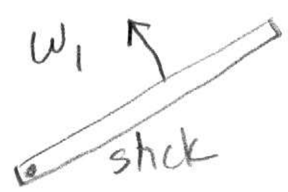
1



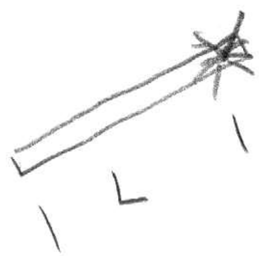
$$I_0 = \frac{1}{12} ML^2$$

$$I_1 = I_0 + M\left(\frac{L}{2}\right)^2$$

$$= \left(\frac{1}{4} + \frac{1}{12}\right) ML^2$$

$$I_1 = \frac{1}{3} ML^2$$

$$L_1 = I_1 \omega_1$$



$$I_2 = I_1 + \left(\frac{1}{3}M\right)L^2$$

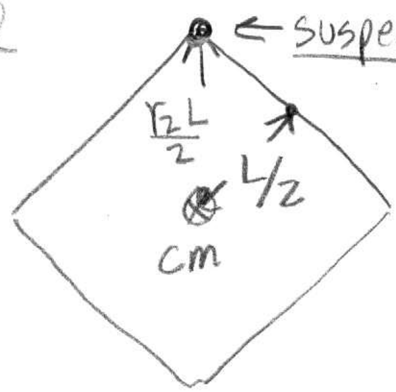
$$= \frac{2}{3} ML^2$$

$$L_1 = L_2 = I_1 \omega_1 = I_2 \omega_2$$

$$\omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{\frac{1}{3}ML^2}{\frac{2}{3}ML^2}\omega_1$$

$$\boxed{\omega_2 = \frac{1}{2}\omega_1}$$

2



I_0 of stick $= \frac{1}{12} ML^2$

I of 1 stick about $\otimes$ is

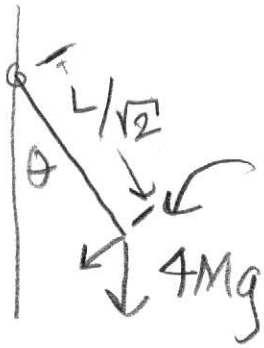
$$= \frac{1}{12} ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3} ML^2$$

4 sticks $= \frac{4}{3} ML^2$

About Suspension Point $I = \frac{4}{3} ML^2 + 4M\left(\frac{L}{\sqrt{2}}\right)^2$

$$= \left(\frac{4}{3} + \frac{4}{2}\right) ML^2$$

$$= \left(\frac{8}{6} + \frac{12}{6}\right) ML^2 = \frac{10}{3} ML^2$$



$$\tau = -4Mg\frac{L}{\sqrt{2}}\theta$$

$$I\ddot{\theta} = -(2\sqrt{2}MgL)\theta$$

$$\omega^2 = \frac{2\sqrt{2}MgL}{I} = \frac{2\sqrt{2}MgL}{\frac{10}{3}ML^2} = \frac{3\sqrt{2}}{5}\frac{g}{L}$$

$$\boxed{T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{3}{5}\sqrt{2}\frac{g}{L}}} = 2\pi\sqrt{\frac{5}{3\sqrt{2}}\frac{L}{g}}}$$

3 (a) $X_s = \frac{F_0}{k} = \frac{10^{-2}}{0.4} = \boxed{2.5 \cdot 10^{-2}\,m}$

(b) $\omega_1 = \sqrt{\omega_0^2 - \frac{1}{4}\gamma^2}$ $\quad \omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{0.4}{0.1}} = 2\ 1/s$

$= \sqrt{2^2 - \frac{1}{4}\cdot(16\cdot10^{-4})}$ $\quad \gamma = \frac{b}{m} = \frac{4\cdot10^{-3}}{0.1} = 4\cdot10^{-2}\ 1/s$

$\omega_1 = 1.9999 \simeq 2\ 1/s$

$$\boxed{Q = \frac{\omega_1}{\gamma} = \frac{2}{4\cdot10^{-2}} = 50}$$

(c) (i) $\ddot{z} + \gamma\dot{z} + \omega_0^2 z = \frac{F_0}{m}e^{i\omega t}$

$$z = z_0 e^{i\omega t}$$

$$z_0 = \frac{F_0}{m\left((\omega_0^2 - \omega^2) + i\gamma\omega\right)}$$

$$\omega_0^2 - \omega^2 = 4 - (1.98) \doteq 0.0796$$

$$\gamma\omega = .04 \cdot 1.98 = 0.0792$$

$$\omega_0^2 - \omega^2 + i\gamma\omega = 0.0796 + 0.0792\, i$$

$$= \sqrt{0.0796^2 + 0.0792^2}\; e^{\underbrace{i\tan^{-1}\left(\frac{0.792}{0.796}\right)}_{\simeq \pi/4}}$$

$$= 0.112\, e^{i\frac{\pi}{4}}$$

$$\boxed{|z_0| = \frac{10^{-2}}{0.1 \times 0.112} = 0.89\text{ m}}$$

(ii) $$z_0 = \frac{F_0}{m\ 0.112\, e^{i\pi/4}}\, e^{i\omega t}$$

$$z_0 = 0.89\, e^{i(\omega t - \pi/4)}$$

$$\omega t - \frac{\pi}{4} = 0$$

$$\boxed{t = \frac{\pi}{4\omega} = \frac{\pi}{4 \cdot 2} = 0.39\text{ s}}$$

4 (a) $L = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

$= \sqrt{120^2 + 90^2}$ cm

$= 30\sqrt{4^2 + 3^2}$ cm

$L = 150$ cm

$T = t_B - t_A = 4$ ns

$\frac{L}{T} = \frac{150 \text{ cm}}{4 \text{ ns}} = 37{,}5 \text{ cm/ns}$

$\boxed{\frac{L}{T} > 30 \text{ cm/ns} \Rightarrow \underline{\text{spacelike}}}$

(b) $(cT)^2 - L^2 = -L'^2$ L' in frame where $T' = 0$

$120^2 - 150^2 = -L'^2$

$-90^2 = -L'^2$

$\boxed{L' = 90 \text{ cm}}$

(c) $\boxed{x_A' = y_A' = t_A' = 0}$ $\beta = \frac{4}{5}$

$\boxed{y_B' = y_B = 90 \text{ cm}}$ $\gamma = (1-\beta^2)^{-1/2} = \left(1 - \frac{16}{25}\right)^{-1/2}$

$= \left(\frac{9}{25}\right)^{-1/2} = \frac{5}{3}$

$$\boxed{\begin{aligned} x_B' &= \gamma(x_B - \beta c t_B) \\ &= \frac{5}{3}\left(120 - \frac{4}{5}\cdot 30 \cdot 4\right) = \frac{5}{3}\cdot\frac{1}{5}\cdot 120 \text{ cm} = 40 \text{ cm} \end{aligned}}$$

$t_B' = \frac{1}{c}\gamma(ct_B - \beta x_B) = \frac{1}{c}\cdot\frac{5}{3}\cdot\frac{1}{5}\,120 = 4$ ns

5 (a) $\frac{1}{2}\cancel{m}c^2 = G\frac{M\cancel{m}}{r_s}$

$$\boxed{r_s = \frac{2GM}{c^2}}$$

(b) $r_s = \frac{2\cdot\left(\frac{2}{3}\cdot 10^{-10}\right)\left(2\cdot 10^{30}\right)}{\left(3\cdot 10^{8}\right)^2}\ \text{m}$

$= \frac{8}{27}\cdot 10^{-10+30-16}\ \text{m}$

$r_s = \frac{8}{27}\cdot 10^{4}\ \text{m}$

$$\boxed{r_s = 3.0\cdot 10^{3}\ \text{m} = 3.0\ \text{km}}$$

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6 (a) $\tau = 10\,y = \gamma(\tau_0 = 1\,y)$

$\boxed{\gamma = 10} = \frac{1}{\sqrt{1-\beta^2}}$

$\gamma^2 = \frac{1}{1-\beta^2}$

$1-\beta^2 = \frac{1}{\gamma^2} \Rightarrow \beta^2 = 1 - \frac{1}{\gamma^2}$

$\beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{100}}\ \left(\simeq 1 - \frac{1}{2}\frac{1}{100}\right)$

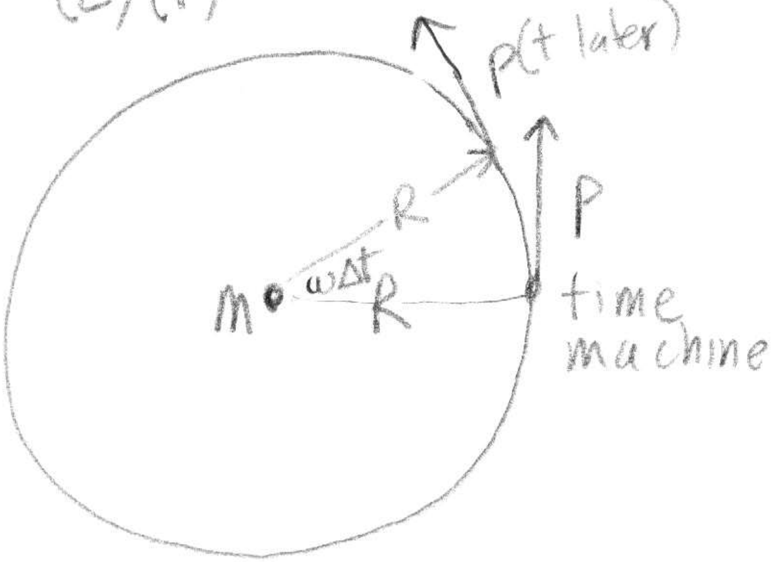
$$\boxed{\beta = 0.995}$$

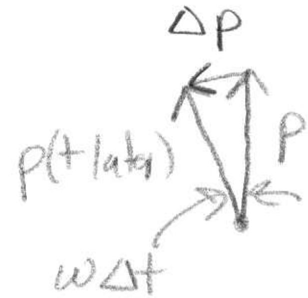
(b) $p = m(v)\,v = \frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\,v \qquad \frac{v}{c} = \beta$

$v = \beta c$

$\boxed{p = \gamma\beta m_0 c} \qquad \gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

(c)(i)





$$\Delta p \cong p\,\omega\Delta t$$

$$\frac{\Delta p}{\Delta t} = p\omega$$

$$\omega = \frac{v}{R} = \frac{\beta c}{R}$$

so $$\frac{dp}{dt} = p \times \frac{\beta c}{R} = \gamma\beta m_0 c\,\frac{\beta c}{R}$$

$$\boxed{\frac{dp}{dt} = \gamma\beta^2\,\frac{m_0 c^2}{R}}$$

(ii) $$F = \frac{dp}{dt}$$

$$\frac{GM\cancel{m_0}}{R^2} = \gamma\beta^2\,\frac{\cancel{m_0}c^2}{R}$$

$$\boxed{R = \frac{1}{\gamma\beta^2}\,\frac{GM}{c^2}}$$

(iii) $$\frac{GM_{sun}}{c^2} = \frac{\left(\frac{2}{3}\cdot 10^{-10}\right)\left(2\cdot 10^{30}\right)}{\left(3\cdot 10^{8}\right)^2}$$

$$= \frac{4}{27}\cdot 10^4 = 1.5\cdot 10^3\,\text{m} = 1.5\,\text{km}$$

$$R = \frac{1}{10 \times (0.995)^2} \cdot 1.5 \text{ km}$$

$$\boxed{R \simeq 150 \text{ m}}$$

since this is smaller than r_s for the sun, the sun would have to collapse to a very small "black hole" to be inside this orbit. Further, our calculation shows you'd really better use general relativity to do the problem.

(iv) $$a = \frac{F}{m_0} = G\frac{M_{sun}}{R^2}$$

$$= \frac{G M_{sun}}{\frac{1}{\gamma^2\beta^4}\frac{G^2 M_{sun}^2}{c^4}} = \gamma^2\beta^4 \frac{c^2}{r_s}$$

$$a = 10^2 (0.995)^4 \frac{(3 \cdot 10^8)^2}{3 \cdot 10^3}$$

$$\simeq 3 \cdot 10^{2+16-3}$$

$$\boxed{a \simeq 3 \cdot 10^{15} \text{ m/s}^2 = 3 \cdot 10^{14} g}$$

(v) a is $\perp$ to velocity. time dilitation causes:

$$a = \frac{a'}{\gamma^2} \qquad a' = \gamma^2 a = (10)^2 \cdot 3 \cdot 10^{15} \text{ m/s}^2$$

$$\boxed{a' = 3 \cdot 10^{17} \text{ m/s}^2}$$

would kill the inhabitants