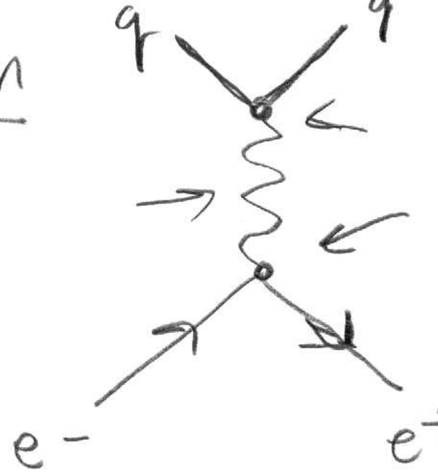
The Z^0

Nature of couplings (focus on annihilation)

Photon



$$\mathcal{M} \propto \frac{\sqrt{\alpha} \cdot \sqrt{\alpha} \cdot e_q}{Q^2}$$

Taking Spin into account:

same
=

$e^-_R e^+_L$ "Right Handed"

$e^-_L e^+_R$ "Left handed"

Z^0 not the same

$$C_R \neq C_L$$

Z^0	ν_e, ν_μ, ν_τ	e, μ, τ	u, c, t	d, s, b
C_L	1	$-1+2\sin^2\theta_W$ -0.538	$1-\frac{4}{3}\sin^2\theta_W$ 1.31	$-1+\frac{2}{3}\sin^2\theta_W$ -0.85
C_R	0	$2\sin^2\theta_W$ $.462$	$-\frac{4}{3}\sin^2\theta_W$ -0.31	$\frac{2}{3}\sin^2\theta_W$ 0.15

For $\bar{e}, \bar{u}, \bar{d}$ etc: $\overline{C_L} = C_R$, $\overline{C_R} = C_L$

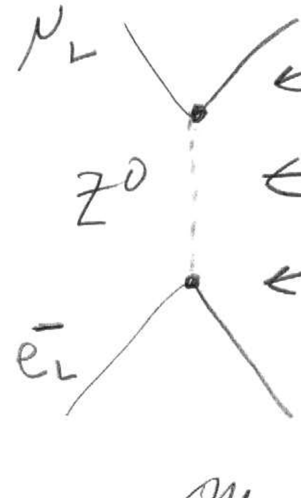
Like the W... Left handed ignored

θ_W is the Weak or Weinberg Mixing Angle

Expresses "Mixing" between $\sin^2\theta_W \simeq 0.231$

Parity Violating (only C_L, $C_R = 0$)

Parity Conserving



C_L

$\leftarrow -1+2\sin^2\theta_W$

$\leftarrow$ propagator

$\leftarrow -1+2\sin^2\theta_W$

C_L

historical

$$G_F = 1.17\cdot 10^{-5}\frac{1}{GeV^2}$$

$$\frac{\overbrace{\sqrt{2}\,G_F}\,M_Z^2}{\underbrace{q^2 - M_Z^2}\,(+iM_Z\Gamma_Z)}$$

neglect

as $q^2 \ll M_Z^2$

$\simeq \sqrt{2}\,G_F$

$$\mathcal{M} = C_L^2\,\frac{\sqrt{2}\,G_F M_Z^2}{E^2 - M_Z^2}$$

neglect Z^0 width

center of mass

Recalling

$$e_L^- e_R^+ \to \gamma^* \to \mu_L^- \mu_R^+ \qquad \mathcal{M} = \frac{4\pi\alpha}{E^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\frac{\alpha^2}{E^2}\left| 1 + C_L^2 \frac{\sqrt{2}\, G_F E^2 M_Z^2}{4\pi\alpha(E^2 - M_Z^2)}\right|^2 (1+\cos\theta^*)^2$$

in case full $iM_Z\Gamma_Z$ included

$$e_L^- e_R^+ \to \mu_R^- \mu_L^+ \Leftarrow \text{final state}$$

"WRONG" Handedness

e.m doesn't care.

one $C_L \to C_R$

& $1+\cos\theta^* \to 1-\cos\theta^*$

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\frac{\alpha^2}{E^2}\left| \underbrace{1 + C_R C_L \frac{\sqrt{2}\, G_F E^2 M_Z^2}{4\pi\alpha(E^2 - M_Z^2)}}_{\text{main thing}}\right|^2 (1-\cos\theta^*)^2$$

$C_R \neq C_L$ messes up cancellation between $+2\cos\theta^*$ and $-2\cos$

must be careful..

$$e_R^- e_L^+ \to \mu_L^- \mu_R^+ \left| 1 + C_R C_L (\quad) \right|^2 (1-\cos\theta^*)^2$$

$\Uparrow \qquad \Downarrow$

$\Uparrow \qquad \Downarrow$

$$e^-_R e^+_L \to \mu^-_R \mu^+_L \qquad \left|1 + c_R^2(\ )\right|^2 (1+\cos\theta^*)^2$$

coefficient of $\cos\theta^*$ term ..

$$\frac{1}{2} \cdot \frac{1}{4}\frac{\alpha^2}{E^2}\left[\underline{-2(C_L^2 - 2C_R C_L + C_R^2)\frac{\sqrt{2}\, G_F E^2 M_Z^2}{\alpha(E^2 - M_Z^2)}} + (C_L^4 - 2C_L^2 C_R^2 + C_R^4)\left| \quad '' \quad \right|^2 \right]$$

↑ average

Key point: • dominates at low energy

$$\alpha \; -\underbrace{(C_L - C_R)^2}_{-1} \frac{G_F E^2}{\alpha}$$

$$\frac{G_F E^2}{\alpha} = 1 \quad ?$$

$$E^2 \simeq \frac{\alpha}{G_F}$$

$$E \simeq \sqrt{\frac{1}{137}} \frac{1}{\sqrt{10^{-5}}} \simeq 25 \text{ GeV}$$

Two accelerators devoted to this in the 1970's

Traditional Parameterization

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left[A_0(1+\cos^2\theta^*) + A_1\cos\theta\right]$$

$$A_1 = (C_L - C_R)^2\, \mathrm{Re}(r(E)) + \frac{1}{2}(C_L^2 - C_R^2)^2 |r(E)|^2$$

$$r(E) = \frac{1}{4\pi}\frac{\sqrt{2}\, G_F M_Z^2}{E^2 - M_Z^2 + iM_Z\Gamma_Z}\frac{E^2}{\alpha}$$ (re-introduced

$$A_0 = 1 + \frac{1}{2}\mathrm{Re}(r)(C_L + C_R)^2 + \frac{1}{4}|r|^2(C_L^2 + C_R^2)^2$$

$$F = \int_0^1 \frac{d\sigma}{d\Omega}d\Omega = \frac{2\pi\alpha^2}{4s}\left[A_0\left(1+\frac{1}{3}\right) + A_1\cdot\frac{1}{2}\right]$$

$$B = \int_{-1}^0 \frac{d\sigma}{d\Omega}d\Omega = \frac{2\pi\alpha^2}{4s}\left[A_0\left(1+\frac{1}{3}\right) - A_1\cdot\frac{1}{2}\right]$$

$$\frac{F-B}{F+B} = \frac{A_1}{\frac{8}{3}A_0} = \frac{3}{8}\cdot\frac{A_1}{A_0}$$

Most interesting these days...

$$E \gg M_Z$$

$$\underset{E\to\infty}{r(E)} = \frac{\sqrt{2}\, G_F M_Z^2}{4\pi\alpha} = \frac{1}{4\sin^2\theta_W \cos^2\theta_W}$$

$\uparrow$ Electroweak Unification $= 1.41$ $(\sqrt{2}\,?)$

$$e_L^- e_R^+ \to \gamma^* \to \mu_L^- \mu_R^+$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\frac{\alpha^2}{E^2}\left|1 + \underbrace{(-1+2\sin^2\theta_W)}_{\substack{\sin^2\theta_W - \cos^2\theta_W \\ -\cos 2\theta_W}}{}^2 \underbrace{\frac{1}{4\sin^2\theta_W\cos^2\theta_W}}_{\sin^2 2\theta_W}\right|^2 (1+\cos\theta^*)^2$$

$$= \frac{1}{4}\frac{\alpha^2}{E^2}\left|1 + \frac{\cos^2 2\theta_W}{\sin^2 2\theta_W}\right|^2 (1+\cos\theta^*)^2$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{1}{4}\frac{\alpha^2}{E^2}\frac{1}{\sin^4 2\theta_W}(1+\cos\theta^*)^2}$$

$$e_L^- e_R^+ \to \mu_R^- \mu_L^+ \quad (e_R^- e_L^+ \to \mu_L^- \mu_R^+)$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\frac{\alpha^2}{E^2}\left|1 + \underbrace{(-1+2\sin^2\theta_W)}_{\sin^2\theta_W - \cos^2\theta_W}\cancel{(2\sin^2\theta_W)}\frac{1}{\underset{2}{\cancel{4\sin^2\theta_W}}\cos^2\theta_W}\right|^2 (1-\cos\theta^*)^2$$

$$= \frac{1}{4}\frac{\alpha^2}{E^2}\left|1 + \frac{\sin^2\theta_W - \cos^2\theta_W}{2\cos^2\theta_W}\right|^2 (1-\cos\theta^*)^2$$

$$= \frac{1}{4}\frac{\alpha^2}{E^2}\left|\frac{1}{2}\left(1 + \frac{\sin^2\theta_W}{\cos^2\theta_W}\right)\right|^2 (1-\cos\theta^*)^2$$

$$= \frac{1}{4}\frac{\alpha^2}{E^2}\,\frac{1}{4}\,\frac{1}{\cos^4\theta_W}\left(1-\cos\theta^*\right)^2$$

$$e^-_R\, e^+_L \to \mu^-_R\, \mu^+_L$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\frac{\alpha^2}{E^2}\left|1 + \frac{4\sin^4\theta_W}{4\sin^2\theta_W\cos^2\theta_W}\right|^2\left(1+\cos\theta^*\right)^2$$

$$= \frac{1}{4}\frac{\alpha^2}{E^2}\left|1 + \frac{\sin^2\theta_W}{\cos^2\theta_W}\right|^2\left(1+\cos\theta^*\right)^2$$

$$= \frac{1}{4}\frac{\alpha^2}{E^2}\,\frac{1}{\cos^4\theta_W}\left(1+\cos\theta^*\right)^2$$

---

unpolarized ...

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\cdot\frac{1}{4}\frac{\alpha^2}{E^2}\left[\left(\frac{1}{\sin^4 2\theta_W} + \frac{1}{\cos^4\theta_W}\right)\left(1+\cos\theta^*\right)^2 + 2\cdot\frac{1}{4}\,\frac{1}{\cos^4\theta_W}\left(1-\cos\theta^*\right)^2\right]$$

$$= \frac{1}{4}\frac{\alpha^2}{E^2}\left[\frac{1}{2}\overbrace{\left(\frac{1}{\sin^4 2\theta_W} + \frac{3}{2\cos^2\theta_W}\right)}^{A_2}\left(1+\cos^2\theta^*\right) + \overbrace{\left(\frac{1}{\sin^4 2\theta_W} + \frac{1}{2\cos^2\theta_W}\right)}^{A_1}\cos\theta^*\right]$$

2.26 > 1 (pure QED)

2.83

$$A_{FB} = \frac{3}{8}\cdot\frac{A_1}{A_2} = \frac{3}{8}\,\frac{2.83}{2.26} = \underline{0.47}$$