a change \( \Delta V \) in the volume \( V_0 \) of any solid or liquid material that is approximately proportional to \( V_0 \) and \( \Delta T \). The quantities \( \alpha \) and \( \beta \) are the coefficients of linear expansion and volume expansion. For solids, \( \beta = 3\alpha \). (See Examples 17.2 through 17.4)

\[ \Delta L = \alpha V_0 \Delta T \]  
(17.6)

\[ \Delta V = \beta V_0 \Delta T \]  
(17.7)

The quantities \( m \) and \( c \) are the mass and specific heat capacity of a body. The change of temperature \( \Delta T \) is proportional to \( \Delta T \). This proportionality can be expressed either in terms of the mass \( m \) and specific heat capacity \( c \) or in terms of the number of moles \( n \) and the molar heat capacity \( C = M \). Here \( M \) is the molar mass and \( m = nM \). (See Examples 17.6 and 17.7)

\[ Q = mc \Delta T \]  
(17.13)

\[ Q = nc \Delta T \]  
(17.14)

When a material is cooled or heated and held so it cannot contract or expand, it is under a tensile stress \( F/A \). (See Example 17.5)

\[ F/A = -Y\alpha \Delta T \]  
(17.12)

Heat is energy in transit from one body to another as a result of a temperature difference. The quantity of heat \( Q \) required to raise the temperature of a quantity of material by a small amount \( \Delta T \) is proportional to \( \Delta T \). This proportionality can be expressed either in terms of the mass \( m \) and specific heat capacity \( c \) or in terms of the number of moles \( n \) and the molar heat capacity \( C = M \). Here \( M \) is the molar mass and \( m = nM \). (See Examples 17.6 and 17.7)

\[ Q = \pm mL \]  
(17.20)
When heat is added to a body, the corresponding $Q$ is positive; when it is removed, $Q$ is negative. The basic principle of calorimetry comes from conservation of energy. In an isolated system whose parts interact by heat exchange, the algebraic sum of the $Q$'s for all parts of the system must be zero. (See Examples 17.8 through 17.11)

Conduction is the transfer of energy of molecular motion within bulk materials without bulk motion of the materials. The heat current $H$ or conduction depends on the area $A$ through which the heat flows, the length $L$ of the heat-flow path, the temperature difference ($T_H - T_C$), and the thermal conductivity $k$ of the material. (See Examples 17.2 through 17.4)

Convection is a complex heat transfer process that involves mass motion from one region to another. It depends on surface area, orientation, and the temperature difference between a body and its surroundings.

Radiation is energy transfer through electromagnetic radiation. The radiation heat current $H$ depends on the surface area $A$, the emissivity $e$ of the surface (a pure number between 0 and 1), and the Kelvin temperature $T$. It involves a fundamental constant $\sigma$ called the Stefan-Boltzmann constant. When a body at temperature $T$ is surrounded by material at temperature $T_0$, the net heat current $H_{\text{net}}$ from the body to its surroundings depends on both $T$ and $T_0$. (See Examples 17.5 and 17.16)

$$H = \frac{dQ}{dt} = kA \frac{T_H - T_C}{L} \quad (17.21)$$

$$H = A\sigma T^4 \quad (17.25)$$

$$H_{\text{net}} = A\sigma (T^4 - T_0^4) \quad (17.26)$$

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Your Notes