Standing Waves

Back to 1-d

$$\frac{\partial^2 y}{\partial^2 x} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$y(x,t) = A\cos(kx \mp \omega t)$$

or, even

$$\underbrace{A_-\cos(kx-\omega t) + A_+}_{\text{can be } A_- \neq A_+}\cos(kx+\omega t)$$

What considerations dictate choice of A_- & A_+ ?

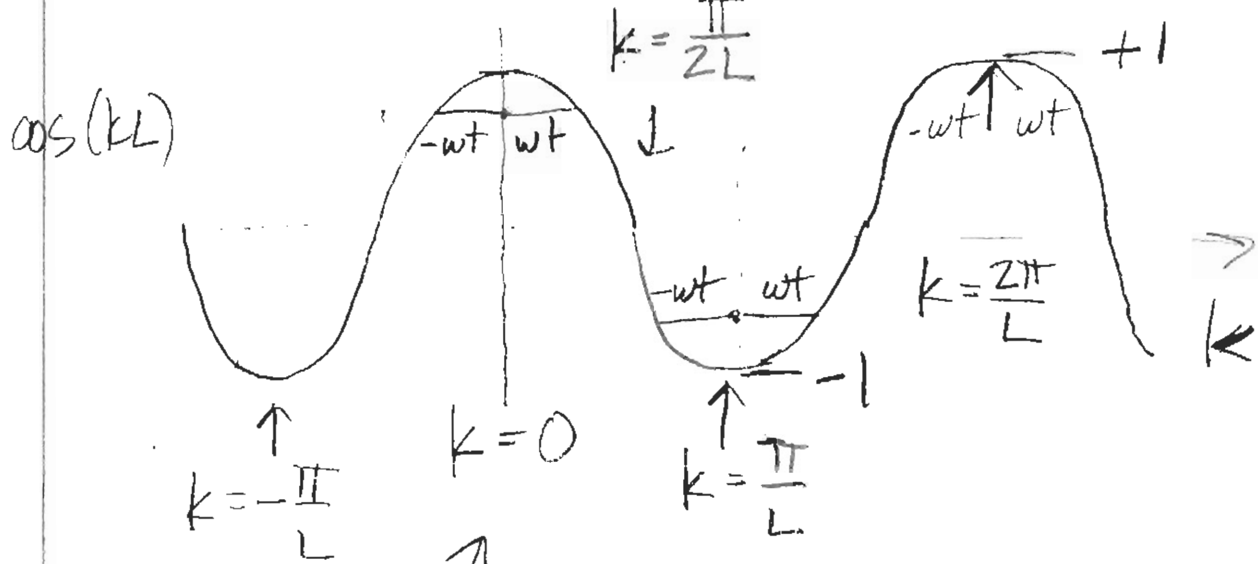
→ Boundary Conditions

→ Initial ($t=0$) Conditions

Famous Case: Standing Waves on a String

need $\cos(kL-\omega t) = \cos(kL+\omega t)$
for all time

Plot ... $t=0$, $\cos kL$

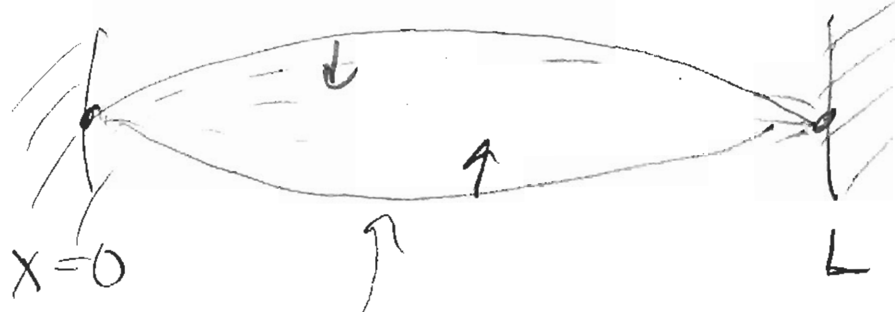


k=0 works, but then no dependence on x at all!

$$k_n = \frac{(\text{integer})\cdot\pi}{L} = \frac{n\pi}{L} \quad n=1,2,\ldots$$

– values add no new information

$$\cos(-kx-\omega t) = \cos(kx+\omega t)$$



the "fundamental"

What is the "wave function"?

<u>key</u>: <u>ends fixed</u> (Boundary Conditions)

Try

$$y(x,t) = A_- \cos(kx-\omega t) + A_+ \cos(kx+\omega t)$$

$$y(0,t) = 0 = A_- \cos(-\omega t) + A_+ \cos(\omega t)$$

all t — cos even, $= \cos(\omega t)$

$$\therefore A_+ = -A_-$$

$$y(x,t) = A_- \left(\cos(kx-\omega t) - \cos(kx+\omega t)\right)$$

$$y(L,t) = 0 = A_- \left(\underbrace{\cos(kL-\omega t) - \cos(kL+\omega t)}_{\text{must be } 0 \text{ for all time!}}\right)$$

cannot be 0

must be 0 for all time!

$$y_n = A_-\left(\cos\left(\frac{n\pi}{L}x - \omega t\right) - \cos\left(\frac{n\pi}{L}x + \omega t\right)\right)$$

$$\cos\left(\frac{n\pi}{L}x\right)\cos(-\omega t) - \sin\left(\frac{n\pi}{L}x\right)\sin(-\omega t)$$

$$-\cos\left(\frac{n\pi}{L}x\right)\cos(\omega t) + \sin\left(\frac{n\pi}{L}x\right)\sin(\omega t)$$

$$\boxed{y_n = 2A_- \sin\left(\frac{n\pi}{L}x\right)\sin(\omega t)}$$

v = property of string

but $\quad v = \frac{\omega}{k} = \frac{\omega}{\frac{n\pi}{L}}$

$$\omega_n = \frac{n\pi}{L} v$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{nv}{2L} = n \cdot f_1$$

$$f_1 = \frac{v}{2L}$$

"fundamental frequency"