Of all the mechanical waves that occur in nature, the most important in our everyday lives are longitudinal waves in a medium, usually air, called sound waves. The reason is that the human ear is tremendously sensitive and can detect sound waves even of very low intensity. Besides their use in spoken communication, our ears allow us to pick up a myriad of cues about our environment, from the welcome sound of a meal being prepared to the warning sound of an approaching car. The ability to hear an unseen nocturnal predator was essential to the survival of our ancestors, so it is no exaggeration to say that we humans owe our existence to our highly evolved sense of hearing.

Up to this point we have described mechanical waves primarily in terms of displacement; however, a description of sound waves in terms of pressure fluctuations is often more appropriate, largely because the ear is primarily sensitive to changes in pressure. We'll study the relations among displacement, pressure fluctuation, and intensity and the connections between these quantities and human sound perception.

When a source of sound or a listener moves through the air, the listener may hear a different frequency than the one emitted by the source. This is the Doppler effect, which has important applications in medicine and technology.
16.1 Sound Waves

The most general definition of sound is that it is a longitudinal wave in a medium. Our main concern in this chapter is with sound waves in air, but sound can travel through any gas, liquid, or solid. You may be all too familiar with the propagation of sound through a solid if your neighbor’s stereo speakers are right next to your wall.

The simplest sound waves are sinusoidal waves, which have definite frequency, amplitude, and wavelength. The human ear is sensitive to waves in the frequency range from about 20 to 20,000 Hz, called the audible range, but we also use the term sound for similar waves with frequencies above (ultrasonic) and below (infrasonic) the range of human hearing.

Sound waves usually travel out in all directions from the source of sound, with an amplitude that depends on the direction and distance from the source. We’ll return to this point in the next section. For now, we concentrate on the idealized case of a sound wave that propagates in the positive x-direction only. As we discussed in Section 15.3, such a wave is described by a wave function \( y(x, t) \), which gives the instantaneous displacement \( y \) of a particle in the medium at position \( x \) at time \( t \). If the wave is sinusoidal, we can express it using Eq. (15.7):

\[
y(x, t) = A \cos(kx - \omega t)
\]

(sound wave propagating in the +x-direction)

Remember that in a longitudinal wave the displacements are parallel to the direction of travel of the wave, so distances \( x \) and \( y \) are measured parallel to each other, not perpendicular as in a transverse wave. The amplitude \( A \) is the maximum displacement of a particle in the medium from its equilibrium position (Fig. 16.1). It is also called the displacement amplitude.

Sound waves may also be described in terms of variations of pressure at various points. In a sinusoidal sound wave in air, the pressure fluctuates above and below atmospheric pressure \( p_0 \), in a sinusoidal variation with the same frequency as the motions of the air particles. The human ear operates by sensing such pressure variations. A sound wave entering the ear canal exerts a fluctuating pressure on one side of the eardrum; the air on the other side of the eardrum, vented to the outside by the Eustachian tube, is at atmospheric pressure. The pressure difference on the two sides of the eardrum sets it into motion. Microphones and similar devices also usually sense pressure differences, not displacements, so it is very useful to develop a relation between these two descriptions.

Let \( p(x, t) \) be the instantaneous pressure fluctuation in a sound wave at any point \( x \) at time \( t \). That is, \( p(x, t) \) is the amount by which the pressure differs from normal atmospheric pressure \( p_0 \). Think of \( p(x, t) \) as the gauge pressure defined in Section 14.2; it can be either positive or negative. The absolute pressure at a point is then \( p_0 + p(x, t) \).

To see the connection between the pressure fluctuation \( p(x, t) \) and the displacement \( y(x, t) \) in a sound wave propagating in the +x-direction, consider an imaginary cylinder of air with cross-sectional area \( S \) and axis along the direction of propagation (Fig. 16.2). When no sound wave is present, the cylinder has length \( \Delta x \) and volume \( V = S \Delta x \), as shown by the shaded volume in Fig. 16.2. When a wave is present, at time \( t \) the end of the cylinder that is initially at \( x \) is displaced by \( y_1 = y(x, t) \), and the end that is initially at \( x + \Delta x \) is displaced by \( y_2 = y(x + \Delta x, t) \); this is shown by the red lines. If \( y_2 > y_1 \), as shown in Fig. 16.2, the cylinder’s volume has increased, which causes a decrease in pressure. If \( y_2 < y_1 \), the cylinder’s volume has decreased and the pressure has increased. If...
16.1 Sound Waves

When \( y_2 = y_1 \), the cylinder is simply shifted to the left or right; there is no volume change and no pressure fluctuation. The pressure fluctuation depends on the difference between the displacement at neighboring points in the medium.

Quantitatively, the change in volume \( \Delta V \) of the cylinder is

\[
\Delta V = S(y_2 - y_1) = S(y(x + \Delta x, t) - y(x, t))
\]

In the limit as \( \Delta x \to 0 \), the fractional change in volume \( dV/V \) is

\[
dV = \lim_{\Delta x \to 0} \frac{S(y(x + \Delta x, t) - y(x, t))}{S\Delta x} = \frac{\partial y(x, t)}{\partial x}
\]

(16.2)

The fractional volume change is related to the pressure fluctuation by the bulk modulus \( B \), which by definition [Eq. (11.13)] is \( B = -p(x, t)/(dV/V) \) (see Section 11.4). Solving for \( p(x, t) \), we have

\[
p(x, t) = -B \frac{\partial y(x, t)}{\partial x}
\]

(16.3)

The negative sign arises because when \( \partial y(x, t)/\partial x \) is positive, the displacement is greater at \( x + \Delta x \) than at \( x \), corresponding to an increase in volume and a decrease in pressure.

When we evaluate \( \partial y(x, t)/\partial x \) for the sinusoidal wave of Eq. (16.1), we find

\[
p(x, t) = BkA \sin(kx - \omega t)
\]

(16.4)

Figure 16.3 shows \( y(x, t) \) and \( p(x, t) \) for a sinusoidal sound wave at \( t = 0 \). It also shows how individual particles of the wave are displaced at this time. While \( y(x, t) \) and \( p(x, t) \) describe the same wave, these two functions are one-quarter cycle out of phase: at any time, the displacement is greatest where the pressure fluctuation is zero, and vice versa. In particular, note that the compressions (points of greatest pressure and density) and rarefactions (points of lowest pressure and density) are points of zero displacement.

**Figure 16.3** Three ways to describe a sound wave. (a) A graph of displacement \( y \) versus position \( x \) [Eq. (16.1)] at \( t = 0 \). (b) A cartoon showing the displacement of individual particles in the gas at \( t = 0 \). Particles are displaced to the right where \( y > 0 \) and to the left where \( y < 0 \). Points where particles pile up are compressions (dark shading), and points where they are pulled apart are rarefactions (light shading). (c) A graph of pressure fluctuation \( p \) versus position \( x \) [Eq. (16.4)] at \( t = 0 \). Note that \( p \) is most positive at a compression and most negative at a rarefaction.
Keep in mind that the graphs in Fig. 16.3 show the wave at only one instant of time. Because the wave is propagating in the +x-direction, as time goes by the wave patterns in the functions \( y(x, t) \) and \( p(x, t) \) move to the right at the wave speed \( v = \omega/k \). Hence the positions of the compressions and rarefactions also move to the right at this same speed. The particles, by contrast, simply oscillate back and forth in simple harmonic motion as shown in Fig. 16.1.

Visualizing the relationship between particle motion and wave motion is not as easy for longitudinal waves as it is for transverse waves on a string. Figure 16.4 will help you understand these motions. To use this figure, tape two index cards edge-to-edge with a gap of 1 mm or so between them, forming a thin slit. Place the cards over the top of the figure with the slit horizontal, and move them downward with constant speed. The portions of the sine curves that are visible through the slit correspond to a row of particles in a medium in which a longitudinal sinusoidal wave is traveling. Each particle undergoes simple harmonic motion about its equilibrium position, with delays or phase shifts that increase continuously along the slit. The regions of maximum compression and expansion move from left to right with constant speed. Moving the card upward simulates a wave traveling from right to left.

Equation (16.4) shows that the quantity \( BkA \) represents the maximum pressure fluctuation. We call this the pressure amplitude, denoted by \( p_{\text{max}} \):

\[
p_{\text{max}} = BkA \quad \text{(sinusoidal sound wave)} \tag{16.5}
\]

The pressure amplitude is directly proportional to the displacement amplitude \( A \), as we might expect, and it also depends on wavelength. Waves of shorter wavelength \( \lambda \) (larger wave number \( k = 2\pi/\lambda \)) have greater pressure variations for a given amplitude because the maxima and minima are squeezed closer together. A medium with a large value of bulk modulus \( B \) requires a relatively large pressure amplitude for a given displacement amplitude because large \( B \) means a less compressible medium; that is, greater pressure change is required for a given volume change.

**Example 16.1**

Amplitude of a sound wave in air

In a sinusoidal sound wave of moderate loudness the maximum pressure variations are of the order of \( 3.0 \times 10^{-2} \) Pa above and below atmospheric pressure \( p_a \) (nominally \( 1.013 \times 10^5 \) Pa at sea level). Find the corresponding maximum displacement if the frequency is \( 1000 \) Hz. In air at normal atmospheric pressure and density, the speed of sound is \( 344 \) m/s and the bulk modulus is \( 1.42 \times 10^5 \) Pa.

**SOLUTION**

**IDENTIFY and SET UP:** We are given the pressure amplitude \( p_{\text{max}} \), wave speed \( v \), frequency \( f \), and bulk modulus \( B \). Our target variable is the displacement \( A \), which is related to \( p_{\text{max}} \) by Eq. (16.5). We also use the relationship \( \omega = \frac{2\pi}{v} \) [Eq. (15.6)] to determine the wave number \( k \) from \( v \) and the angular frequency \( \omega = 2\pi f \).

**EXECUTE:** From Eq. (16.5), the maximum displacement is \( A = p_{\text{max}}/Bk \). From Eq. (15.6), the wave number is

\[
k = \frac{\omega}{v} = \frac{2\pi f}{v} = \frac{(2\pi \text{ rad})(1000 \text{ Hz})}{344 \text{ m/s}} = 18.3 \text{ rad/m}
\]

Then

\[
A = \frac{p_{\text{max}}}{Bk} = \frac{3.0 \times 10^{-2} \text{ Pa}}{(1.42 \times 10^5 \text{ Pa})(18.3 \text{ rad/m})} = 1.2 \times 10^{-8} \text{ m}
\]

**EVALUATE:** This displacement amplitude is only about \( \frac{1}{10^6} \) the size of a human cell. Remember that the ear actually senses pressure fluctuations; it detects these minuscule displacements only indirectly.
Amplitude of a sound wave in the inner ear

When a sound wave enters the ear, it sets the eardrum into oscillation, which in turn causes oscillation of the three tiny bones in the middle ear called the ossicles (Fig. 16.5). This oscillation is finally transmitted to the fluid-filled inner ear; the motion of the fluid disturbs hair cells within the inner ear, which transmit nerve impulses to the brain with the information that a sound is present. The moving part of the eardrum has an area of about 43 mm², and the area of the stirrup (the smallest of the ossicles) where it connects to the inner ear is about 3.2 mm². For the sound in the previous example, determine a) the pressure amplitude and b) the displacement amplitude of the wave in the fluid of the inner ear. The speed of sound in this fluid is about 1500 m/s.

**IDENTIFY:** Although the sound wave is now traveling in fluid (mostly water) rather than gas, the same principles and relationships among the properties of the wave apply.

**SET UP:** We can safely neglect the mass of the ossicles (about 58 mg = 5.8 × 10⁻³ kg), so the force exerted by the ossicles on the fluid in the inner ear is the same as the force exerted on the eardrum and ossicles by the sound wave in air. (We used this same idea in Chapters 4 and 5 when we said that the tension is the same at either end of a massless rope.) Hence the pressure amplitude \( p_{\text{max}} \) is greater in the inner ear than in the outside air because the same force is exerted on a smaller area (the area of the stirrup versus the area of the eardrum).

Given the pressure amplitude in the inner ear, we find the displacement amplitude using Eq. (16.5). The values of \( B \) and \( k \) are different than in the air. To determine \( k \), note that the wave in the inner ear has the same angular frequency \( \omega \) as the wave in the air because the air, eardrum, ossicles, and inner-ear fluid all oscillate together. But because the wave speed \( v \) is greater in the inner ear than in the air (1500 m/s versus 344 m/s), the wave number \( k = \omega / v \) is greater as well.

**EXECUTE:** a) Using the area of the eardrum and pressure amplitude found in Example 16.1, the maximum force exerted by the sound wave in air on the eardrum is \( F_{\text{max}} = p_{\text{max (air)}} S_{\text{eardrum}} \). Hence the pressure amplitude in the inner ear fluid is

\[
p_{\text{max (inner ear)}} = \frac{F_{\text{max}}}{S_{\text{stirrup}}} = \frac{p_{\text{max (air)}} S_{\text{eardrum}}}{S_{\text{stirrup}}} = \frac{(3.0 \times 10^{-2} \text{ Pa})(43 \text{ mm}^2)}{3.2 \text{ mm}^2} = 0.40 \text{ Pa}
\]

b) To find the maximum displacement, we again use the relationship \( A = \frac{p_{\text{max}}}{Bk} \) as in Example 16.1. The fluid in the inner ear is mostly water, which has a much greater bulk modulus than air because water is much more difficult to compress. From Table 11.2 the compressibility of water (unfortunately also called \( k \)) equals 45.8 × 10⁻¹¹ Pa⁻¹, so \( B_{\text{fluid}} = 1/(45.8 \times 10^{-11} \text{ Pa}^{-1}) = 2.18 \times 10^{10} \text{ Pa} \).

We determine the value of the wave number \( k \) using the value of \( \omega \) from Example 16.1 and \( v = 1500 \text{ m/s} \) for the inner-ear fluid. Hence

\[
k_{\text{inner ear}} = \frac{\omega}{v_{\text{inner ear}}} = \frac{6283 \text{ rad/s}}{1500 \text{ m/s}} = 4.2 \text{ rad/m}
\]

When we put everything together, the maximum displacement of the fluid in the inner ear is

\[
A_{\text{inner ear}} = \frac{p_{\text{max (inner ear)}}}{B_{\text{fluid}} k_{\text{inner ear}}} = \frac{0.40 \text{ Pa}}{(2.18 \times 10^{10} \text{ Pa})(4.2 \text{ rad/m})} = 4.4 \times 10^{-11} \text{ m}
\]
EVALUATE: The result in part (a) shows that the effect of the ossicles is to increase the pressure amplitude in the inner ear by a factor of \((43 \text{ mm}^2)/(3.2 \text{ mm}^2) = 13\). This amplification factor helps give the human ear its great sensitivity.

The displacement amplitude in the inner ear is even smaller than in the air. What really matters in the inner ear, however, is the pressure amplitude, since the pressure variations within the fluid cause the forces that set the hair cells into motion.

### Perception of Sound Waves

The physical characteristics of a sound wave are directly related to the perception of that sound by a listener. For a given frequency, the greater the pressure amplitude of a sinusoidal sound wave, the greater the perceived loudness. The relationship between pressure amplitude and loudness is not a simple one, and it varies from one person to another. One important factor is that the ear is not equally sensitive to all frequencies in the audible range. A sound at one frequency may seem louder than one of equal pressure amplitude at a different frequency. At 1000 Hz the minimum pressure amplitude that can be perceived with normal hearing is about \(3 \times 10^{-5}\) Pa; to produce the same loudness at 200 Hz or 15,000 Hz requires about \(3 \times 10^{-4}\) Pa. Perceived loudness also depends on the health of the ear. A loss of sensitivity at the high-frequency end usually happens naturally with age but can be further aggravated by excessive noise levels. Studies have shown that many young rock musicians have suffered permanent ear damage and have hearing that is typical of persons 65 years of age. Portable stereo headsets used at high volume pose similar threats to hearing. Be careful!

The frequency of a sound wave is the primary factor in determining the pitch of a sound, the quality that lets us classify the sound as “high” or “low.” The higher the frequency of a sound (within the audible range), the higher the pitch that a listener will perceive. Pressure amplitude also plays a role in determining pitch. When a listener compares two sinusoidal sound waves with the same frequency but different pressure amplitudes, the one with the greater pressure amplitude is usually perceived as louder but also as slightly lower in pitch.

Musical sounds have wave functions that are more complicated than a simple sine function. The pressure fluctuation in the sound wave produced by a clarinet is shown in Fig. 16.6a. The pattern is so complex because the column of air in a wind instrument like a clarinet vibrates at a fundamental frequency and at many harmonics at the same time. (In Section 15.8, we described this same behavior for a string that has been plucked, bowed, or struck. We’ll examine the physics of wind instruments in Section 16.5.) The sound wave produced in the surrounding air has a similar amount of each harmonic, that is, a similar harmonic content. Figure 16.6b shows the harmonic content of the sound of a clarinet. The mathematical process of translating a pressure-time graph like Fig. 16.6a into a graph of harmonic content like Fig. 16.6b is called Fourier analysis.

Two tones produced by different instruments might have the same fundamental frequency (and thus the same pitch) but sound different because of the presence of different amounts of the various harmonics. The difference is called tone color, quality, or timbre and is often described in subjective terms such as reedy, golden, round, mellow, and tinny. A tone that is rich in harmonics, like the clarinet tone in Figs. 16.6a and 16.6b usually sounds thin and “stringy” or “reedy,” while a tone containing mostly a fundamental, like the alto recorder tone in Figs. 16.6c and 16.6d is more mellow and flutelike. The same principle applies to the human voice, which is another example of a wind instrument; the vowels “a” and “e” sound different because of differences in harmonic content.
Another factor in determining tone quality is the behavior at the beginning (attack) and end (decay) of a tone. A piano tone begins with a thump and then dies away gradually. A harpsichord tone, in addition to having different harmonic content, begins much more quickly with a click, and the higher harmonics begin before the lower ones. When the key is released, the sound also dies away much more rapidly with a harpsichord than with a piano. Similar effects are present in other musical instruments. With wind and string instruments the player has considerable control over the attack and decay of the tone, and these characteristics help to define the unique characteristics of each instrument.

Unlike the tones made by musical instruments or the vowels in human speech, **noise** is a combination of all frequencies, not just frequencies that are integer multiples of a fundamental frequency. (An extreme case is “white noise,” which contains equal amounts of all frequencies across the audible range.) Examples include the sound of the wind and the hissing sound you make in saying the consonant “s.”

**Test Your Understanding**

You use an electronic signal generator to produce a sinusoidal sound wave in air. You then increase the frequency of the wave while keeping the pressure amplitude constant. What effect does this have on the displacement amplitude of the sound wave?

---

**16.2 | Speed of Sound Waves**

We found in Section 15.4 that the speed $v$ of a transverse wave on a string depends on the string tension $F$ and the linear mass density $\mu$: $v = \sqrt{F/\mu}$. What, we may ask, is the corresponding expression for the speed of sound waves in a gas or liquid? On what properties of the medium does the speed depend?

We can make an educated guess about these questions by remembering a claim that we made in Section 15.4: For mechanical waves in general, the expression for the wave speed is of the form

$$v = \sqrt{\frac{\text{(restoring force returning the system to equilibrium)}}{\text{(inertia resisting the return to equilibrium)}}}$$

A sound wave in a bulk fluid causes compressions and rarefactions of the fluid, so the restoring-force term in the above expression must be related to how easy or difficult it is to compress the fluid. This is precisely what the bulk modulus $B$ of the medium tells us. According to Newton’s second law, inertia is related to mass. The “massiveness” of a bulk fluid is described by its density, or mass per unit volume, $\rho$. (The corresponding quantity for a string is the mass per unit length, $\mu$.) Hence we expect that the speed of sound waves should be of the form $v = \sqrt{B/\rho}$.

To check our guess, we’ll derive the speed of sound (longitudinal) waves in a fluid in a pipe. This is a situation of some importance, since all musical wind instruments are fundamentally pipes in which a longitudinal wave (sound) propagates in a fluid (air). Human speech works on the same principle; sound waves propagate in your vocal tract, which is basically an air-filled pipe connected to the lungs at one end (your larynx) and to the outside air at the other end (your mouth). The steps in our derivation are completely parallel to those we used in Section 15.4 to find the speed of transverse waves, so you’ll find it useful to review that section.
A sound wave propagating in a fluid confined to a tube. (a) Fluid in equilibrium. (b) At time t after the piston begins moving to the right at speed \( v_s \), the fluid between the piston and point \( P \) is in motion. The speed of sound waves is \( v \).

Figure 16.7 shows a fluid (either liquid or gas) with density \( \rho \) in a pipe with cross-sectional area \( A \). In the equilibrium state, the fluid is under a uniform pressure \( p \). In Fig. 16.7a the fluid is at rest. We take the \( x \)-axis along the length of the pipe. This is also the direction in which we make a longitudinal wave propagate, so the displacement \( y \) is also measured along the pipe, just as in Section 16.1 (see Fig. 16.2).

At time \( t = 0 \) we start the piston at the left end moving toward the right with constant speed \( v_s \). This initiates a wave motion that travels to the right along the length of the pipe, in which successive sections of fluid begin to move and become compressed at successively later times.

Figure 16.7b shows the fluid at time \( t \). All portions of fluid to the left of point \( P \) are moving to the right with speed \( v_s \), and all portions to the right of \( P \) are still at rest. The boundary between the moving and stationary portions travels to the right with a speed equal to the speed of propagation or wave speed \( v \). At time \( t \) the piston has moved a distance \( v_s t \), and the boundary has advanced a distance \( vt \). As with a transverse disturbance in a string, we can compute the speed of propagation from the impulse-momentum theorem.

The quantity of fluid set in motion in time \( t \) is the amount that originally occupied a section of the cylinder with length \( vt \), cross-sectional area \( A \), and volume \( vtA \). The mass of this fluid is \( \rho vtA \), and its longitudinal momentum (that is, momentum along the length of the pipe) is

\[
\text{Longitudinal momentum} = (\rho vtA) v_y
\]

Next we compute the increase of pressure, \( \Delta p \), in the moving fluid. The original volume of the moving fluid, \( Av_t \), has decreased by an amount \( Av_t t \). From the definition of the bulk modulus \( B \), Eq. (11.13) in Section 11.5,

\[
B = -\frac{\text{Pressure change}}{\text{Fractional volume change}} = -\frac{-\Delta p}{-Av_t/tAv_t}
\]

\[
\Delta p = B \frac{v_s}{v}
\]

The pressure in the moving fluid is \( p + \Delta p \) and the force exerted on it by the piston is \( (p + \Delta p)A \). The net force on the moving fluid (see Fig. 16.7b) is \( \Delta pA \), and the longitudinal impulse is

\[
\text{Longitudinal impulse} = \Delta pAt = B \frac{v_s}{v} At
\]

Because the fluid was at rest at time \( t = 0 \), the change in momentum up to time \( t \) is equal to the momentum at that time. Applying the impulse-momentum theorem, we find

\[
B \frac{v_s}{v} At = \rho vtAv_y
\]

When we solve this expression for \( v \), we get

\[
v = \sqrt{\frac{B}{\rho}} \quad \text{(speed of a longitudinal wave in a fluid)}
\]

(16.7)

which agrees with our educated guess. Thus the speed of propagation of a longitudinal pulse in a fluid depends only on the bulk modulus \( B \) and the density \( \rho \) of the medium.
While we derived Eq. (16.7) for waves in a pipe, it also applies to longitudinal waves in a bulk fluid. Thus the speed of sound waves traveling in air or water is determined by this equation.

When a longitudinal wave propagates in a solid rod or bar, the situation is somewhat different. The rod expands sideways slightly when it is compressed longitudinally, while a fluid in a pipe with constant cross section cannot move sideways. Using the same kind of reasoning that led us to Eq. (16.7), we can show that the speed of a longitudinal pulse in the rod is given by

\[ v = \sqrt{\frac{Y}{\rho}} \quad \text{(speed of a longitudinal wave in a solid rod)} \quad (16.8) \]

where \( Y \) is Young’s modulus, defined in Section 11.4.

**CAUTION** Equation (16.8) applies only to a rod or bar whose sides are free to bulge and shrink a little as the wave travels. It does not apply to longitudinal waves in a bulk solid, since in these materials, sideways motion in any element of material is prevented by the surrounding material. The speed of longitudinal waves in a bulk solid depends on the density, the bulk modulus, and the shear modulus; a full discussion is beyond the scope of this book.

As with the derivation for a transverse wave on a string, Eqs. (16.7) and (16.8) are valid for sinusoidal and other periodic waves, not just for the special case discussed here.

Table 16.1 lists the speed of sound in several bulk materials. Sound waves travel more slowly in lead than in aluminum or steel because lead has a lower bulk modulus and shear modulus and a higher density.

**Example 16.3**

**Wavelength of sonar waves**

A ship uses a sonar system to detect underwater objects (Fig. 16.8). The system emits underwater sound waves and measures the time interval for the reflected wave (echo) to return to the detector.

Determine the speed of sound waves in water using Eq. (16.7) and find the wavelength of a 262-Hz wave.

**SOLUTION**

**IDENTIFY and SET UP:** To use Eq. (16.7), we find the bulk modulus of water from the compressibility (Table 11.2) and the density \( \rho = 1.00 \times 10^3 \text{ kg/m}^3 \). Given the speed and the frequency \( f = 262 \text{ Hz} \), we find the wavelength from the relationship \( v = f \lambda \).

**EXECUTE:** From Table 11.2 we find that the compressibility of water, which is the reciprocal of the bulk modulus, is \( k = 45.8 \times 10^{-11} \text{ Pa}^{-1} \). Thus \( B = (1/45.8) \times 10^{11} \text{ Pa} \). We obtain

\[ v = \frac{B}{\rho} = \sqrt{\frac{(1/45.8) \times 10^{11} \text{ Pa}}{1.00 \times 10^3 \text{ kg/m}^3}} = 1480 \text{ m/s} \]

The wavelength is given by

\[ \lambda = \frac{v}{f} = \frac{1480 \text{ m/s}}{262 \text{ s}^{-1}} = 5.65 \text{ m} \]
The calculated value of $v$ agrees well with the experimental value in Table 16.1. Though water is far denser than air ($\rho$ is larger), it is also far more incompressible ($B$ is larger) and the speed $v = \sqrt{\frac{B}{\rho}}$ turns out to be more than four times the speed of sound in air at ordinary temperatures.

We found in Example 15.1 (Section 15.2) that a wave of this frequency in air has a wavelength of 1.31 m. The speed of sound in water is greater than in air, so the wavelength $\lambda = \frac{v}{f}$ must be greater as well; this is just as we calculated.

Dolphins emit high-frequency sound waves (typically 100,000 Hz) and use the echoes for guidance and for hunting. The corresponding wavelength in water is 1.48 cm. With this high-frequency “sonar” system they can sense objects that are roughly as small as the wavelength (but not much smaller). Ultrasonic imaging is a medical technique that uses exactly the same physical principle; sound waves of very high frequency and very short wavelength, called ultrasound, are scanned over the human body, and the “echoes” from interior organs are used to create an image. With ultrasound of frequency 5 MHz = $5 \times 10^6$ Hz, the wavelength in water (the primary constituent of the body) is 0.3 mm, and features as small as this can be discerned in the image. Ultrasound is used for the study of heart-valve action, detection of tumors, and prenatal examinations (Fig. 16.9). Ultrasound is more sensitive than x rays in distinguishing various kinds of tissues and does not have the radiation hazards associated with x rays.

**Example 16.4**

**Speed of a longitudinal wave**

What is the speed of longitudinal waves in a lead rod?

**SOLUTION**

**IDENTIFY:** Waves of this kind are made by clamping the rod in place and striking one end face-on with a hammer. The amplitude of the resulting waves is so small as to be nearly invisible to the naked eye. But the question is about the speed of the waves, which does not depend on the amplitude. Note that we cannot simply use the value for lead from Table 16.1, as that value refers to the speed of sound in a bulk material, not a rod.

**SET UP:** Equation (16.8) applies to this situation. We find the values of Young’s modulus $Y$ and the density $\rho$ from Tables 11.1 and 14.1, respectively.

**EXECUTE:** We find $Y = 1.6 \times 10^{11}$ Pa and $\rho = 11.3 \times 10^3$ kg/m$^3$ (that is, 11.4 times the density of water), so

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{1.6 \times 10^{11} \text{ Pa}}{11.3 \times 10^3 \text{ kg/m}^3}} = 1.2 \times 10^3 \text{ m/s}$$

**EVALUATE:** This is more than three times the speed of sound in air, but slower than the speed of sound in bulk lead (see Table 16.1). The reason is that for lead the bulk modulus is greater than Young’s modulus.

**Speed of Sound in Gases**

Most of the sound waves that we encounter on a daily basis propagate in air. To use Eq. (16.7) to find the speed of sound waves in air, we must keep in mind that the bulk modulus of a gas depends on the pressure of the gas; the greater the pressure applied to a gas to compress it, the more it resists further compression and hence the greater the bulk modulus. (That’s why specific values of the bulk modulus for gases are not given in Table 11.1.) The expression for the bulk modulus of a gas for use in Eq. (16.7) is

$$B = \gamma p_0$$

where $p_0$ is the equilibrium pressure of the gas. The quantity $\gamma$ (the Greek letter “gamma”) is called the *ratio of heat capacities*. It is a dimensionless number that
characterizes the thermal properties of the gas. (We’ll learn more about this quantity in Chapter 19.) As an example, the ratio of heat capacities for air is $\gamma = 1.40$.

At normal atmospheric pressure $p_0 = 1.01 \times 10^5$ Pa, so $B = (1.40) \left( 1.013 \times 10^5 \text{ Pa} \right) = 1.42 \times 10^5$ Pa. This value is miniscule compared to the bulk modulus of a typical solid (Table 11.1), which is approximately $10^{10}$ to $10^{11}$ Pa. This shouldn’t be surprising: it’s simply a statement that air is far easier to compress than steel.

The density $\rho$ of a gas also depends on the pressure, which in turn depends on the temperature. It turns out that the ratio $B/\rho$ for a given type of ideal gas does not depend on the pressure at all, only the temperature. From Eq. (16.7), this means that the speed of sound in a gas is fundamentally a function of temperature $T$:

$$v = \sqrt{\frac{\gamma RT}{M}} \quad \text{(speed of sound in an ideal gas)} \quad (16.10)$$

This expression incorporates several quantities that you may recognize from your study of ideal gases in chemistry and which we will study in Chapters 17, 18, and 19. The temperature $T$ is the absolute temperature in kelvins (K), equal to the Celsius temperature plus 273.15; thus 20.00°C corresponds to $T = 293.15$ K. The quantity $M$ is the molar mass, or mass per mole of the substance of which the gas is composed. The gas constant $R$ has the same value for all gases. The current best numerical value of $R$ is

$$R = 8.314472(15) \text{ J/mol} \cdot \text{K}$$

which for practical calculations we can write as 8.314 J/mol·K.

For any particular gas, $\gamma$, $R$, and $M$ are constants, and the wave speed is proportional to the square root of the absolute temperature. We will see in Chapter 18 that Eq. (16.10) is almost identical to the expression for the average speed of molecules in an ideal gas. This shows that sound speeds and molecular speeds are closely related; exploring that relationship in detail would be beyond our scope.

### Example 16.5 Speed of sound in air

Compute the speed of sound waves in air at room temperature ($T = 20^\circ \text{C}$) and find the range of wavelengths in air to which the human ear (which can hear frequencies in the range of 20–20,000 Hz) is sensitive. The mean molar mass for air (a mixture of principally nitrogen and oxygen) is $28.8 \times 10^{-3}$ kg/mol and the ratio of heat capacities is $\gamma = 1.40$.

**SOLUTION**

**IDENTIFY and SET UP:** We use Eq. (16.10) to find the sound speed and the relation $v = \lambda f$ to determine the wavelength that corresponds to each frequency.

**EXECUTE:** At $T = 20^\circ \text{C} = 293$ K we find

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$= \sqrt{\frac{(1.40) (8.314 \text{ J/mol} \cdot \text{K}) (293 \text{ K})}{28.8 \times 10^{-3} \text{ kg/mol}}} = 344 \text{ m/s}$$

Using this value of $v$ and the expression $\lambda = \frac{v}{f}$, we find that at $20^\circ \text{C}$ a 20-Hz note corresponds to a wavelength of 17 m and a 20,000-Hz note corresponds to a wavelength of 1.7 cm.

**EVALUATE:** Our calculated value of $v$ agrees with the measured speed of sound at this temperature to within 0.3%.

It’s interesting to note that bats can hear much higher frequencies. Like dolphins, bats use high-frequency sound waves for navigation. A typical frequency is 100 kHz; the corresponding wavelength in air at $20^\circ \text{C}$ is about 3.4 mm, small enough for them to detect the flying insects they eat.
In this discussion we have ignored the molecular nature of a gas and have treated it as a continuous medium. A gas is actually composed of molecules in random motion, separated by distances that are large in comparison with their diameters. The vibrations that constitute a wave in a gas are superposed on the random thermal motion. At atmospheric pressure, a molecule travels an average distance of about \(10^{-7}\) m between collisions, while the displacement amplitude of a faint sound may be only \(10^{-9}\) m. We can think of a gas with a sound wave passing through as being comparable to a swarm of bees; the swarm as a whole oscillates slightly while individual insects move about through the swarm, apparently at random.

**Test Your Understanding**

Calculate the speed of longitudinal waves along a steel railroad track. Explain why you can hear the sound of an approaching train from listening to the rails before you can hear it in the air.

### 16.3 | Sound Intensity

Traveling sound waves, like all other traveling waves, transfer energy from one region of space to another. We saw in Section 15.5 that a useful way to describe the energy carried by a sound wave is through the wave intensity \(I\), equal to the time average rate at which energy is transported per unit area across a surface perpendicular to the direction of propagation. In particular, we will express the intensity of a sound wave in terms of the displacement amplitude \(A\) or pressure amplitude \(p_{\text{max}}\).

For simplicity, let us consider a sound wave propagating in the \(+x\)-direction so that we can use our expressions for Section 16.1 for the displacement \(y(x, t)\) and pressure fluctuation \(p(x, t)\)—Eqs. (16.1) and (16.4), respectively. In Section 6.4 we saw that power equals the product of force and velocity [see Eq. (6.18)]. So the power per unit area in this sound wave equals the product of \(p(x, t)\) (force per unit area) and the particle velocity \(v_p(x, t)\). The particle velocity \(v_p(x, t)\) is the velocity at time \(t\) of that portion of the wave medium at coordinate \(x\). Using Eqs. (16.1) and (16.4), we find

\[
v_p(x, t) = \frac{\partial y(x, t)}{\partial t} = \omega A \sin(kx - \omega t)
\]

\[
p(x, t)v_p(x, t) = [BkA \sin(kx - \omega t)][\omega A \sin(kx - \omega t)]
= BkA^2 \sin^2(kx - \omega t)
\]

**CAUTION** Remember that the velocity of the wave as a whole is not the same as the particle velocity. While the wave continues to move in the direction of propagation, individual particles in the wave medium merely slosh back and forth, as shown in Fig. (16.1). Furthermore, the maximum speed of a particle of the medium can be very different from the wave speed.

The intensity is, by definition, the time average value of \(p(x, t)v_p(x, t)\). For any value of \(x\) the average value of the function \(\sin^2(kx - \omega t)\) over one period \(T = 2\pi/\omega\) is \(1/2\), so

\[
I = \frac{1}{2} BkA^2
\]  

(16.11)
By using the relations \( \omega = vk \) and \( v^2 = B/\rho \), we can transform Eq. (16.11) into the form

\[
I = \frac{1}{2} \sqrt{\rho Bo^2 A^2} \quad \text{(intensity of a sinusoidal sound wave)} \quad (16.12)
\]

This equation shows why in a stereo system, a low-frequency woofer has to vibrate with much larger amplitude than a high-frequency tweeter to produce the same sound intensity.

It is usually more useful to express \( I \) in terms of the pressure amplitude \( p_{\text{max}} \). Using Eq. (16.5) and the relation \( \omega = vk \), we find

\[
I = \frac{\omega p_{\text{max}}^2}{2Bk} = \frac{v p_{\text{max}}^2}{2B} \quad (16.13)
\]

By using the wave speed relation \( v^2 = B/\rho \), we can also write Eq. (16.13) in the alternative forms

\[
I = \frac{p_{\text{max}}^2}{2\rho v} = \frac{p_{\text{max}}^2}{2\sqrt{\rho B}} \quad \text{(intensity of a sinusoidal sound wave)} \quad (16.14)
\]

You should verify these expressions (see Exercise 16.16). Comparison of Eqs. (16.12) and (16.14) shows that sinusoidal sound waves of the same intensity but different frequency have different displacement amplitudes \( A \) but the same pressure amplitude \( p_{\text{max}} \). This is another reason why it is usually more convenient to describe a sound wave in terms of pressure fluctuations, not displacement.

The total average power carried across a surface by a sound wave equals the product of the intensity at the surface and the surface area if the intensity over the surface is uniform. The average total sound power emitted by a person speaking in an ordinary conversational tone is about \( 10^{-3} \) W, while a loud shout corresponds to about \( 3 \times 10^{-2} \) W. If all the residents of New York City were to talk at the same time, the total sound power would be about 100 W, equivalent to the electric power requirement of a medium-sized light bulb. On the other hand, the power required to fill a large auditorium or stadium with loud sound is considerable (see Example 16.9.)

If the sound source emits waves in all directions equally, the intensity decreases with increasing distance \( r \) from the source according to the inverse-square law: the intensity is proportional to \( 1/r^2 \). We discussed this law and its consequences in Section 15.5.

The inverse-square relationship does not apply indoors because sound energy can also reach a listener by reflection from the walls and ceiling. Indeed, part of the architect's job in designing an auditorium is to tailor these reflections so that the intensity is as nearly constant as possible over the entire auditorium.

---

**Problem-Solving Strategy: Sound Intensity**

**Identify the relevant concepts:** The relationships between intensity and amplitude of a sound wave are rather straightforward. Quite a few other quantities are involved in these relationships, however, so it's particularly important to decide which is your target variable.
**Example 16.6**

**Intensity of a sound wave in air**

Find the intensity of the sound wave in Example 16.1, with $p_{\text{max}} = 3.0 \times 10^{-2}$ Pa. Assume the temperature is 20°C so that the density of air is $\rho = 1.20 \text{ kg/m}^3$ and the speed of sound is $v = 344 \text{ m/s}$.

**SOLUTION**

**IDENTIFY and SET UP:** We are given the pressure amplitude $p_{\text{max}}$, density $\rho$, and wave speed $v$, and want to find the intensity $I$. This is done most easily with Eq. (16.14).

**EXECUTE:** From Eq. (16.14),

$$I = \frac{p^2_{\text{max}}}{2\rho v} = \frac{(3.0 \times 10^{-2} \text{ Pa})^2}{2(1.20 \text{ kg/m}^3)(344 \text{ m/s})}$$

$$= 1.1 \times 10^{-6} \text{ W/(s m}^2) = 1.1 \times 10^{-6} \text{ W/m}^2$$

**Example 16.7**

**Same intensity, different frequencies**

A 20-Hz sound wave has the same intensity as the 1000-Hz sound wave in Examples 16.1 and 16.6. What are the displacement amplitude and pressure amplitude of the 20-Hz sound wave?

**SOLUTION**

**IDENTIFY and SET UP:** Given the intensity, we can use Eq. (16.12) to determine the displacement amplitude $A$. Note that we are not given the value of $B$. However, $\rho$ and $B$ depend only on the properties of the medium, not the amplitude or frequency, so their values will cancel out if we equate the intensities at 20 Hz and at 1000 Hz. We can also use Eq. (16.14) to find the pressure amplitude at 20 Hz.

**EXECUTE:**

Inspection of Eq. (16.12) shows that if a wave in a given medium (same $\rho$ and $B$) has the same intensity $I$ at two different frequencies, then the product $\omega A$ must have the same value for both frequencies. From Example 16.1, $A = 1.2 \times 10^{-8} \text{ m}$ at 1000 Hz, so

$$(20 \text{ Hz})A_{20} = (1000 \text{ Hz})(1.2 \times 10^{-8} \text{ m})$$

$$A_{20} = 6.0 \times 10^{-7} \text{ m} = 0.60 \mu\text{m}$$

Do you understand why we didn’t have to convert the frequencies to angular frequencies?

Since the intensity is the same for both frequencies, Eq. (16.14) shows that the pressure amplitude $p_{\text{max}}$ must also be the same for both. Hence $p_{\text{max}} = 3.0 \times 10^{-2}$ Pa for $f = 20$ Hz.

**EVALUATE:** Our result reinforces the idea that pressure amplitude offers a more convenient description of a sound wave than displacement amplitude. Note also that using Eq. (16.5) and $k = \omega/v$, we get $p_{\text{max}} = BkA = (B/v)\omega A$; the bulk modulus $B$ and wave speed $v$ depend only on the medium, so we again conclude that the product $\omega A$ must have the same value for both frequencies.
A high-altitude sound wave

At an altitude of 11,000 m, near the cruising altitude of jetliners (Fig. 16.10), the atmosphere is cold and thin: the temperature is \(-57^\circ C\), the pressure is \(2.26 \times 10^4\) Pa, and the density is \(0.364 \text{ kg/m}^3\). What would be the intensity of a 1000-Hz sound wave with the same displacement amplitude as the wave at sea level in Examples 16.1 and 16.6?

**Example 16.8**

**Problem:**

How do sound waves differ in the upper atmosphere?

**Solution:**

**IDENTIFY and SET UP:** We can use Eq. (16.12) to determine the intensity from the density, bulk modulus, frequency, and amplitude. We are not given the value of \(B\), but we can calculate it using Eq. (16.9).

**EXECUTE:** Using Eq. (16.9) and the value \(\gamma = 1.40\) for air, we find \(B = \gamma p_0 = (1.40)(2.26 \times 10^4 \text{ Pa}) = 3.16 \times 10^5 \text{ Pa}\). Then, from Eq. (16.12),

\[
I = \frac{1}{2} \sqrt{\left(0.364 \text{ kg/m}^3\right)(2.26 \times 10^4 \text{ Pa})} \times (2\pi)^2(1000 \text{ Hz})^2(1.2 \times 10^{-8} \text{ m})^2
\]

\[
= 3.1 \times 10^{-1} \text{ W/m}^2
\]

**EVALUATE:** This intensity is only 28% as great as that of a wave with the same amplitude at sea level \((3.1 \times 10^{-1} \text{ W/m}^2\) versus \(1.1 \times 10^{-6} \text{ W/m}^2\)). Hence a loudspeaker oscillating with a certain amplitude would produce a substantially less intense sound at high altitude. At even higher altitude, the atmosphere fades into the near-vacuum of interplanetary space and the density and pressure (and hence bulk modulus) decrease to zero. The intensity likewise decreases, and outer space is nearly silent.

For an outdoor concert we want the sound intensity at a distance of 20 m from the speaker array to be 1 W/m\(^2\). Assuming that the sound waves have the same intensity in all directions, what acoustic power output is needed from the speaker array?

**Example 16.9**

**Problem:**

“Play it loud!”

**Solution:**

**IDENTIFY:** This example uses the definition of intensity as power per unit area. Here the total power is the target variable, and the area in question is a hemisphere centered on the speaker array.

**SET UP:** We make the assumptions that the speakers are near ground level and that the acoustic power is spread uniformly over a hemisphere 20 m in radius (that is, we assume that none of the acoustic power is directed into the ground). The surface area of this hemisphere is equal to \((1/2)(4\pi)(20 \text{ m})^2\), or about 2500 m\(^2\). The required power is the product of this area and the intensity.

**EXECUTE:** The speaker array power is

\[
(1 \text{ W/m}^2)(2500 \text{ m}^2) = 2500 \text{ W} = 2.5 \text{ kW}
\]

**EVALUATE:** The electrical power input to the speaker would need to be considerably larger because the efficiency of such devices is not very high (typically a few percent for ordinary speakers, and up to 25% for horn-type speakers).

The Decibel Scale

Because the ear is sensitive over a broad range of intensities, a logarithmic intensity scale is usually used. The sound intensity level \(\beta\) of a sound wave is defined by the equation

\[
\beta = (10 \text{ dB}) \log \frac{I}{I_0} \quad \text{(definition of sound intensity level)} \quad (16.15)
\]
In this equation, $I_0$ is a reference intensity, chosen to be $10^{-12}$ W/m², approximately the threshold of human hearing at 1000 Hz. Recall that “log” means the logarithm to base 10. Sound intensity levels are expressed in decibels, abbreviated dB. A decibel is $\frac{1}{10}$ of a bel, a unit named for Alexander Graham Bell (the inventor of the telephone). The bel is inconveniently large for most purposes, and the decibel is the usual unit of sound intensity level.

If the intensity of a sound wave equals $I_0$ or $10^{-12}$ W/m², its sound intensity level is 0 dB. An intensity of 1 W/m² corresponds to 120 dB. Table 16.2 gives the sound intensity levels in decibels of several familiar sounds. You can use Eq. (16.15) to check the value of sound intensity level $P$ given for each intensity in the table.

<table>
<thead>
<tr>
<th>Source or Description of Sound</th>
<th>Sound Intensity Level, $P$ (dB)</th>
<th>Intensity, $I$ (W/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Military jet aircraft 30 m away</td>
<td>140</td>
<td>$10^2$</td>
</tr>
<tr>
<td>Threshold of pain</td>
<td>120</td>
<td>1</td>
</tr>
<tr>
<td>Riveter</td>
<td>95</td>
<td>$3.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>Elevated train</td>
<td>90</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Busy street traffic</td>
<td>70</td>
<td>$10^{-5}$</td>
</tr>
<tr>
<td>Ordinary conversation</td>
<td>65</td>
<td>$3.2 \times 10^{-6}$</td>
</tr>
<tr>
<td>Quiet automobile</td>
<td>50</td>
<td>$10^{-7}$</td>
</tr>
<tr>
<td>Quiet radio in home</td>
<td>40</td>
<td>$10^{-8}$</td>
</tr>
<tr>
<td>Average whisper</td>
<td>20</td>
<td>$10^{-10}$</td>
</tr>
<tr>
<td>Rustle of leaves</td>
<td>10</td>
<td>$10^{-11}$</td>
</tr>
<tr>
<td>Threshold of hearing at 1000 Hz</td>
<td>0</td>
<td>$10^{-12}$</td>
</tr>
</tbody>
</table>

Because the ear is not equally sensitive to all frequencies in the audible range, some sound-level meters weight the various frequencies unequally. One such scheme leads to the so-called dBA scale; this scale deemphasizes the low and very high frequencies, where the ear is less sensitive than at midrange frequencies.

**Example 16.10**

Temporary deafness

A ten-minute exposure to 120-dB sound will typically shift your threshold of hearing at 1000 Hz from 0 dB up to 28 dB for a while. Ten years of exposure to 92-dB sound will cause a permanent shift up to 28 dB. What intensities correspond to 28 dB and 92 dB?

**SOLUTION**

**IDENTIFY and SET UP:** We solve Eq. (16.15) to find the intensity $I$ (the target variable) for each value of the sound intensity level $P$. When $P = 28$ dB,

$$I = I_0 10^{P/10} = (10^{-12} \text{ W/m}^2) 10^{28/10} = (10^{-12} \text{ W/m}^2) 10^{2.8} = 6.3 \times 10^{-10} \text{ W/m}^2$$

Similarly, for $P = 92$ dB,

$$I = (10^{-12} \text{ W/m}^2) 10^{92/10} = 1.6 \times 10^{-3} \text{ W/m}^2$$

**EXECUTE:** We rearrange Eq. (16.15) by dividing both sides by 10 dB and then using the relationship $10^{P_{10}} = x$:

$$I = I_0 10^{P_{10}}$$

Be careful!
A bird sings in a meadow

Consider an idealized model with a bird (treated as a point source) emitting constant sound power, with intensity inversely proportional to the square of the distance from the bird. By how many decibels does the sound intensity level drop when you move twice as far away from the bird?

**SOLUTION**

**IDENTIFY:** Because the decibel scale is logarithmic, the difference between two sound intensity levels (the target variable) corresponds to the ratio of the corresponding intensities. The ratio of the intensities comes from the inverse-square law.

**SET UP:** We label the two points 1 and 2 (see Fig. 16.11). We use Eq. (16.15), the definition of sound intensity level, twice (once at each point). We use Eq. (15.26), the statement of the inverse-square law, to relate the intensities at the two points.

**EXECUTE:** The difference in sound intensity level, \( \beta_2 - \beta_1 \), is given by

\[
\beta_2 - \beta_1 = (10 \text{ dB}) \left[ \log \frac{I_2}{I_0} - \log \frac{I_1}{I_0} \right]
\]

\[
= (10 \text{ dB}) \left[ (\log I_2 - \log I_0) - (\log I_1 - \log I_0) \right]
\]

\[
= (10 \text{ dB}) \log \frac{I_2}{I_1}
\]

Now we use the reciprocal of Eq. (15.26): \( I_2/r_2^2 = r_1^2/I_1 \), so

\[
\beta_2 - \beta_1 = (10 \text{ dB}) \log \frac{r_1^2}{r_2^2} = (10 \text{ dB}) \log \frac{r_1^2}{(2r_1)^2} = (10 \text{ dB}) \log \frac{1}{4} = -6.0 \text{ dB}
\]

A decrease in intensity of a factor of 4 corresponds to a 6-dB decrease in sound intensity level.

**EVALUATE:** Our result is negative, which tells us (correctly) that the sound intensity level is less at point 2 than at point 1. The 6-dB difference doesn’t depend on the value of the sound intensity level at point 1. If point 1 is relatively close to the bird so that \( \beta_1 = 56 \) dB, then at a point twice as far away, \( \beta_2 = 50 \) dB; if point 1 is more distant from the bird so that \( \beta_1 = 28 \) dB, then at a point twice as far away, \( \beta_2 = 22 \) dB.

It’s interesting to note that the perceived loudness of a sound is not directly proportional to its intensity. As an example, most people usually interpret an increase of 8 to 10 dB in sound intensity level (corresponding to an intensity that increases by a factor of 6 to 10) as a doubling of loudness.

**Test Your Understanding**

You double the intensity of a sound wave in air while leaving the frequency unchanged. (The pressure, density, and temperature of the air remain unchanged as well.) What effect does this have on the displacement amplitude, pressure amplitude, bulk modulus, sound speed, and sound intensity level?
Standing Sound Waves and Normal Modes

When longitudinal (sound) waves propagate in a fluid in a pipe with finite length, the waves are reflected from the ends in the same way that transverse waves on a string are reflected at its ends. The superposition of the waves traveling in opposite directions again forms a standing wave. Just as for transverse standing waves on a string (Section 15.7), standing sound waves (normal modes) in a pipe can be used to create sound waves in the surrounding air. This is the operating principle of the human voice as well as many musical instruments, including woodwinds, brasses, and pipe organs.

Transverse waves on a string, including standing waves, are usually described only in terms of the displacement of the string. But, as we have seen, sound waves in a fluid may be described either in terms of the displacement of the fluid or in terms of the pressure variation in the fluid. To avoid confusion, we’ll use the terms displacement node and displacement antinode to refer to points where particles of the fluid have zero displacement and maximum displacement, respectively.

We can demonstrate standing sound waves in a column of gas using an apparatus called Kundt’s tube (Fig. 16.12). A horizontal glass tube a meter or so long is closed at one end and has a flexible diaphragm at the other end that can transmit vibrations. A nearby loudspeaker is driven by an audio oscillator and amplifier; this produces sound waves that force the diaphragm to vibrate sinusoidally with a frequency that we can vary. The sound waves within the tube are reflected at the other, closed end of the tube. We spread a small amount of light powder uniformly along the bottom of the tube. As we vary the frequency of the sound, we pass through frequencies at which the amplitude of the standing waves becomes large enough for the powder to be swept along the tube at those points where the gas is in motion. The powder therefore collects at the displacement nodes (where the gas is not moving). Adjacent nodes are separated by a distance equal to \( \lambda/2 \), and we can measure this distance. Given the wavelength, we can use this experiment to determine the wave speed: We read the frequency \( f \) from the

\[
 Certain \text{ sound frequencies produce a standing wave in tube:} \\
N = \text{displacement nodes} \\
A = \text{displacement antinodes}
\]

16.12 Demonstrating standing sound waves using a Kundt’s tube. The blue shading represents the density of the gas at an instant when the gas pressure at the displacement nodes is a maximum or a minimum.
oscillator dial, and we can then calculate the speed $v$ of the waves from the relation $v = \frac{\lambda}{T}$.

Figure 16.13 will help you to visualize standing sound waves; it is analogous to Fig. 16.4 for longitudinal traveling waves. Again tape two index cards together edge-to-edge, with a gap of a millimeter or two forming a thin slit. Place the card over the diagram with the slit horizontal and move it vertically with constant velocity. The portions of the sine curves that appear in the slit correspond to the oscillations of the particles in a longitudinal standing wave. Each particle moves with longitudinal simple harmonic motion about its equilibrium position.

Figure 16.14 is an enlarged version of a portion of Fig. 16.13 centered on a displacement node. Note that particles on opposite sides of a displacement node oscillate in opposite phase. When these particles approach each other, the gas between them is compressed and the pressure rises; when they recede from each other, there is an expansion and the pressure drops. Hence at a displacement node the gas undergoes the maximum amount of compression and expansion, and the variations in pressure and density above and below the average have their maximum value. By contrast, particles on opposite sides of a displacement antinode oscillate in phase; the distance between the particles is nearly constant, and there is no variation in pressure or density at a displacement antinode.

We use the term pressure node to describe a point in a standing sound wave at which the pressure and density do not vary and the term pressure antinode to describe a point at which the variations in pressure and density are greatest. Using these terms, we can summarize our observations about standing sound waves as follows: a pressure node is always a displacement antinode, and a pressure antinode is always a displacement node. Figure 16.12 depicts a standing sound wave at an instant at which the pressure variations are greatest; the blue shading shows that the density and pressure of the gas have their maximum and minimum values at the displacement nodes (labeled $N$).

In a standing longitudinal wave, a displacement node $N$ is a pressure antinode (a point where the pressure fluctuates the most) and a displacement antinode $A$ is a pressure node (a point where the pressure does not fluctuate at all).
When reflection takes place at a closed end of a pipe (an end with a rigid barrier or plug), the displacement of the particles at this end must always be zero, analogous to a fixed end of a string. Thus a closed end of a pipe is a displacement node and a pressure antinode; the particles do not move, but the pressure variations are maximum. An open end of a pipe is a pressure node because it is open to the atmosphere, where the pressure is constant. Because of this, an open end is always a displacement antinode, in analogy to a free end of a string; the particles oscillate with maximum amplitude, but the pressure does not vary. (Strictly speaking, the pressure node actually occurs somewhat beyond an open end of a pipe. But if the diameter of the pipe is small in comparison to the wavelength, which is true for most musical instruments, this effect can safely be neglected.) Thus longitudinal waves in a column of fluid are reflected at the closed and open ends of a pipe in the same way that transverse waves in a string are reflected at fixed and free ends, respectively.

---

### Conceptual Example 16.12
**The sound of silence**

A directional loudspeaker aims a sound wave of wavelength \( \lambda \) at a wall (Fig. 16.15). At what distances from the wall could you stand and hear no sound at all?

**SOLUTION**

Your ear detects pressure variations in the air; increases or decreases in the pressure outside your eardrum cause it to move slightly in or out, a motion that generates an electrical signal that is sent to the brain. (If you’ve ever had trouble getting your ears to “pop” on a drive up into the mountains or on an airline flight, you’re familiar with just how sensitive your ears are to pressure changes.) Hence you will hear no sound if your ear is at a pressure node, which is a displacement antinode. The wall is a displacement antinode; the distance from a node to an adjacent antinode is \( \lambda /4 \), and the distance from one antinode to the next is \( \lambda /2 \) (Fig. 16.15). Hence the distances \( d \) from the wall at which no sound will be heard are

\[
d = \frac{\lambda}{4} \\
(\text{first displacement antinode and pressure node.})
\]

\[
d = \frac{\lambda}{4} + \frac{\lambda}{2} = \frac{3\lambda}{4} \\
(\text{second displacement antinode and pressure node})
\]

\[
d = \frac{3\lambda}{4} + \frac{\lambda}{2} = \frac{5\lambda}{4} \\
(\text{third displacement antinode and pressure node})
\]

and so on. If the loudspeaker is not highly directional, this effect is hard to notice because of multiple reflections of sound waves from the floor, ceiling, and other points on the walls.

---

### Organ Pipes and Wind Instruments

The most important application of standing sound waves is the production of musical tones by wind instruments. Organ pipes are one of the simplest examples (Fig. 16.16). Air is supplied by a blower, at a gauge pressure typically of the order of \( 10^3 \) Pa (\( 10^{-2} \) atm), to the bottom end of the pipe (Fig. 16.17). A stream of air emerges from the narrow opening at the edge of the horizontal surface and is
directed against the top edge of the opening, which is called the mouth of the pipe. The column of air in the pipe is set into vibration, and there is a series of possible normal modes, just as with the stretched string. The mouth always acts as an open end; thus it is a pressure node and a displacement antinode. The other end of the pipe (at the top in Fig. 16.17) may be either open or closed.

In Fig. 16.18, both ends of the pipe are open, so both ends are pressure nodes and displacement antinodes. An organ pipe that is open at both ends is called an open pipe. The fundamental frequency \( f_1 \) corresponds to a standing-wave pattern with a displacement antinode at each end and a displacement node in the middle (Fig. 16.18a). The distance between adjacent antinodes is always equal to one half-wavelength, and in this case that is equal to the length \( L \) of the pipe; \( \lambda/2 = L \). The corresponding frequency, obtained from the relation \( f = v/\lambda \), is

\[
f_1 = \frac{v}{2L} \quad \text{(open pipe)}
\]

Figures 16.18b and 16.18c show the second and third harmonics (first and second overtones); their vibration patterns have two and three displacement nodes, respectively. For these, a half-wavelength is equal to \( L/2 \) and \( L/3 \), respectively, and the frequencies are twice and three times the fundamental, respectively. That is, \( f_2 = 2f_1 \) and \( f_3 = 3f_1 \). For every normal mode of an open pipe the length \( L \)}
must be an integer number of half-wavelengths, and the possible wavelengths \( \lambda_n \) are given by

\[
L = n \frac{\lambda_n}{2} \quad \text{or} \quad \lambda_n = \frac{2L}{n} \quad (n = 1, 2, 3, \ldots) \quad \text{(open pipe)} \quad (16.17)
\]

The corresponding frequencies \( f_n \) are given by \( f_n = v/\lambda_n \), so all the normal-mode frequencies for a pipe that is open at both ends are given by

\[
f_n = \frac{nv}{2L} \quad (n = 1, 2, 3, \ldots) \quad \text{(open pipe)} \quad (16.18)
\]

The value \( n = 1 \) corresponds to the fundamental frequency, \( n = 2 \) to the second harmonic (or first overtone), and so on. Alternatively, we can say

\[
f_n = nf_1 \quad (n = 1, 2, 3, \ldots) \quad \text{(open pipe)} \quad (16.19)
\]

with \( f_1 \) given by Eq. (16.16).

Figure 16.19 shows a pipe that is open at the left end but closed at the right end. This is called a stopped pipe. The left (open) end is a displacement antinode (pressure node), but the right (closed) end is a displacement node (pressure antinode). The distance between a node and the adjacent antinode is always one quarter-wavelength. Figure 16.19a shows the lowest-frequency mode; the length of the pipe is a quarter-wavelength \((L = \lambda_1/4)\). The fundamental frequency is \( f_1 = v/\lambda_1 \), or

\[
f_1 = \frac{v}{4L} \quad \text{(stopped pipe)} \quad (16.20)
\]

This is one-half the fundamental frequency for an open pipe of the same length. In musical language, the pitch of a closed pipe is one octave lower (a factor of 2 in frequency) than that of an open pipe of the same length. Figure 16.19b shows the next mode, for which the length of the pipe is three-quarters of a wavelength, corresponding to a frequency \( 3f_1 \). For Fig. 16.19c, \( L = 5\lambda/4 \) and the frequency is \( 5f_1 \).

The possible wavelengths are given by

\[
L = n \frac{\lambda_n}{4} \quad \text{or} \quad \lambda_n = \frac{4L}{n} \quad (n = 1, 3, 5, \ldots) \quad \text{(stopped pipe)} \quad (16.21)
\]

The normal-mode frequencies are given by \( f_n = v/\lambda_n \), or

\[
f_n = \frac{nv}{4L} \quad (n = 1, 3, 5, \ldots) \quad \text{(stopped pipe)} \quad (16.22)
\]

or

\[
f_n = nf_1 \quad (n = 1, 3, 5, \ldots) \quad \text{(stopped pipe)} \quad (16.23)
\]

with \( f_1 \) given by Eq. (16.20). We see that the second, fourth, and all even harmonics are missing. In a pipe that is closed at one end, the fundamental frequency is \( f_1 = v/4L \), and only the odd harmonics in the series \((3f_1, 5f_1, \ldots)\) are possible.

A final possibility is a pipe that is closed at both ends, with displacement nodes and pressure antinodes at both ends. This wouldn’t be of much use as a musical instrument because there would be no way for the vibrations to get out of the pipe.
Example 16.13  

A tale of two pipes

On a day when the speed of sound is 345 m/s, the fundamental frequency of a stopped organ pipe is 220 Hz. a) How long is this stopped pipe? b) The second overtone of this pipe has the same wavelength as the third harmonic of an open pipe. How long is the open pipe?

**Solution**

**Identify and Set Up:** Since this is a stopped pipe (open at one end and closed at the other), the normal-mode frequencies are given by Eq. (16.22). We use this to determine the length \( L \) from the frequency in part (a). In part (b), we must make a comparison to an open pipe, for which the frequencies are given by Eq. (16.18).

**Execute:**

a) For a stopped pipe, \( f = \frac{v}{4L} \), so the length of the stopped pipe is

\[
L_{\text{stopped}} = \frac{v}{4f_1} = \frac{345 \text{ m/s}}{4(220 \text{ s}^{-1})} = 0.392 \text{ m}
\]

b) The frequency of the first overtone of a stopped pipe is \( f_3 = 3f_1 \), and the frequency of the second overtone is \( f_5 = 5f_1 \):

\[
f_5 = 5f_1 = 5(220 \text{ Hz}) = 1100 \text{ Hz}
\]

If the wavelengths are the same, the frequencies are the same, so the frequency of the third harmonic of the open pipe is also 1100 Hz. The third harmonic of an open pipe is at \( 3f_1 = \frac{3v}{2L_{\text{open}}} \). If this equals 1100 Hz, then

\[
1100 \text{ Hz} = 3\left(\frac{345 \text{ m/s}}{2L_{\text{open}}}\right) \quad \text{and} \quad L_{\text{open}} = 0.470 \text{ m}
\]

**Evaluate:** The stopped pipe is 0.392 m long and has a fundamental frequency of 220 Hz; the open pipe is longer, 0.470 m, but has a higher fundamental frequency of \( (1100 \text{ Hz})/3 = 367 \text{ Hz} \). If this seems like a contradiction, you should again compare Figs. 16.18a and 16.19a.

In an organ pipe in actual use, several modes are always present at once; the motion of the air is a superposition of these modes. This situation is analogous to a string that is struck or plucked, as in Fig. 15.25. Just as for a vibrating string, a complex standing wave in the pipe produces a traveling sound wave in the surrounding air with a harmonic content similar to that of the standing wave. A very narrow pipe produces a sound wave rich in higher harmonics, which we hear as a thin and “stringy” tone; a fatter pipe produces mostly the fundamental mode, heard as a softer, more flute-like tone. The harmonic content also depends on the shape of the pipe’s mouth.

We have talked about organ pipes, but this discussion is also applicable to other wind instruments. The flute and the recorder are directly analogous. The most significant difference is that those instruments have holes along the pipe. Opening and closing the holes with the fingers changes the effective length \( L \) of the air column and thus changes the pitch. Any individual organ pipe, by comparison, can play only a single note. The flute and recorder behave as open pipes, while the clarinet acts as a stopped pipe (closed at the reed end, open at the bell).

Equations (16.18) and (16.22) show that the frequencies of any wind instrument are proportional to the speed of sound \( v \) in the air column inside the instrument. As Eq. (16.10) shows, \( v \) depends on temperature; it increases when temperature increases. Thus the pitch of all wind instruments rises with increasing temperature. An organ that has some of its pipes at one temperature and others at a different temperature is bound to sound out of tune.

**Test Your Understanding**

If you connect a hose to one end of a metal pipe and blow compressed air into it, the pipe produces a musical tone. If instead you blow compressed helium into the pipe at the same pressure and temperature, the pipe produces a higher tone. Why?
16.5 | Resonance

Many mechanical systems have normal modes of oscillation. As we have seen, these include columns of air (as in an organ pipe) and stretched strings (as in a guitar; see Section 15.8). In each mode, every particle of the system oscillates with simple harmonic motion at the same frequency as the mode. Air columns and stretched strings have an infinite series of normal modes, but the basic concept is closely related to the simple harmonic oscillator, discussed in Chapter 13, which has only a single normal mode (that is, only one frequency at which it oscillates after being disturbed).

Suppose we apply a periodically varying force to a system that can oscillate. The system is then forced to oscillate with a frequency equal to the frequency of the applied force (called the driving frequency). This motion is called a forced oscillation. We talked about forced oscillations of the harmonic oscillator in Section 13.8, and we suggest that you review that discussion. In particular, we described the phenomenon of mechanical resonance. A simple example of resonance is pushing Cousin Throckmorton on a swing. The swing is a pendulum; it has only a single normal mode, with a frequency determined by its length. If we push the swing periodically with this frequency, we can build up the amplitude of the motion. But if we push with a very different frequency, the swing hardly moves at all.

Resonance also occurs when a periodically varying force is applied to a system with many normal modes. An example is shown in Fig. 16.20a. An open organ pipe is placed next to a loudspeaker that is driven by an amplifier and emits pure sinusoidal sound waves of frequency \( f \), which can be varied by adjusting the amplifier. The air in the pipe is forced to vibrate with the same frequency \( f \) as the driving force provided by the loudspeaker. In general the amplitude of this motion is relatively small, and the air inside the pipe will not move in any of the normal-mode patterns shown in Fig. 16.18. But if the frequency \( f \) of the force is close to one of the normal-mode frequencies, the air in the pipe moves in the normal-mode

---

![Diagram](image)

16.20 (a) The air in an open pipe is forced to oscillate at the same frequency as the sinusoidal sound waves coming from the loudspeaker. (b) The resonance curve of the open pipe graphs the amplitude of the standing sound wave in the pipe as a function of the driving frequency.
pattern for that frequency, and the amplitude can become quite large. Figure 16.20b shows the amplitude of oscillation of the air in the pipe as a function of the driving frequency \( f \). The shape of this graph is called the resonance curve of the pipe; it has peaks where \( f \) equals the normal-mode frequencies of the pipe. The detailed shape of the resonance curve depends on the geometry of the pipe.

If the frequency of the force is precisely equal to a normal-mode frequency, the system is in resonance, and the amplitude of the forced oscillation is maximum. If there were no friction or other energy-dissipating mechanism, a driving force at a normal-mode frequency would continue to add energy to the system, and the amplitude would increase indefinitely. In such an idealized case the peaks in the resonance curve of Fig. 16.20b would be infinitely high. But in any real system there is always some dissipation of energy, or damping, as we discussed in Section 13.8; the amplitude of oscillation in resonance may be large, but it cannot be infinite.

The “sound of the ocean” you hear when you put your ear next to a large seashell is due to resonance. The noise of the outside air moving past the seashell is a mixture of sound waves of almost all audible frequencies, which forces the air inside the seashell to oscillate. The seashell behaves like an organ pipe, with a set of normal-mode frequencies; hence the inside air oscillates most strongly at those frequencies, producing the seashell’s characteristic sound. To hear a similar phenomenon, uncap a full bottle of your favorite beverage and blow across the open top. The noise is provided by your breath blowing across the top, and the “organ pipe” is the column of air inside the bottle above the surface of the liquid. If you take a drink and repeat the experiment, you will hear a lower tone because the “pipe” is longer and the normal-mode frequencies are lower.

Resonance also occurs when a stretched string is forced to oscillate (Section 15.8). Suppose that one end of a stretched string is held fixed while the other is given a transverse sinusoidal motion with small amplitude, setting up standing waves. If the frequency of the driving mechanism is not equal to one of the normal-mode frequencies of the string, the amplitude at the antinodes is fairly small. However, if the frequency is equal to any one of the normal-mode frequencies, the string is in resonance, and the amplitude at the antinodes is very much larger than that at the driven end. The driven end is not precisely a node, but it lies much closer to a node than to an antinode when the string is in resonance. The photographs in Fig. 15.20 were made this way, with the left end of the string fixed and the right end oscillating vertically with small amplitude; large-amplitude standing waves resulted when the frequency of oscillation of the right end was equal to the fundamental frequency or to one of the first three overtones.

It is easy to demonstrate resonance with a piano. Push down the damper pedal (the right-hand pedal) so that the dampers are lifted and the strings are free to vibrate, and then sing a steady tone into the piano. When you stop singing, the piano seems to continue to sing the same note. The sound waves from your voice excite vibrations in the strings that have natural frequencies close to the frequencies (fundamental and harmonics) present in the note you sang.

A more spectacular example is a singer breaking a wine glass with her amplified voice. A good-quality wine glass has normal-mode frequencies that you can hear by tapping it. If the singer emits a loud note with a frequency corresponding exactly to one of these normal-mode frequencies, large-amplitude oscillations can build up and break the glass (Fig. 16.21).

Resonance is a very important concept, not only in mechanical systems but in all areas of physics. In Chapter 31 we will see examples of resonance in electric circuits.
Example 16.14  

An organ-guitar duet

A stopped organ pipe is sounded near a guitar, causing one of the strings to vibrate with large amplitude. We vary the tension of the string until we find the maximum amplitude. The string is 80% as long as the stopped pipe. If both the pipe and the string vibrate at their fundamental frequency, calculate the ratio of the wave speed on the string to the speed of sound in air.

**SOLUTION**

**IDENTIFY:** The large response of the string is an example of resonance. It occurs because the organ pipe and the guitar string have the same fundamental frequency.

**SET UP:** Letting the subscripts a and s stand for the air in the pipe and the string, respectively, the condition for resonance is $f_{ka} = f_{ks}$. Equation (16.20) gives the fundamental frequency for a stopped pipe, while the fundamental frequency for a guitar string held at both ends is given by Eq. (15.32). These expressions involve the wave speed in air ($v_a$) and on the string ($v_s$) and the lengths of the pipe and string; we are given that $L_a = 0.80L_s$, and our target variable is the ratio $v_s/v_a$.

**EXECUTE:** From Eqs. (16.20) and (15.32), $f_{ka} = v_a/4L_a$ and $f_{ks} = v_s/2L_s$. Setting these equal to each other, we find

$$\frac{v_s}{4L_a} = \frac{v_s}{2L_s}$$

Substituting $L_a = 0.80L_s$ and rearranging, we get

$$\frac{v_s}{v_a} = 0.40$$

**EVALUATE:** As an example, if the speed of sound in air is 345 m/s, the wave speed on the string is $(0.40)(345 \text{ m/s}) = 138 \text{ m/s}$. Note that while the standing waves in the pipe and on the string have the same frequency, they have different wavelengths $\lambda = v/f$ because the two media have different wave speeds $v$. Which standing wave has the greater wavelength?

Test Your Understanding

Suppose the open pipe in Fig. 16.21 is 60.0 cm long. If the speed of sound in air is 345 m/s, what frequency emitted by the speaker produces the greatest response in the pipe?

16.6  |  Interference of Waves

Wave phenomena that occur when two or more waves overlap in the same region of space are grouped under the heading interference. As we have seen, standing waves are a simple example of an interference effect: two waves traveling in opposite directions in a medium combine to produce a standing wave pattern with nodes and antinodes that do not move.

Figure 16.22 shows an example of another type of interference that involves waves that spread out in space. Two speakers, driven in phase by the same amplifier, emit identical sinusoidal sound waves with the same constant frequency. We place a microphone at point $P$ in the figure, equidistant from the speakers. Wave crests emitted from the two speakers at the same time travel equal distances and arrive at point $P$ at the same time; hence the waves arrive in phase, and there is constructive interference. The total wave amplitude at $P$ is twice the amplitude from each individual wave, and we can measure this combined amplitude with the microphone.

Now let’s move the microphone to point $Q$, where the distances from the two speakers to the microphone differ by a half-wavelength. Then the two waves arrive a half-cycle out of step, or out of phase; a positive crest from one speaker arrives at the same time as a negative crest from the other. Destructive interference takes place, and the amplitude measured by the microphone is much smaller.
16.6 | Interference of Waves

than when only one speaker is present. If the amplitudes from the two speakers are
equal, the two waves cancel each other out completely at point Q, and the total
amplitude there is zero.

**CAUTION** Although this situation bears some resemblance to standing
waves in a pipe, the total wave in Fig. 16.22 is a traveling wave, not a standing
wave. To see why, recall that in a standing wave there is no net flow of energy
in any direction. By contrast, in Fig. 16.22 there is an overall flow of energy from
the speakers into the surrounding air; this is characteristic of a traveling wave.
The interference between the waves from the two speakers simply causes the
energy flow to be channeled into certain directions (for example, toward P)
and away from other directions (for example, away from Q). You can see
another difference between Fig. 16.22 and a standing wave by considering a
point, such as Q, where destructive interference occurs. Such a point is both a
displacement node and a pressure node because there is no wave at all at this
point. Compare this to a standing wave, in which a pressure node is a displace-
ment antinode and vice versa.

Constructive interference occurs wherever the distances traveled by the two
waves differ by a whole number of wavelengths, 0, λ, 2λ, 3λ, . . . ; in all these
cases the waves arrive at the microphone in phase (Fig. 16.23a). If the distances
from the two speakers to the microphone differ by any half-integer number of
wavelengths, \( \lambda/2, 3\lambda/2, 5\lambda/2, \ldots \), the waves arrive at the microphone out of
phase and there will be destructive interference (Fig. 16.23b). In this case, little or
no sound energy flows toward the microphone directly in front of the speakers.
The energy is instead directed to the sides, where constructive interference occurs.

---

**16.23** Two speakers driven by the same amplifier, emitting waves in phase. Only the
waves directed toward the microphone are shown, and they are separated for clarity.
(a) Constructive interference occurs when the path difference is 0, λ, 2λ, 3λ, . . .
(b) Destructive interference occurs when the path difference is \( \lambda/2, 3\lambda/2, 5\lambda/2, \ldots \).
Loudspeaker interference

Two small loudspeakers, A and B (Fig. 16.24), are driven by the same amplifier and emit pure sinusoidal waves in phase. If the speed of sound is 350 m/s, a) for what frequencies does constructive interference occur at point P? b) for what frequencies does destructive interference occur at point P?

SOLUTION

IDENTIFY: The nature of the interference at P depends on the difference in path lengths from points A and B to P and how this difference compares to the wavelength.

SET UP: We calculate the path lengths from A to P and from B to P using the Pythagorean theorem. Constructive interference occurs when the difference in path lengths equals a whole number of wavelengths, while destructive interference occurs when the path length difference is a half-integer number of wavelengths. To find the corresponding frequencies, we use the relationship \( v = f \lambda \).

EXECUTE: The distance from speaker A to point P is \( \sqrt{(2.00 \text{ m})^2 + (4.00 \text{ m})^2} = 4.47 \text{ m} \), and the distance from speaker B to point P is \( \sqrt{(1.00 \text{ m})^2 + (4.00 \text{ m})^2} = 4.12 \text{ m} \). The path difference is \( d = 4.47 \text{ m} - 4.12 \text{ m} = 0.35 \text{ m} \).

a) Constructive interference occurs when the path difference is \( d = 0, \lambda, 2\lambda, \ldots \) or \( d = n\lambda \). The possible frequencies are

\[
\frac{nv}{d} = \frac{350 \text{ m/s}}{0.35 \text{ m}} = \frac{350 \text{ m/s}}{0.35 \text{ m}} \quad (n = 1, 2, 3, \ldots)
\]

\[= 1000 \text{ Hz}, 2000 \text{ Hz}, 3000 \text{ Hz}, \ldots\]

b) Destructive interference occurs when the path difference is \( d = \lambda/2, 3\lambda/2, 5\lambda/2, \ldots \text{ or } d = n\lambda/2 \). The possible frequencies are

\[
\frac{nv}{2d} = \frac{350 \text{ m/s}}{2(0.35 \text{ m})} = \frac{350 \text{ m/s}}{2(0.35 \text{ m})} \quad (n = 1, 2, 3, \ldots)
\]

\[= 500 \text{ Hz}, 1500 \text{ Hz}, 2500 \text{ Hz}, \ldots\]

EVALUATE: As we increase the frequency, the sound at point P alternates between large and small amplitudes; the maxima and minima occur at the frequencies we have found. It can be hard to notice this effect in an ordinary room because of multiple reflections from the walls, floor, and ceiling. Such an experiment is best done either outdoors or in an anechoic chamber, which has walls that absorb almost all sound and thereby eliminate reflections.

Experiments closely analogous to the one in Example 16.15, but using light, have provided both strong evidence for the wave nature of light and a means of measuring its wavelengths. We will discuss these experiments in detail in Chapter 35.

Interference effects are used to control noise from very loud sound sources such as gas-turbine power plants or jet engine test cells. The idea is to use additional sound sources that in some regions of space interfere destructively with the unwanted sound and cancel it out. Microphones in the controlled area feed signals back to the sound sources, which are continuously adjusted for optimum cancellation of noise in the controlled area.

Test Your Understanding

Suppose the frequency from the two loudspeakers in Fig. 16.22 is 250 Hz, so that there is neither constructive nor destructive interference at P. What will the microphone detect if it is moved toward the amplifier along the line in Fig. 16.24 labeled 4.00 m?
In Section 16.6 we talked about interference effects that occur when two different waves with the same frequency overlap in the same region of space. Now let's look at what happens when we have two waves with equal amplitude but slightly different frequencies. This occurs, for example, when two tuning forks with slightly different frequencies are sounded together, or when two organ pipes that are supposed to have exactly the same frequency are slightly "out of tune."

Consider a particular point in space where the two waves overlap. The displacements of the individual waves at this point are plotted as functions of time in Fig. 16.25a. The total length of the time axis represents one second, and the frequencies are 16 Hz (blue graph) and 18 Hz (red graph). Applying the principle of superposition, we add the two displacements at each instant of time to find the total displacement at that time. The result is the graph of Fig. 16.25b. At certain times the two waves are in phase; their maxima coincide and their amplitudes add. But because of their slightly different frequencies, the two waves cannot be in phase at all times. Indeed, at certain times (like \( t = 0.50 \) s in Fig. 16.25) the two waves are exactly out of phase. The two waves then cancel each other, and the total amplitude is zero.

The resultant wave in Fig. 16.25b looks like a single sinusoidal wave with a varying amplitude that goes from a maximum to zero and back. In this example the amplitude goes through two maxima and two minima in one second, so the frequency of this amplitude variation is 2 Hz. The amplitude variation causes variations of loudness called beats, and the frequency with which the loudness varies is called the beat frequency. In this example the beat frequency is the difference of the two frequencies. If the beat frequency is a few hertz, we hear it as a waver or pulsation in the tone.

We can prove that the beat frequency is always the difference of the two frequencies \( f_a \) and \( f_b \). Suppose \( f_a \) is larger than \( f_b \); the corresponding periods are \( T_a \) and \( T_b \), with \( T_a < T_b \). If the two waves start out in phase at time \( t = 0 \), they are again in phase when the first wave has gone through exactly one more cycle than the second. This happens at a value of \( t \) equal to \( T_{\text{beat}} \), the period of the beat. Let \( n \) be the number of cycles of the first wave in time \( T_{\text{beat}} \); then the number of cycles of the second wave in the same time is \((n - 1)\), and we have the relations

\[
T_{\text{beat}} = nT_a \quad \text{and} \quad T_{\text{beat}} = (n - 1)T_b
\]

Eliminating \( n \) between these two equations, we find

\[
T_{\text{beat}} = \frac{T_a T_b}{T_b - T_a}
\]
The reciprocal of the beat period is the beat frequency, \( f_{\text{beat}} = 1/T_{\text{beat}} \), so
\[
f_{\text{beat}} = \frac{T_b - T_a}{T_a T_b} = \frac{1}{T_a} - \frac{1}{T_b}
\]
and finally
\[
f_{\text{beat}} = f_a - f_b \quad \text{(beat frequency)} \quad \text{(16.24)}
\]

As claimed, the beat frequency is the difference of the two frequencies. In using
Eq. (16.24), remember that \( f_a \) is the higher frequency.

An alternative way to derive Eq. (16.24) is to write functions to describe the
curves in Fig. 16.25a and then add them. Suppose that at a certain position the two
waves are given by
\[
y_a(t) = A \sin 2\pi f_a t \quad \text{and} \quad y_b(t) = -A \sin 2\pi f_b t.
\]
We use the trigonometric identity
\[
\sin a - \sin b = 2 \sin \frac{1}{2}(a - b) \cos \frac{1}{2}(a + b)
\]
We can then express the total wave \( y(t) = y_a(t) + y_b(t) \) as
\[
y_a(t) + y_b(t) = \left[ 2A \sin \frac{1}{2}(2\pi)(f_a - f_b) t \right] \cos \frac{1}{2}(2\pi)(f_a + f_b) t
\]
The amplitude factor (the quantity in brackets) varies slowly with frequency
\( \frac{1}{2}(f_a - f_b) \). The cosine factor varies with a frequency equal to the average
frequency \( \frac{1}{2}(f_a + f_b) \). The square of the amplitude factor, which is proportional to
the intensity that the ear hears, goes through two maxima and two minima per
cycle. So the beat frequency \( f_{\text{beat}} \) that is heard is twice the quantity \( \frac{1}{2}(f_a - f_b) \), or
just \( f_a - f_b \) in agreement with Eq. (16.24).

Beats between two tones can be heard up to a beat frequency of about 6 or 7
Hz. Two piano strings or two organ pipes differing in frequency by 2 or 3 Hz
sound wavery and "out of tune," although some organ stops contain two sets of
pipes deliberately tuned to beat frequencies of about 1 to 2 Hz for a gently undu-
lating effect. Listening for beats is an important technique in tuning
all musical
instruments.

At frequency differences greater than about 6 or 7 Hz, we no longer hear individual
beats, and the sensation merges into one of consonance or dissonance,
depending on the frequency ratio of the two tones. In some cases the ear perceives
a tone called a difference tone, with a pitch equal to the beat frequency of the two
tones. For example, if you listen to a whistle that produces sounds at 1800 Hz and
1900 Hz when blown, you will hear not only these tones but also a much lower
100-Hz tone.

The engines on multiengine propeller aircraft have to be synchronized so that
the sounds don't cause annoying beats, which are heard as loud throbbing sounds
(Fig. 16.26). On some planes this is done electronically; on others the pilot does it
by ear, just like tuning a piano.

**Test Your Understanding**

One tuning fork vibrates at 440 Hz, while a second tuning fork vibrates at an
unknown frequency. When both tuning forks are sounded simultaneously, you
hear a tone that rises and falls in intensity three times per second. What do you
conclude about the frequency of the second tuning fork?
The Doppler Effect

You’ve probably noticed that when a car approaches you with its horn sounding, the pitch seems to drop as the car passes. This phenomenon, first described by the 19th-century Austrian scientist Christian Doppler, is called the Doppler effect. When a source of sound and a listener are in motion relative to each other, the frequency of the sound heard by the listener is not the same as the source frequency. A similar effect occurs for light and radio waves; we’ll return to this later in this section.

To analyze the Doppler effect for sound, we’ll work out a relation between the frequency shift and the velocities of source and listener relative to the medium (usually air) through which the sound waves propagate. To keep things simple, we consider only the special case in which the velocities of both source and listener lie along the line joining them. Let \( v_s \) and \( v_l \) be the velocity components along this line for the source and the listener, respectively, relative to the medium. We choose the positive direction for both \( v_s \) and \( v_l \) to be the direction from the listener \( L \) to the source \( S \). The speed of sound relative to the medium, \( v \), is always considered positive.

Moving Listener

Let’s think first about a listener \( L \) moving with velocity \( v_l \) toward a stationary source \( S \) (Fig. 16.27). The source emits a sound wave with frequency \( f_s \) and wavelength \( \lambda = \frac{v}{f_s} \). The figure shows several wave crests, separated by equal distances \( \lambda \). The wave crests approaching the moving listener have a speed of propagation relative to the listener of \( v + v_l \). So the frequency \( f_L \) with which the crests arrive at the listener’s position (that is, the frequency the listener hears) is

\[
\begin{align*}
\lambda &= \frac{v}{f_s} \\

f_L &= \left(\frac{v + v_l}{v}\right)f_s = \left(1 + \frac{v_l}{v}\right)f_s
\end{align*}
\]

or

\[
\begin{align*}
\lambda &= \frac{v}{f_s} \\

f_L &= \left(\frac{v + v_l}{v}\right)f_s = \left(1 + \frac{v_l}{v}\right)f_s
\end{align*}
\]

(moving listener, stationary source)

So a listener moving toward a source \( (v_l > 0) \), as in Fig. 16.27, hears a higher frequency (higher pitch) than does a stationary listener. A listener moving away from the source \( (v_l < 0) \) hears a lower frequency (lower pitch).
Wave crests emitted by a moving source are crowded together in front of the source (to the right of this source) and stretched out behind it (to the left of this source).

The Doppler effect explains why the siren on a fire engine or ambulance has a high pitch \( f_L > f_S \) when it is approaching you \( (v_S < 0) \) and a low pitch \( f_L < f_S \) when it is moving away \( (v_S > 0) \).

**Moving Source and Moving Listener**

Now suppose the source is also moving, with velocity \( v_S \) (Fig. 16.28). The wave speed relative to the wave medium (air) is still \( v \); it is determined by the properties of the medium and is not changed by the motion of the source. But the wavelength is no longer equal to \( v/f_S \). Here’s why. The time for emission of one cycle of the wave is the period \( T = 1/f_S \). During this time, the wave travels a distance \( vT = v/f_S \) and the source moves a distance \( v_S T = v_S/f_S \). The wavelength is the distance between successive wave crests, and this is determined by the relative displacement of source and wave. As Fig. 16.28 shows, this is different in front of and behind the source. In the region to the right of the source in Fig. 16.28 (that is, in front of the source), the wavelength is

\[
\lambda = \frac{v}{f_S} - \frac{v_S}{f_S} = \frac{v - v_S}{f_S}
\]

(wavelength in front of a moving source) \( \text{(16.27)} \)

In the region to the left of the source (that is, behind the source), it is

\[
\lambda = \frac{v + v_S}{f_S}
\]

(wavelength behind a moving source) \( \text{(16.28)} \)

The waves in front of and behind the source are compressed and stretched out, respectively, by the motion of the source (Fig. 16.29).

To find the frequency heard by the listener behind the source, we substitute Eq. (16.28) into the first form of Eq. (16.25):

\[
f_L = \frac{v + v_L}{\lambda} = \frac{v + v_L}{(v + v_S)/f_S}
\]

\[
f_L = \frac{v + v_L}{v + v_S}f_S
\]

(Doppler effect, moving source and moving listener)
This expresses the frequency \( f_L \) heard by the listener in terms of the frequency \( f_S \) of the source.

Equation (16.29) includes all possibilities for motion of source and listener (relative to the medium) along the line joining them. If the listener happens to be at rest in the medium, \( v_L \) is zero. When both source and listener are at rest or have the same velocity relative to the medium, then \( v_L = v_S \) and \( f_L = f_S \). Whenever the direction of the source or listener velocity is opposite to the direction from the listener toward the source (which we have defined as positive), the corresponding velocity to be used in Eq. (16.29) is negative.

**Problem-Solving Strategy**

**IDENTIFY** the relevant concepts: The Doppler effect is relevant whenever the source of waves, the wave detector (listener), or both are in motion.

**SET UP** the problem using the following steps:
1. Establish a coordinate system. Define the positive direction to be the direction from the listener to the source, and make sure you know the signs of all relevant velocities. A velocity in the direction from the listener toward the source is positive; a velocity in the opposite direction is negative. Also, the velocities must all be measured relative to the air in which the sound is traveling.
2. Use consistent notation to identify the various quantities: subscript \( S \) for source, \( L \) for listener.
3. Determine which unknown quantities are your target variables.

**EXECUTE** the solution:
1. Use Eq. (16.29) to relate the frequencies at the source and the listener, the sound speed, and the velocities of the source and the listener. If the source is moving, you can find the wavelength measured by the listener using Eq. (16.27) or (16.28).
2. When a wave is reflected from a surface, either stationary or moving, the analysis can be carried out in two steps. In the first, the surface plays the role of listener; the frequency with which the wave crests arrive at the surface is \( f_L \). Then think of the surface as a new source, emitting waves with this same frequency \( f_L \). Finally, determine what frequency is heard by a listener detecting this new wave.

**EVALUATE** your answer: Ask whether your final result makes sense. If the source and the listener are moving toward each other, \( f_L > f_S \); if they are moving apart, \( f_L < f_S \). If the source and the listener have no relative motion, \( f_L = f_S \).

---

**Example 16.16**

**Doppler effect I: Wavelengths**

A police siren emits a sinusoidal wave with frequency \( f_S = 300 \text{ Hz} \). The speed of sound is 340 m/s. a) Find the wavelength of the waves if the siren is at rest in the air. b) If the siren is moving at 30 m/s (108 km/h, or 67 mi/h), find the wavelengths of the waves ahead of and behind the source.

**SOLUTION**

**IDENTIFY:** The Doppler effect is not involved in part (a), since neither the source nor the listener is moving. In part (b), the source is in motion and we must invoke the Doppler effect.

**SET UP:** We use the relationship \( v = \lambda f \) to determine the wavelength when the police siren is at rest. When it is in motion, we find the wavelength on either side of the siren using Eqs. (16.27) and (16.28).

**16.30** Wavelengths ahead of and behind the police siren when the siren is moving through the air at 30 m/s.
EXECUTE: a) When the source is at rest,

\[ \lambda = \frac{v}{f_s} = \frac{340 \, \text{m/s}}{300 \, \text{Hz}} = 1.13 \, \text{m} \]

b) The situation is shown in Fig. 16.30. From Eq. (16.27), in front of the siren,

\[ \lambda = \frac{v - v_s}{f_s} = \frac{340 \, \text{m/s} - 30 \, \text{m/s}}{300 \, \text{Hz}} = 1.03 \, \text{m} \]

From Eq. (16.28), behind the siren,

\[ \lambda = \frac{v + v_s}{f_s} = \frac{340 \, \text{m/s} + 30 \, \text{m/s}}{300 \, \text{Hz}} = 1.23 \, \text{m} \]

EVALUATE: The wavelength is less in front of the siren and greater behind the siren, as it should be.

---

**Example 16.17**

**Doppler effect II: Frequencies**

If a listener \( L \) is at rest and the siren in Example 16.16 is moving away from \( L \) at 30 m/s (Fig. 16.31), what frequency does the listener hear?

**SOLUTION**

**IDENTIFY and SET UP:** Our target variable is the listener's frequency \( f_L \). We know \( f_s = 300 \, \text{Hz} \) from Example 16.16, and we have \( v_L = 0 \) and \( v_s = 30 \, \text{m/s} \). (The source velocity \( v_s \) is positive because the siren is moving in the same direction as the direction from listener to source.)

**EXECUTE:** From Eq. (16.29),

\[ f_L = \frac{v}{v + v_s} f_s = \frac{340 \, \text{m/s}}{340 \, \text{m/s} + 30 \, \text{m/s}} (300 \, \text{Hz}) = 276 \, \text{Hz} \]

**EVALUATE:** The source and listener are moving apart, so the frequency \( f_L \) heard by the listener is less than the frequency \( f_s \) emitted by the source.

Here's an alternative approach we can use to check our result. From Example 16.16, the wavelength behind the source (which is where the listener in Fig. 16.31 is located) is 1.23 m, so

\[ f_L = \frac{v}{\lambda} = \frac{340 \, \text{m/s}}{1.23 \, \text{m}} = 276 \, \text{Hz} \]

Even though the source is moving, the wave speed \( v \) relative to the stationary listener is unchanged.

---

**Example 16.18**

**Doppler effect III: A moving listener**

If the siren is at rest and the listener is moving toward the left at 30 m/s (Fig. 16.32), what frequency does the listener hear?

**SOLUTION**

**IDENTIFY and SET UP:** The key difference between this example and Example 16.17 is that the source is at rest (so \( v_s = 0 \)) and the listener is in motion. The positive direction (from listener to source) is still from left to right, so \( v_L = -30 \, \text{m/s} \).

**EXECUTE:** From Eq. (16.27),

\[ f_L = \frac{v + v_L}{v} f_s = \frac{340 \, \text{m/s} + (-30 \, \text{m/s})}{340 \, \text{m/s}} (300 \, \text{Hz}) = 274 \, \text{Hz} \]

**EVALUATE:** Again the frequency heard by the listener is less than the source frequency. Note that the relative velocity of source and listener is the same as in the previous example, but the Doppler shift is different because the velocities relative to the air are different.
Example 16.19  
**Doppler effect IV: Moving source, moving listener**

If the siren is moving away from the listener with a speed of 45 m/s relative to the air and the listener is moving toward the siren with a speed of 15 m/s relative to the air (Fig. 16.33), what frequency does the listener hear?

**SOLUTION**

**IDENTIFY and SET UP:** Now both the listener and the source are in motion, with \( v_L = 15 \text{ m/s} \) and \( v_s = 45 \text{ m/s} \). (Both velocities are positive because both velocity vectors point in the direction from listener to source.)

**EXECUTE:** Once again using Eq. (16.27), we find

\[
\begin{align*}
\frac{v + v_L}{v + v_s} f_s &= \frac{340 \text{ m/s} + 15 \text{ m/s}}{340 \text{ m/s} + 45 \text{ m/s}} (300 \text{ Hz}) \\
&= 277 \text{ Hz}
\end{align*}
\]

**EVALUATE:** The frequency heard by the listener is again less than the source frequency, but the value is different than in the previous two examples, even though the source and listener move away from each other at 30 m/s in all three cases. The sign of the Doppler shift of frequency (that is, whether \( f_L \) is less than or greater than \( f_s \)) depends on how the source and the listener are moving relative to each other; to determine the value of the Doppler shift of frequency, you must know the velocities of source and listener relative to the air.

Example 16.20  
**Doppler effect V: A double Doppler shift**

The police car with its 300-Hz siren is moving toward a warehouse at 30 m/s, intending to crash through the door. What frequency does the driver of the police car hear reflected from the warehouse?

**SOLUTION**

**IDENTIFY:** In this situation there are two Doppler shifts, as shown in Fig. 16.34. In the first shift, the warehouse is the stationary “listener.” The frequency of sound reaching the warehouse, which we call \( f_w \), is greater than 300 Hz because the source is approaching. In the second shift, the warehouse acts as a source of sound with frequency \( f_w \), and the listener is the driver of the police car; she hears a frequency greater than \( f_w \) because she is approaching the source.

**SET UP:** To determine \( f_w \), we use Eq. (16.27) with \( f_L \) replaced by \( f_w \). For this part of the problem, \( v_L = v_w = 0 \) (the warehouse is at rest) and \( v_s = -30 \text{ m/s} \) (the siren is moving in the negative direction from source to listener).

To determine the frequency heard by the driver, which is our target variable, we again use Eq. (16.27) but now with \( f_s \) replaced by \( f_w \). For this second part of the problem, \( v_s = 0 \) because the stationary warehouse is the source and the velocity of the listener (the driver) is \( v_L = +30 \text{ m/s} \). (The listener’s velocity is positive because it is in the direction from listener to source.)
EXECUTE: The frequency reaching the warehouse is
\[
\frac{f_w}{v + v_s} \cdot f_s = \frac{340 \text{ m/s}}{340 \text{ m/s} + (-30 \text{ m/s})} (300 \text{ Hz}) = 329 \text{ Hz}
\]

Then the frequency heard by the driver is
\[
\frac{v + v_s}{v} \cdot f_w = \frac{340 \text{ m/s} + 30 \text{ m/s}}{340 \text{ m/s}} (329 \text{ Hz}) = 358 \text{ Hz}
\]

EVALUATE: Because there are two Doppler shifts, the reflected sound heard by the driver has an even higher frequency than the sound heard by a stationary listener in the warehouse.

**Doppler Effect for Electromagnetic Waves**

In the Doppler effect for sound, the velocities \( v_r \) and \( v_s \) are always measured relative to the air or whatever medium we are considering. There is also a Doppler effect for electromagnetic waves in empty space, such as light waves or radio waves. In this case there is no medium that we can use as a reference to measure velocities, and all that matters is the relative velocity of source and receiver. (By contrast, the Doppler effect for sound does not depend simply on this relative velocity, as discussed in Example 16.19.)

To derive the expression for the Doppler frequency shift for light, we have to use the special theory of relativity. We will discuss this in Chapter 37, but for now we quote the result without derivation. The wave speed is the speed of light, usually denoted by \( c \), and it is the same for both source and receiver. In the frame of reference in which the receiver is at rest, the source is moving away from the receiver with velocity \( v \). (If the source is approaching the receiver, \( v \) is negative.) The source frequency is again \( f_s \). The frequency \( f_R \) measured by the receiver R (the frequency of arrival of the waves at the receiver) is then given by

\[
f_R = \sqrt{\frac{c - v}{c + v}} f_s \quad \text{(Doppler effect for light)}
\]

When \( v \) is positive, the source is moving directly away from the receiver and \( f_R \) is always less than \( f_s \); when \( v \) is negative, the source is moving directly toward the receiver and \( f_R \) is greater than \( f_s \). The qualitative effect is the same as for sound, but the quantitative relationship is different.

A familiar application of the Doppler effect for radio waves is the radar device mounted on the side window of a police car to check other cars’ speeds. The electromagnetic wave emitted by the device is reflected from a moving car, which acts as a moving source, and the wave reflected back to the device is Doppler-shifted in frequency. The transmitted and reflected signals are combined to produce beats, and the speed can be computed from the frequency of the beats. Similar techniques (“Doppler radar”) are used to measure wind velocities in the atmosphere.

The Doppler effect is also used to track satellites and other space vehicles. In Fig. 16.35 a satellite emits a radio signal with constant frequency \( f_s \). As the satellite orbits past, it first approaches and then moves away from the receiver; the frequency \( f_R \) of the signal received on earth changes from a value greater than \( f_s \) to a value less than \( f_s \) as the satellite passes overhead.

The Doppler effect for electromagnetic waves, including visible light, is important in astronomy. Astronomers compare wavelengths of light from distant stars to those emitted by the same elements on earth. For example, in a binary star system, in which two stars orbit about their common center of mass, the light is
Doppler-shifted to higher frequencies when a star is moving toward an observer on earth and to lower frequencies when it's moving away. Measurements of the frequency shifts reveal information about the orbits and masses of the stars that comprise the binary system.

Light from most galaxies is shifted toward the longer-wavelength or red end of the visible spectrum, an effect called the red shift. This is often described as a Doppler shift resulting from motion of these galaxies away from us. However, from the point of view of the general theory of relativity, it is something much more fundamental: it is associated with the expansion of space itself. Distant galaxies have large red shifts because their light has been in transit for a long time and has shared in the expansion of all the space through which it moved. Extrapolating this expansion backward to over $10^{10}$ years ago leads to the “big bang” picture. From this point of view, the big bang was not an explosion in space but the initial rapid expansion of space itself.

**Test Your Understanding**

You are at an outdoor concert with a wind blowing at 10 m/s from the performers toward you. What is the value of $v_S$? Of $v_L$? Is the sound you hear Doppler-shifted? If so, is it shifted to lower or higher frequencies?

---

**16.9 | Shock Waves**

You may have experienced “sonic booms” caused by an airplane flying overhead faster than the speed of sound. We can see qualitatively why this happens from Fig. 16.36. Let $v_S$ denote the speed of the airplane relative to the air, so that it is always positive. The motion of the airplane through the air produces sound; if $v_S$ is less than the speed of sound $v$, the waves in front of the airplane are crowded together with a wavelength given by Eq. (16.27):

$$\lambda = \frac{v - v_S}{f_S}$$

As the speed $v_S$ of the airplane approaches the speed of sound $v$, the wavelength approaches zero and the wave crests pile up on each other (Fig. 16.36a). The airplane must exert a large force to compress the air in front of it; by Newton’s third law, the air exerts an equally large force back on the airplane. Hence there is a large increase in aerodynamic drag (air resistance) as the airplane approaches the speed of sound, a phenomenon known as the “sound barrier.”

When $v_S$ is greater in magnitude than $v$, the source of sound is supersonic, and Eqs. (16.27) and (16.29) for the Doppler effect no longer describe the sound wave in front of the source. Figure 16.36b shows a cross section of what happens. As the airplane moves, it displaces the surrounding air and produces sound. A series of wave crests is emitted from the nose of the airplane; each spreads out in a circle centered at the position of the airplane when it emitted the crest. After a time $t$ the crest emitted from point $S_1$ has spread to a circle with radius $vt$, and the airplane has moved a greater distance $v_S t$ to position $S_2$. You can see that the circular crests interfere constructively at points along the green line that makes an angle $\alpha$ with the direction of the airplane velocity, leading to a very-large-amplitude wave crest along this line. This large-amplitude crest is called a **shock wave** (Fig. 16.36c).
(a) As the speed of the source of sound $S$ approaches the speed of sound, the wave crests begin to pile up in front of $S$. (b) A shock wave forms when the speed of the source is greater than the speed of sound. (c) Photograph of shock waves produced by a T-38 jet aircraft moving at 1.1 times the speed of sound. Separate shock waves are generated by the nose, wings, and tail. The angles of these waves vary because the air is accelerated and decelerated as it moves relative to the airplane, so the relative speed of the airplane and air is different at different points on the airplane.

From the right triangle in Fig. 16.36b we can see that the angle $\alpha$ is given by

$$\sin \alpha = \frac{UT}{V_{st}} = \frac{V}{V_S} \quad \text{(shock wave)} \quad (16.31)$$

In this relation, $V_S$ is the speed of the source (the magnitude of its velocity) relative to the air and is always positive. The ratio $V_S/V$ is called the Mach number. It is greater than unity for all supersonic speeds, and $\sin \alpha$ in Eq. (16.31) is the reciprocal of the Mach number. The first person to break the sound barrier was Capt. Chuck Yeager of the U.S. Air Force, flying the Bell X-1 at Mach 1.06 on October 14, 1947 (Fig. 16.37).
The actual situation is three-dimensional; the shock wave forms a \textit{cone} around the direction of motion of the source. If the source (possibly a supersonic jet airplane or a rifle bullet) moves with constant velocity, the angle $\alpha$ is constant, and the shock-wave cone moves along with the source. It's the arrival of this shock wave that causes the sonic boom you hear after a supersonic airplane has passed by. The larger the airplane, the stronger the sonic boom; the shock wave produced at ground level by the (now retired) Concorde supersonic airliner flying at 12,000 m (40,000 ft) causes a sudden jump in air pressure of about 20 Pa. In front of the shock-wave cone, there is no sound. Inside the cone a stationary listener hears the Doppler-shifted sound of the airplane moving away.

\textbf{CAUTION} We emphasize that a shock wave is produced \textit{continuously} by any object that moves through the air at supersonic speed, not only at the instant that it "breaks the sound barrier." The sound waves that combine to form the shock wave, as in Fig. 16.36b, are created by the motion of the object itself, not by any sound source that the object may carry. The cracking noises of a bullet and of the tip of a circus whip are due to their supersonic motion. A supersonic jet airplane may have very loud engines, but these do not cause the shock wave. Indeed, a space shuttle makes a very loud sonic boom when coming in \textit{for a landing}; its engines are out of fuel at this point, so it is a supersonic glider.

Shock waves have applications outside of aviation. They are used to break up kidney stones and gallstones without invasive surgery, using a technique with the impressive name \textit{extracorporeal shock-wave lithotripsy}. A shock wave produced outside the body is focused by a reflector or acoustic lens so that as much of it as possible converges on the stone. When the resulting stresses in the stone exceed its tensile strength, it breaks into small pieces and can be eliminated. This technique requires accurate determination of the location of the stone, which may be done using ultrasonic imaging techniques (see Example 16.3 in Section 16.2).

\begin{example}
\textbf{Sonic boom of the Concorde}

The Concorde is flying at Mach 1.75 at an altitude of 8000 m, where the speed of sound is 320 m/s. How long after the plane passes directly overhead will you hear the sonic boom?

\textbf{SOLUTION}

\textbf{IDENTIFY:} The shock wave forms a cone trailing backward from the airplane, so the problem is really asking for how much time elapses from when the Concorde flies overhead to when the shock wave reaches you.

\textbf{SET UP:} Figure 16.38 shows the situation just as the shock wave reaches you at point L. A time $t$ (our target variable) has elapsed since the Concorde passed overhead, during which the airliner flying at speed $v_s$ has traveled a distance $v_s t$. We use trigonometry to solve for $t$.

\textbf{EXECUTE:} From Eq. (16.31) the angle $\alpha$ of the shock cone is

$$\alpha = \arcsin \frac{1}{1.75} = 34.8^\circ$$

The speed of the plane is the speed of sound multiplied by the Mach number:

$$v_s = (1.75)(320 \text{ m/s}) = 560 \text{ m/s}$$

From Fig. 16.38 we have

$$\tan \alpha = \frac{8000 \text{ m}}{v_s t}$$

$$t = \frac{8000 \text{ m}}{(560 \text{ m/s})(\tan 34.8^\circ)} = 20.5 \text{ s}$$
\end{example}
**EVALUATE:** You hear the boom 20.5 s after the Concorde passes overhead, and at that time it has traveled \((560 \text{ m/s})(20.5 \text{ s}) = 11.5 \text{ km}\) past the straight-overhead point.

In this calculation we assumed that the speed of sound is the same at all altitudes, so \(\alpha = \arcsin \frac{v_l}{v_s}\) is a constant and the shock wave forms a perfect cone. In fact, the speed of sound decreases with increasing altitude. How would this affect the result?

16.38 You hear a sonic boom when the shock wave reaches you at L (not just when the plane breaks the sound barrier). A listener to the right of L has not yet heard the sonic boom but will shortly; a listener to the left of L has already heard the sonic boom and now hears the Doppler-shifted sound of the airplane.

**Test Your Understanding**

Large meteors have been known to produce sonic booms as they descend at supersonic speeds through the earth’s atmosphere. If the shock wave from such a meteor has angle \(\alpha = 4.0^\circ\), what is the Mach number of the meteor?
Summary

Sound consists of longitudinal waves in a medium. A sinusoidal sound wave is characterized by its frequency $f$ and wavelength $\lambda$ (or angular frequency $\omega$ and wave number $k$) and by its displacement amplitude $A$. The pressure amplitude $p_{\text{max}}$ is directly proportional to the displacement amplitude; the proportionality constant is the product of the wave number and the bulk modulus $B$ of the wave medium. (See Examples 16.1 and 16.2)

The loudness of a sound depends on its amplitude and frequency, while the pitch depends primarily on its frequency. The tone quality or timbre depends on the harmonic content and the attack and decay characteristics.

The speed of a longitudinal (sound) wave in a fluid depends on the bulk modulus $B$ and density $\rho$. If the fluid is an ideal gas, the speed can be expressed in terms of the temperature $T$, molar mass $M$, and ratio of heat capacities $\gamma$ of the gas. The speed of longitudinal waves in a solid rod depends on the density and Young’s modulus $Y$. (See Examples 16.3 through 16.5)

The intensity $I$ of a sound wave is the time average rate at which energy is transported by the wave, per unit area. For a sinusoidal wave, the intensity can be expressed in terms of the displacement amplitude $A$ or the pressure amplitude $p_{\text{max}}$. (See Examples 16.6 through 16.9)

The sound intensity level $\beta$ of a sound wave is a logarithmic measure of its intensity. It is measured relative to $I_0$, an arbitrary intensity defined to be $10^{-12}$ W/m². Sound intensity levels are expressed in decibels (dB). (See Examples 16.10 and 16.11)

Standing sound waves can be set up in a pipe or tube. A closed end is a displacement node and a pressure antinode; an open end is a displacement antinode and a pressure node. For a pipe of length $L$ open at both ends, the normal-mode frequencies are integer multiples of the sound speed divided by $2L$. For a stopped pipe (one that is open at only one end), the normal-mode frequencies are the odd multiples of the sound speed divided by $4L$. (See Examples 16.12 and 16.13)
A system with normal modes of oscillation can be driven to oscillate at any frequency. A maximum response, or resonance, occurs if the driving frequency is close to one of the normal-mode frequencies of the system. (See Example 16.14)

When two or more waves overlap in the same region of space, the resulting effects are called interference. The resulting amplitude can be either larger or smaller than the amplitude of each individual wave, depending on whether the waves are in phase (constructive interference) or out of phase (destructive interference). (See Example 16.15)

Beats are heard when two tones with slightly different frequencies \( f_a \) and \( f_b \) are sounded together. The beat frequency \( f_{\text{beat}} \) is the difference between \( f_a \) and \( f_b \).

\[
f_{\text{beat}} = f_a - f_b \quad (\text{beat frequency})
\]

The Doppler effect for sound is the frequency shift that occurs when there is motion of a source of sound, a listener, or both, relative to the medium. The source and listener frequencies \( f_s \) and \( f_l \) are related by the source and listener velocities \( v_S \) and \( v_L \) relative to the medium and to the speed of sound \( v \).

\[
f_l = \frac{v + v_L}{v + v_S} f_s \quad (\text{Doppler effect, moving source and moving listener})
\]

A sound source moving with a speed \( v_L \) greater than the speed of sound \( v \) creates a shock wave. The wave front is a cone with angle \( \alpha \).

\[
\sin \alpha = \frac{v}{v_S} \quad (\text{shock wave})
\]

**Key Terms**

<table>
<thead>
<tr>
<th>term</th>
<th>page</th>
</tr>
</thead>
<tbody>
<tr>
<td>sound, 592</td>
<td></td>
</tr>
<tr>
<td>audible range, 592</td>
<td></td>
</tr>
<tr>
<td>ultrasonic, 592</td>
<td></td>
</tr>
<tr>
<td>infrasonic, 592</td>
<td></td>
</tr>
<tr>
<td>displacement amplitude, 592</td>
<td></td>
</tr>
<tr>
<td>pressure amplitude, 594</td>
<td></td>
</tr>
<tr>
<td>loudness, 596</td>
<td></td>
</tr>
<tr>
<td>pitch, 596</td>
<td></td>
</tr>
<tr>
<td>timbre, 596</td>
<td></td>
</tr>
<tr>
<td>noise, 597</td>
<td></td>
</tr>
<tr>
<td>sound intensity level, 605</td>
<td></td>
</tr>
<tr>
<td>decibel, 606</td>
<td></td>
</tr>
<tr>
<td>displacement node, 608</td>
<td></td>
</tr>
<tr>
<td>displacement antinode, 608</td>
<td></td>
</tr>
<tr>
<td>pressure node, 609</td>
<td></td>
</tr>
<tr>
<td>pressure antinode, 609</td>
<td></td>
</tr>
<tr>
<td>resonance, 614</td>
<td></td>
</tr>
<tr>
<td>resonance curve, 615</td>
<td></td>
</tr>
<tr>
<td>beats, 619</td>
<td></td>
</tr>
<tr>
<td>beat frequency, 619</td>
<td></td>
</tr>
<tr>
<td>Doppler effect, 621</td>
<td></td>
</tr>
<tr>
<td>supersonic, 627</td>
<td></td>
</tr>
<tr>
<td>shock wave, 627</td>
<td></td>
</tr>
<tr>
<td>Mach number, 628</td>
<td></td>
</tr>
</tbody>
</table>

**Your Notes**