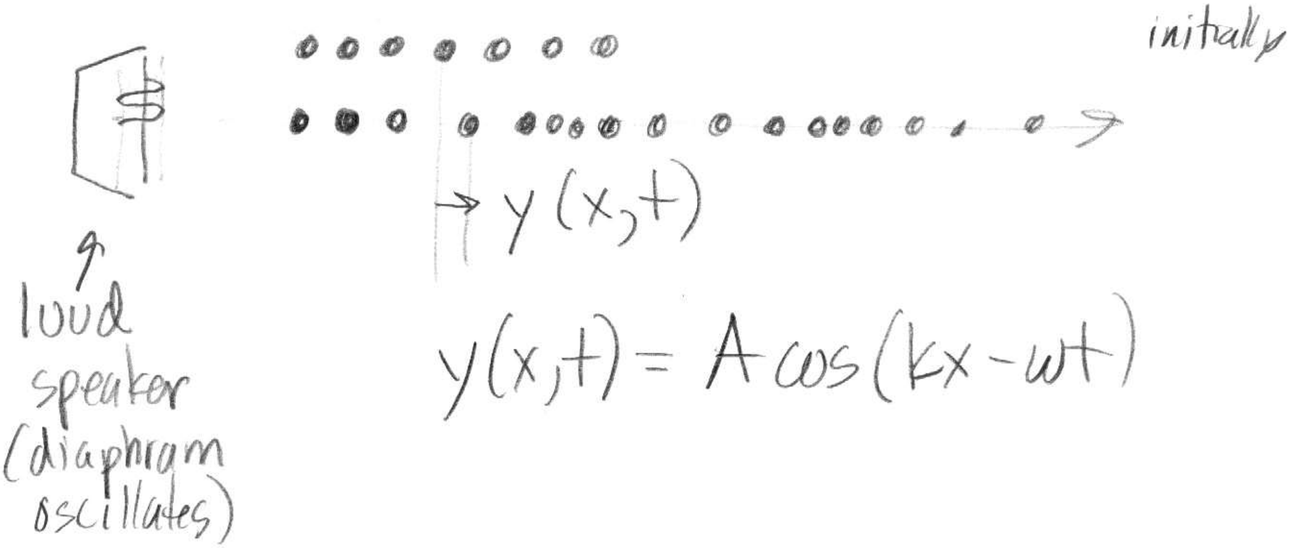
Pressure Waves (Sound)

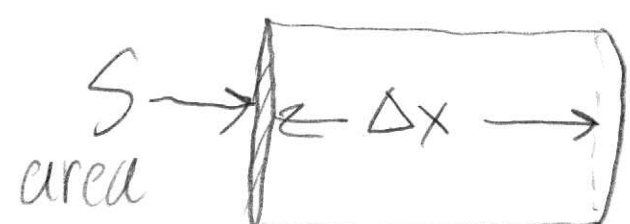


$$y(x,t) = A\cos(kx - \omega t)$$

Relate to pressure increment Δp

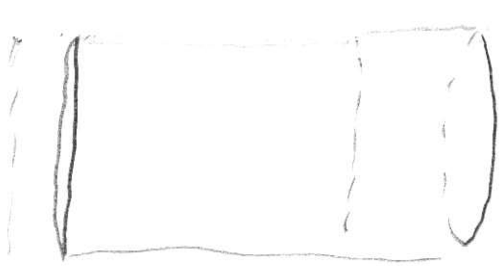
(Δp: above atmospheric)

cylinder of air, no wave:



$y(x,0) = 0$ $\quad$ $y(x+\Delta x, 0) = 0$

later



$y(x,t)$ $\quad$ $\longrightarrow y(x+\Delta x, t)$

idea: volume change depends on different displacements, $\frac{\partial y}{\partial x}$

$$\Delta V = S\cdot\left(y(x+\Delta x,t) - y(x,t)\right)$$

$$\frac{\Delta V}{V} = \frac{S\left(y(x+\Delta x,t) - y(x,t)\right)}{S\,\Delta x}$$

$$\boxed{\frac{\Delta V}{V} = \frac{\partial y}{\partial x}(x,t)}$$

total pressure (absolute)

Gas (Isothermal not right, exactly)

$$pV = nRT$$

$$V = nRT\,\frac{1}{p}$$

$$\Delta V = -nRT\,\frac{\Delta p}{p^2}$$

maybe skip

$$\frac{\Delta V}{V} = \frac{-nRT\,\frac{\Delta p}{p^2}}{nRT\,\frac{1}{p}}$$

$$\frac{\Delta V}{V} = -\frac{\Delta p}{p}$$ ← gauge pressure

change in pres

$$-\frac{\Delta p}{\left(\frac{\Delta V}{V}\right)} = p$$ ← absolute pressure

N/m^2 — isothermal gas

in general

$\equiv$ Bulk Modulus

$= B$ N/m^2

$$\Delta p = -\frac{\Delta V}{V} \times B$$

($B = p$ for isothermal ideal gas)

$$\Delta p \equiv p_G(x,t) = -B \cdot \frac{\partial y}{\partial x}$$

$$p_G(x,t) = -B \frac{\partial}{\partial x}\left(\cos(kx-\omega t)\right)$$

$$p_G(x,t) = BkA \sin(kx-\omega t)$$
$$y(x,t) = A\cos(kx-\omega t)$$

when $\cos = \pm 1$, $\sin = 0$

$\cos = 0$, $\sin = \pm 1$

→ large displacement, no pressure change

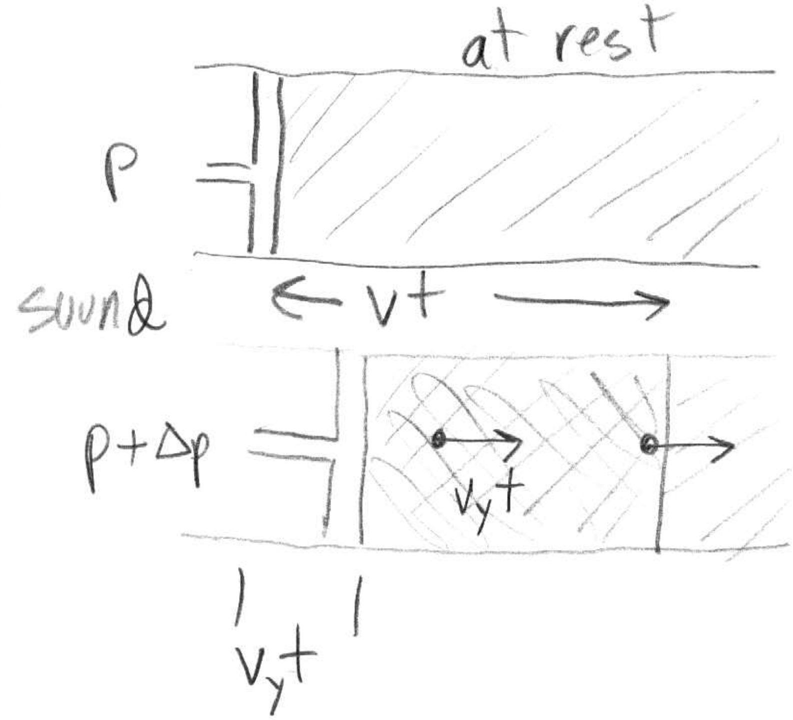
pipe → 0 displacement, large $\Delta p = p_G$

$$p_{G\,max} = BkA$$
$$= B \times \frac{2\pi}{\lambda} \cdot A$$

<u>Air</u>: (not isothermal)

$$B = 1.4 \cdot 10^5 \text{ Pa}$$

Speed of Sound $\propto \sqrt{\dfrac{\text{spring constant}}{\text{mass}}}$



mass : $\rho \cdot vt \cdot A$

momentum : $(\rho \cdot v \cdot t \cdot A)\, v_y$

impulse : $\Delta p \cdot A \cdot t$

"springiness" : $B = \dfrac{-\Delta p}{(\Delta V / V)}$

$$= \frac{-\Delta p}{-A v_y t / A v t}$$

$$\Delta p = \frac{v_y}{v} B$$

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