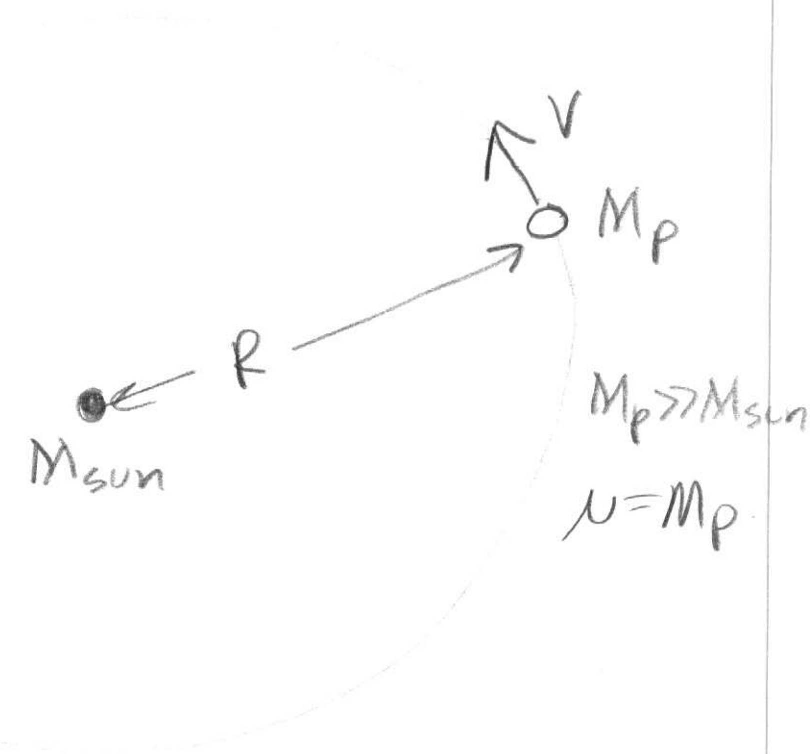
Slingshot Effect

First ... look at circular motion and ponder it ....

Planet about the sun ..

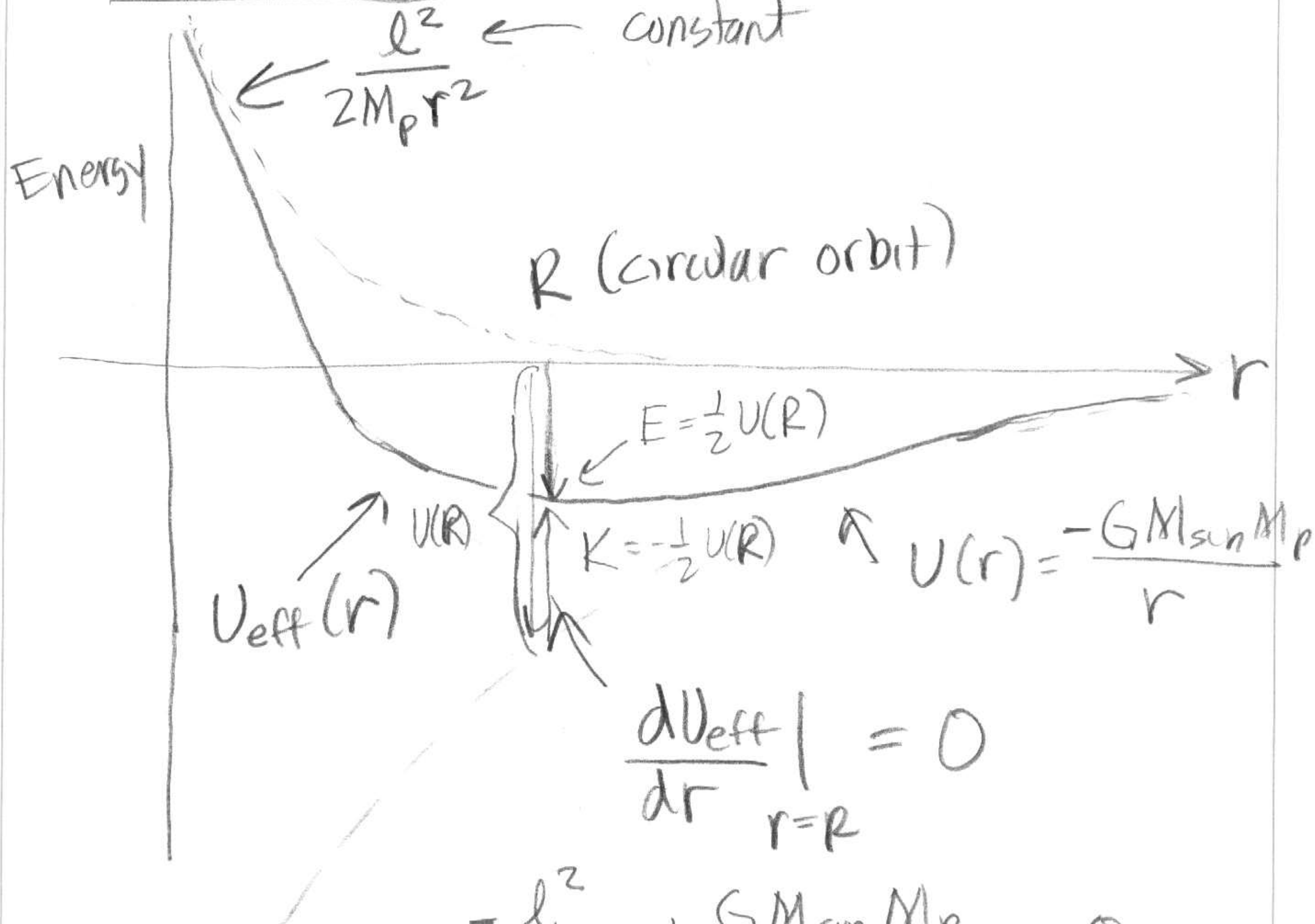


Simply: $$-M_p \cdot \frac{V^2}{R} = -G\frac{M_{sun} M_p}{R^2}$$

so, $$K = \frac{1}{2} M_p V^2 = \frac{1}{2} \cdot \underbrace{G\frac{M_{sun} M_p}{R}}_{-U(R)}$$

$$K = \frac{1}{2} \cdot (-U(R))$$

Visualize:



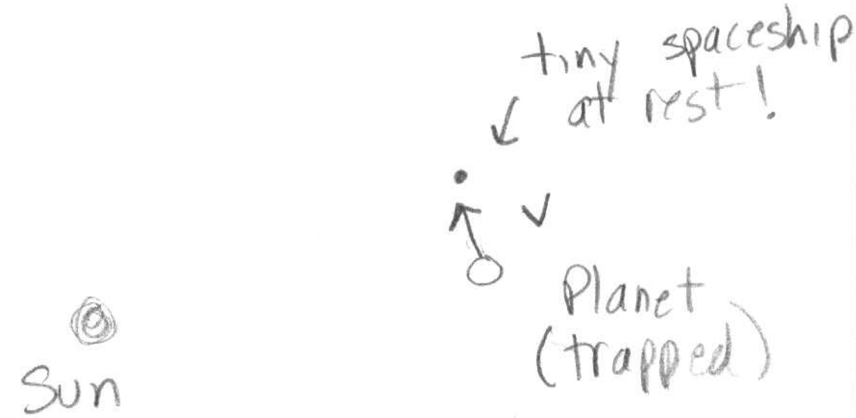
$$\left.\frac{dU_{eff}}{dr}\right|_{r=R} = 0$$

$$\frac{-l^2}{M_p R^3} + \frac{GM_{sun}M_p}{R^2} = 0$$

$$\frac{(M_p R v)^2}{M_p R^3} = \frac{GM_{sun}M_p}{R^2}$$

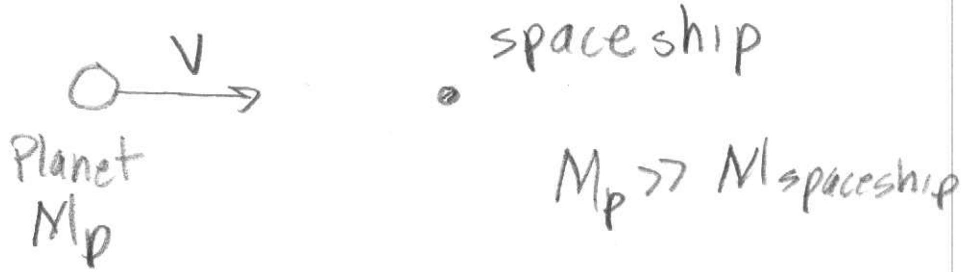
same as before! → $\frac{1}{2}M_p v^2 = \frac{1}{2}\frac{GM_{sun}M_p}{R}$

Second How to use a planet to escape the sun....



Collision

initial



final

(Best Case!)

spaceship... is it bound?

$$K_{spaceship} = \frac{1}{2} M_{spaceship} (2v)^2 = 4 \cdot \frac{1}{2} M_s v^2$$

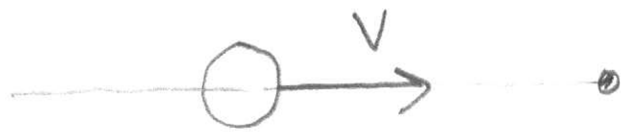
$$K_s = 4 \times \frac{1}{2} M_s v^2 = 4 \times \left(-\frac{1}{2} U(R)\right)$$

$$E = K_s + U(R) = -2U(R) + U(R) = -U(R)$$

$$E = -\left(-\frac{G M_s M_{sun}}{R}\right) = +\frac{G M_s M_{sun}}{R} > 0$$

$\Rightarrow$ spaceship can escape the sun!

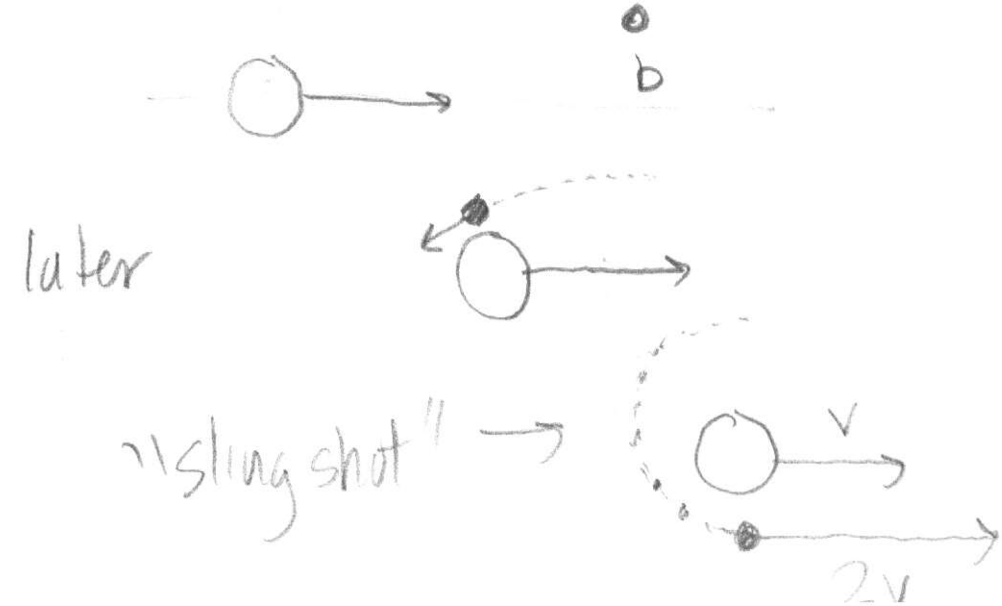
<u>HOW</u>:



aim for the center..

crash land!

Want to plunk yourself down off center a bit..



Full Solution

$$E = \frac{1}{2}\mu\dot{r}^2 + \frac{\ell^2}{2\mu r^2} + U(r) = \text{constant}$$

$$\dot{r} = \frac{dr}{dt} = \pm\left[\frac{2}{\mu}\left(E - \frac{\ell^2}{2\mu r^2} - U(r)\right)\right]^{1/2}$$

$$\ell = mr^2\dot{\theta} \quad \Rightarrow \quad \dot{\theta} = \frac{d\theta}{dt} = \frac{\ell}{\mu r^2}$$

constant

$$\frac{\frac{d\theta}{dt}}{\frac{dr}{dt}} = \frac{d\theta}{dr} = \frac{\frac{\ell}{\mu r^2}}{\pm\left[\frac{2}{\mu}\left(E - \frac{\ell^2}{2\mu r^2} - U(r)\right)\right]^{1/2}}$$

$$\frac{d\theta}{dr} = \frac{\ell}{\pm r\left[\mu^2 r^2 \cdot \frac{2}{\mu}\left(E - \frac{\ell^2}{2\mu r^2} - U(r)\right)\right]^{1/2}}$$

$$= \pm \frac{\ell}{r\left[2\mu r^2\left(E - \frac{\ell^2}{2\mu r^2} - U(r)\right)\right]^{1/2}}$$

$U(r) = -\frac{C}{r}$

$$\frac{d\theta}{dr} = \pm \frac{\ell}{r\left[(2\mu E)r^2 + (2\mu C)r - \ell^2\right]^{1/2}}$$

$\pm$ / clockwise/counterclockwise.