\[ \mathbf{F}_2 = k \frac{q_1 q_2}{r_{21}^2} \mathbf{r}_{21} \]  

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\[ r_{21} = \left| \mathbf{r}_2 - \mathbf{r}_1 \right| \]

\[ \mathbf{F}_1 = -\mathbf{F}_2 \]

\[ \text{newtons or dynes (I), meter}^{-2} \text{ or centimeters}^{-2} (I) \text{ new} \]

\[ \text{newton} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \quad \text{dyne} = \frac{\text{g} \cdot \text{cm}}{\text{s}^2} \]

\[ q_1, q_2 \rightarrow \text{measured in units of charge} \]

\[ \text{can be independent of } \pi, \text{ so the constant } k \text{ is introduced to make units turn out OK} \]

"practical units" → measure q in "Coulombs" → related to batteries, electrolysis

origin not really physics, so k is very important.
newtons = \frac{k \cdot (\text{Coulomb})^2}{(\text{meter})^2}

\begin{equation}
[k] = \frac{m^2 \cdot N}{C^2}
\end{equation}

\begin{equation}
k = 9 \cdot 10^9 \frac{m^2 \cdot N}{C^2} \quad \text{(from measurement)}
\end{equation}

History has left \( k \) with another name,

\begin{equation}
k = \frac{1}{4 \pi \varepsilon_0}
\end{equation}

\varepsilon_0 = "\text{permittivity of vacuum}"

ancient concept.

"modern physics units"

⇒ choose \( k = 1 \) and redefine charge

⇒ kilograms \rightarrow \text{grams}

⇒ meters \rightarrow \text{centimeters}

⇒ seconds \rightarrow \text{seconds}

⇒ newtons \rightarrow \text{dynes}

⇒ \( q \) in "electrostatic units"

or esu

\begin{equation}
esu \equiv \frac{1}{3} \cdot 10^{-9} \text{ Coulombs}
\end{equation}

\begin{equation}
1 \text{ Coulomb} = 3 \cdot 10^9 \text{ esu}
\end{equation}
SI:

\[ F_{21} = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r_{21}^2} \]

Newtons, Coulombs, meters, kg

Gaussian cgs:

\[ \vec{F}_{21} = \frac{91 q_1 q_2}{r_{21}^2} \vec{F}_{21} \]

Dynes, esu, centimeters, grams

In either case, there is a minimum unit of charge that has never been subdivided...

charge on \( "e" \) = 1.6 \times 10^{-19} \text{ C}

a proton = \( -e \), experimentally tested to 1 part in \( 10^{20} \).

The charge on an electron is exactly \( -e \), experimentally tested to 1 part in \( 10^{20} \).

Electrons and protons are otherwise very different. For example, protons have a mass \( \approx 2000 \times \) that of electrons, protons have a size, \( \approx 10^{-13} \text{ cm} \), but electrons do not!
Also, in a hydrogen atom, the electron moves at \( \approx 10^{-2} \times \) speed of light, proton \( \approx 10^{-5} \times \) speed of light, but nonetheless their charges still cancel.

**Electric charge is independent of velocity**

Charges add up.

Electrostatic Forces add up... presence of a third charge does not influence behavior of first two.

\[
\begin{align*}
q_1 & \quad \hat{r}_{21} & \quad r_{12} = 10 \text{ cm} & \quad q_2 \\
5 \text{ esu} & \quad 0 \quad \rightarrow \quad F_1 & \quad -20 \text{ esu} & \quad F_2
\end{align*}
\]

\[
F_2 = \frac{q_1 q_2}{r_{21}^2}
\]

**Gaussian c.g.s.**

\[
\begin{align*}
F_2 &= \frac{5 \cdot (-20)}{(10^2)^2} = (-10^{-2} \text{ dynes}) \cdot F_{12} \\
\end{align*}
\]

\(\text{SI}:\ 1 \text{ dyne} = 1 \frac{\text{ gm \cdot cm}}{\text{s}^2} = 1 \times 10^{-3} \times 10^{-2} \frac{\text{ kg \cdot m}}{\text{s}^2}
\]

\[
\vec{F}_2 = (-10^{-5}) \hat{r}_{21} \text{ N}
\]