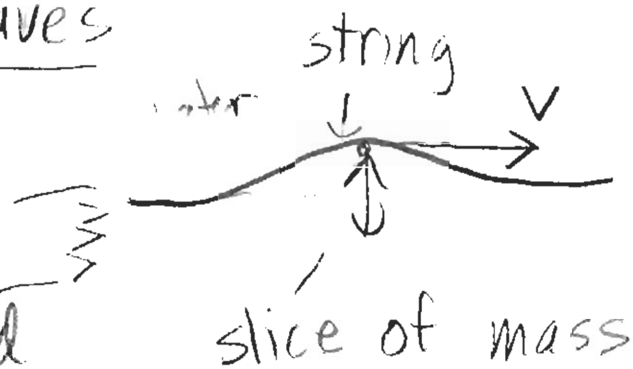
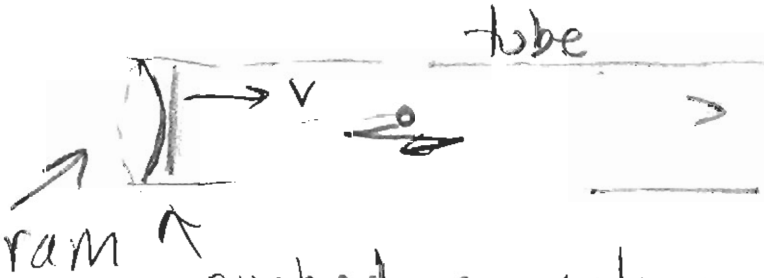
Types of Waves

Transverse:



slice of mass moves $\perp$ or transverse to axis

Longitudinal (sound)



- pushed in, makes a compression
- drawn back, a "rarefaction
- slices of mass move $\parallel$, or longitudinal, to axis.

Earthquakes: speed of longitudinal part ("P-wave") faster than transverse ("S-wave")

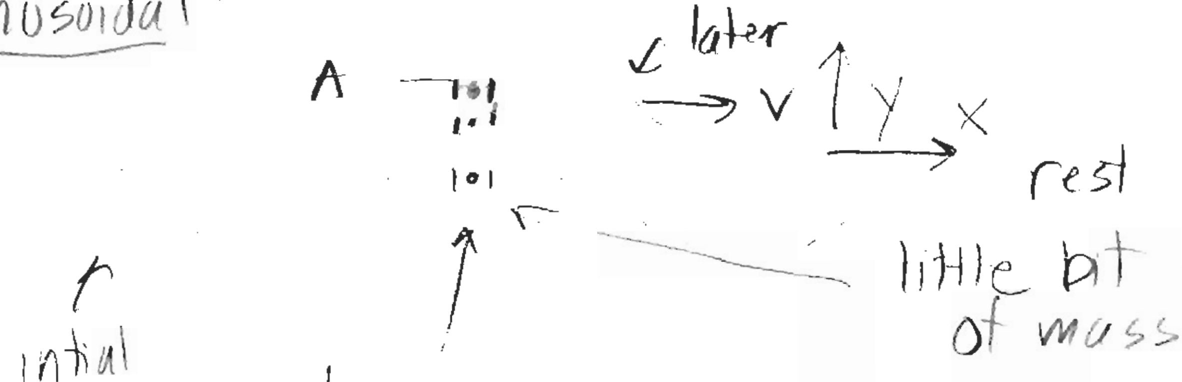
P-wave wakes you, animals up

Pulse . one segment

Periodic ; keep doing it (Sound)

Complication : both time and space.

Sinusoidal :

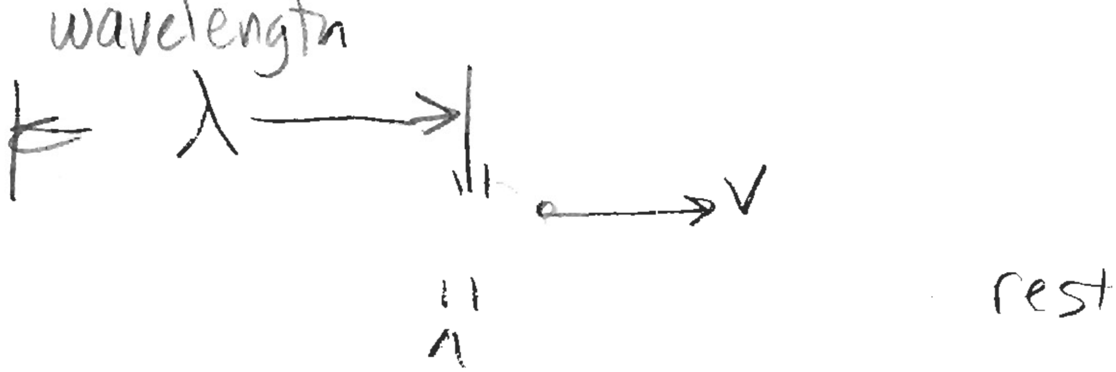


when wave moves left, little bit of mass does not move left, but down. (Transverse)

mass : $y(t) = A\cos(\omega t)$

what will that ω turn out to be.

→ will be related to the wavelength, and the speed v



$A\cos(\omega t)$ (or $\cos(-\omega t)$)

period $T = \frac{2\pi}{\omega}$

$\cos(\omega T) = \cos(2\pi) = 1$
$= \cos(0)$

T is also time it takes for the next crest to go distance λ:

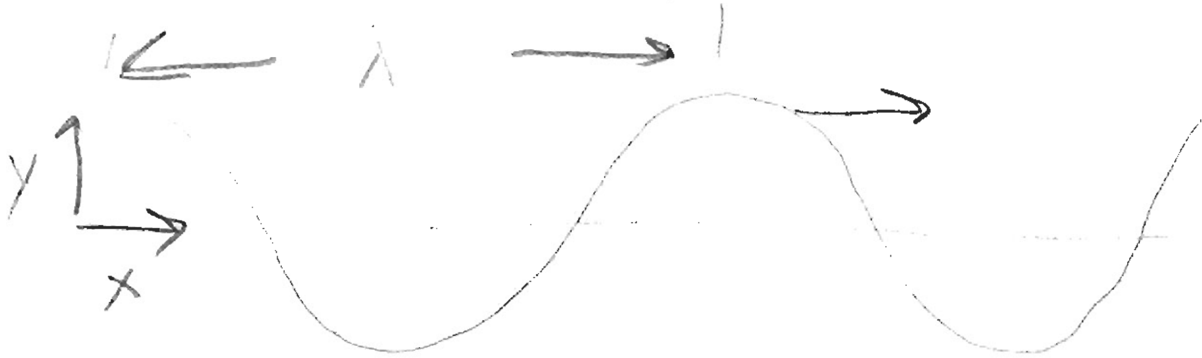
$$v \cdot T = \lambda$$

$$v \cdot \frac{2\pi}{\omega} = \lambda$$

$$\boxed{v = \lambda \cdot \frac{\omega}{2\pi} = \lambda \cdot f}$$

ω in $\frac{\text{radians}}{\text{sec}}$ / length $\frac{1}{\text{time}}$

Wave function: $y(x,t)$



snapshot at $t=0$

$$y(x, t=0) = A\cos\left(\pm \frac{2\pi x}{\lambda}\right)$$

why?

study particular x

$$y(x=0, t) = A\cos(\pm \omega t)$$

Combine

$$y(x,t) = A\cos\left(\pm\left(\frac{2\pi x}{\lambda} \pm \omega t\right)\right)$$

irrelevant not irrelevant

"constant phase"

$$\frac{2\pi x}{\lambda} \pm \omega t = \text{constant}$$

$$x = \mp \underbrace{\frac{\omega\lambda}{2\pi}}_{v!} t + \text{constant}$$

$$x = \mp vt + \text{constant}$$

$$A\cos\left(\frac{2\pi x}{\lambda} + \omega t\right) \rightarrow \text{left moving wave}$$

$$A\cos\left(\frac{2\pi x}{\lambda} - \omega t\right) \rightarrow \text{right moving wave}$$

right later $\longrightarrow v$

<u>often</u>: $\frac{2\pi}{\lambda} = k$ wave vector

$$v = \frac{\omega}{k}$$

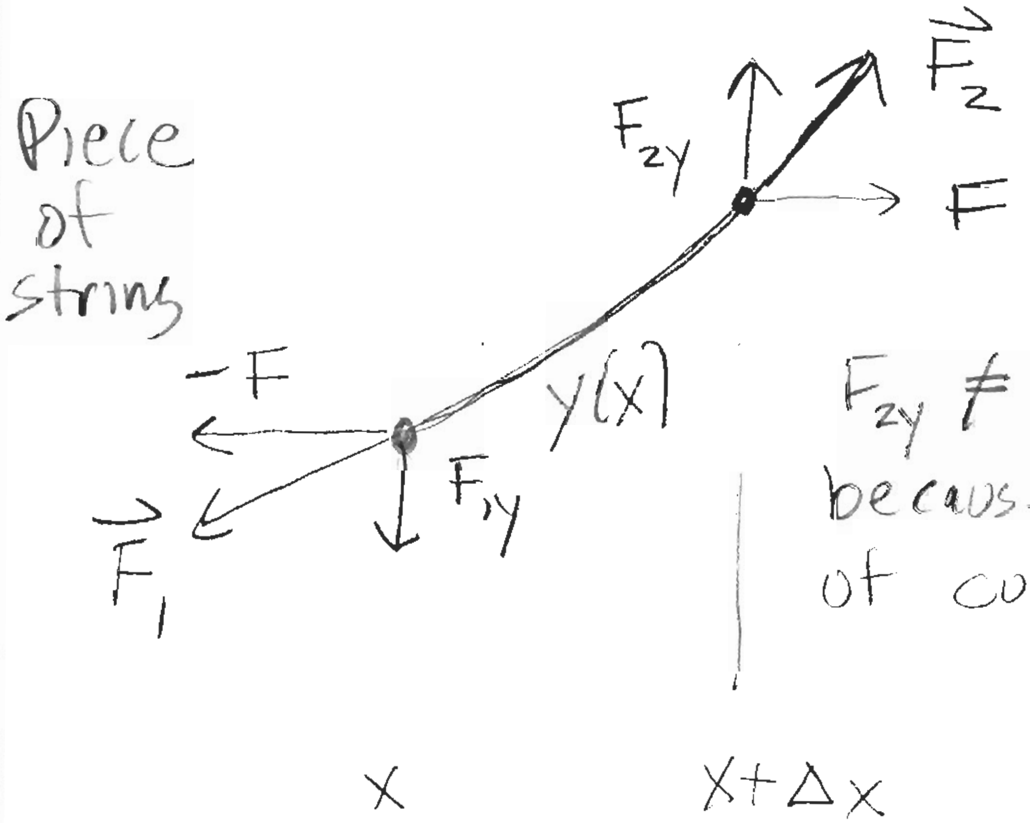
What determines v?

similar to $\omega \rightarrow$ SHO

$$\omega = \sqrt{\frac{k}{m}}$$
$\leftarrow$ spring constant
$\leftarrow$ mass

$$v = \sqrt{\frac{F}{\mu}}$$
$\leftarrow$ force (Tension in string)
$\leftarrow$ mass per unit length

Derivation involves second derivatives



$F_{2y} \neq F_{1y}$ because of curvature

$$\frac{F_{1y}}{F} = -\left(\frac{\partial y}{\partial x}\right)_{x} \qquad \frac{F_{2y}}{F} = \left(\frac{dy}{dx}\right)_{x+\Delta x}$$

$$F_{1y} + F_{2y} = F \cdot \left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_{x}\right]$$

$$= (\mu \cdot \Delta x)\,\frac{\partial^2 y}{\partial t^2}$$

$\uparrow$
mass/length

or $$\frac{\partial^2 y}{\partial x^2} = \frac{F}{\mu}\,\frac{\partial^2 y}{\partial t^2}$$

$$y(x,t) = A\cos(kx \pm \omega t)$$

$$\frac{\partial y}{\partial x} = -kA\sin(kx \pm \omega t)$$

$$\frac{\partial^2 y}{\partial x^2} = -k^2 A\cos(kx \pm \omega t) \qquad \frac{\partial^2 y}{\partial t^2} = -\omega^2 A\cos(\ )$$

$$\frac{\partial^2 y}{\partial x^2} = \left(\frac{k^2}{\omega^2}\right)\frac{\partial^2 y}{\partial t^2}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$\frac{1}{v^2} = \frac{F}{\mu}$$

$$v = \sqrt{\frac{\mu}{F}}$$