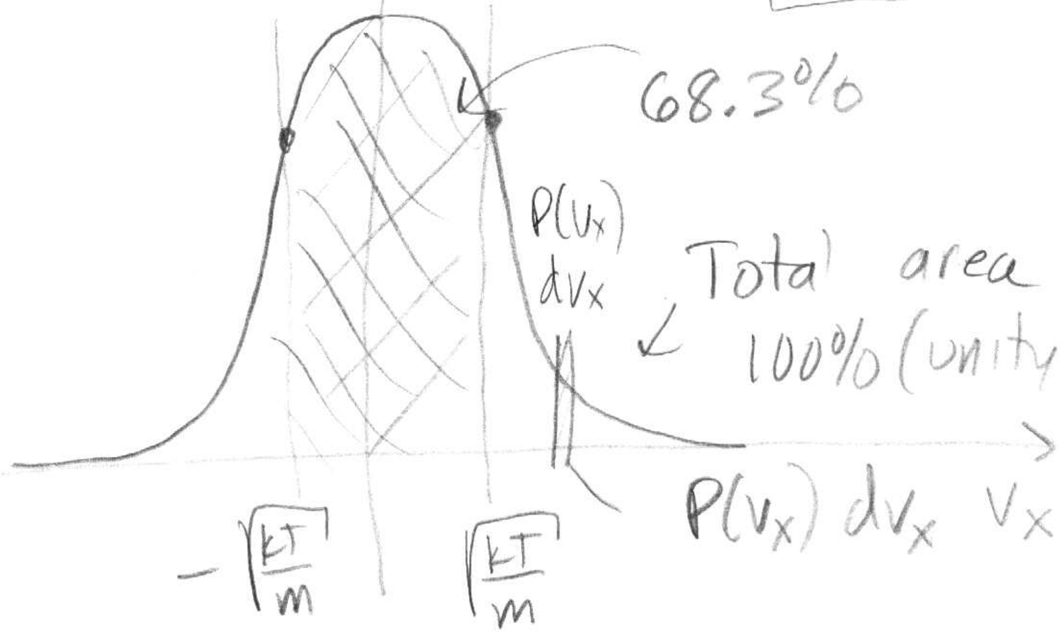
Look at one variable.

$$P(v_x, v_y, v_z) = P(v_x)\,P(v_y)\,P(v_z)$$

$$P(v_x) = \left(\frac{m}{2\pi kT}\right)^{1/2} e^{-\frac{mv_x^2}{2kT}} \qquad \int P(v_x)\,dv_x = 1$$

normalization constant

This is a <u>bell</u> <u>curve</u>, $\boxed{\text{mean } 0}$ $\boxed{\sigma = (kT/m)^{1/2}}$



$$(v_x)_{avg} = 0 = (v_y)_{avg} = (v_z)_{avg}$$

<u>but</u> $(v^2)_{avg} \neq 0$!

$$(v_x)_{avg} = \int_{-\infty}^{\infty} v_x\,P(v_x)\,dv_x = \boxed{0 \text{ by symmetry}}$$

draw

$$P(v_x, v_y, v_z) = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}(v_x^2+v_y^2+v_z^2)}$$

$$P(v_x, v_y, v_z)\, dv_x\, dv_y\, dv_z =$$

$$\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}(v_x^2+v_y^2+v_z^2)}\, dv_x\, dv_y\, dv_z$$

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

direction of $\vec{v}$

$$dv_x\, dv_y\, dv_z = v^2\, dv \cdot d\phi_v \sin\theta_v\, d\theta_v$$

$$\underbrace{P(v)\, dv\, d\phi_v \sin\theta_v\, d\theta_v = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}} v^2\, dv\, d\phi_v \sin\theta_v\, d\theta_v}$$

does not depend on direction

$$\int_0^{2\pi}\int_0^{\pi} d\phi_v \sin\theta_v\, d\theta_v = 4\pi$$

$$P(v)\, dv = \underbrace{4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}}_{P(v)}\, dv$$

Plot

$$(v^2)_{avg} = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int^{\infty} dv\, v^2 v^2 e^{-\frac{mv^2}{2kT}}\, dv$$

$$\left(v^2\right)_{avg} = \frac{3kT}{m}, \quad \left(v^2\right)_{avg}^{1/2} = \sqrt{\frac{3kT}{m}}$$

$$v_{avg} = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty dv \, v \cdot v^2 e^{\frac{-mv^2}{2kT}}$$

$$v_{avg} = \sqrt{\frac{8kT}{\pi}} \neq \left(v^2\right)_{avg}^{1/2} = \sqrt{\frac{3kT}{m}}$$