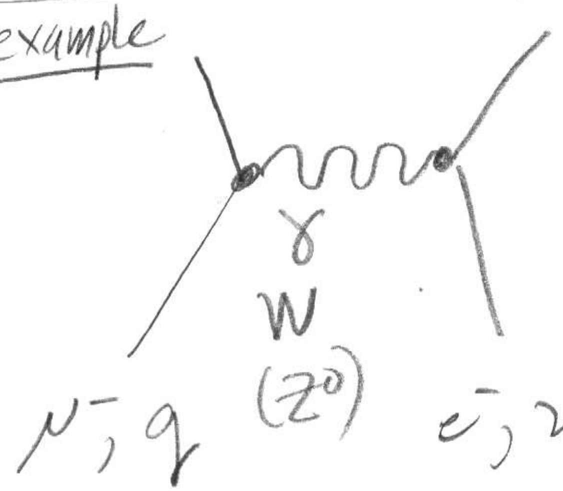
To go further, must understand parton-parton cross sections

example



Let's now always work in CM frame.

$$a + b \rightarrow c + d$$

$$m_a \; m_b \rightarrow m_c \; m_d$$

elastic: $m_a = m_c$, $m_b = m_d$, etc.

Golden Rule:

$$d\sigma = \frac{2\pi}{\hbar v_i} |M_{if}|^2 \rho_f$$

$\rho_f \rightarrow$ density of final states

ultrarelativistic limit: $v_i = c = 1$ (relative)

$$\rho_f = \frac{p_f^2 \, d\Omega_f}{(2\pi\hbar)^3} \frac{dp_f}{dE} \qquad E = \text{total energy}$$

$\hbar = 1$ $\qquad = \sqrt{s}$ c.m

$$p_f = E/2 \quad \text{ultrarelativistic}$$

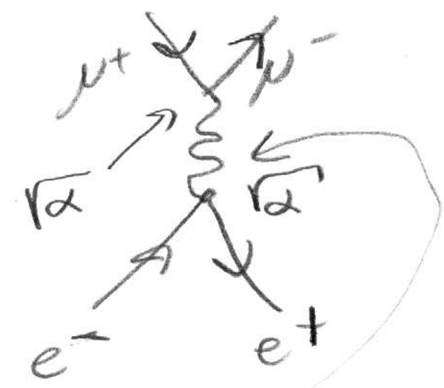
$$\frac{dp_f}{dE} = \frac{1}{2}$$

$$\rho_f = \frac{(E/2)^2 \, d\Omega}{(2\pi)^3} \cdot \frac{1}{2} = \frac{E^2}{2^6 \pi^3}$$

$$\frac{d\sigma}{d\Omega} = \frac{2\pi}{2^6 \pi^3} |\mathcal{M}_{if}|^2 E^2$$

$$= \frac{1}{2^5 \pi^2} |\mathcal{M}_{if}|^2 E^2$$

First look at $e^+e^- \to \mu^+\mu^-$



$$\mathcal{M}_{if} = \frac{4\pi\alpha}{E^2} \leftarrow q^2, \text{ really}$$

$$\boxed{\text{NEGLECTING ANGULAR DEPENDENCE}}$$

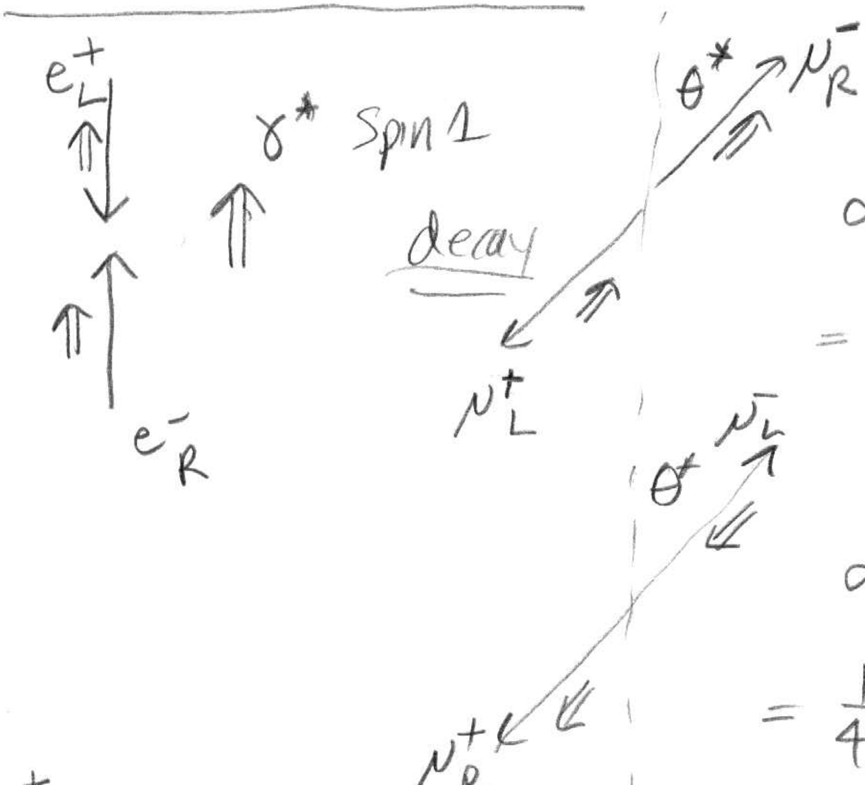
photon:
$(\sqrt{s}, 0, 0, 0)$
center of momentum

$q^2 = s = E^2$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \frac{\alpha^2}{E^4} E^2$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{1}{2} \frac{\alpha^2}{E^2}}$$

Massless Annihilation



$\propto |d^1_{11}(\theta^*)|^2$

$= \frac{1}{4}(1+\cos\theta^*)^2$

$\propto |d^1_{1-1}(\theta^*)|^2$

$= \frac{1}{4}|1-\cos\theta^*|^2$

sum $\frac{1}{2}(1+\cos^2\theta^*)$

another $\frac{1}{2}(1+\cos^2\theta^*)$

↑

2 of 4 states;

$\boxed{\frac{2}{4}} \cdot \cancel{2} \cdot \frac{1}{\cancel{2}}(1+\cos^2\theta^*)$

average over initial

sum over final factor of 2 is properly part of density of states

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{1}{4}\frac{\alpha^2}{E^2}(1+\cos^2\theta^*)}$$

CROSSING

$a + b \rightarrow c + d$? $\mathcal{M}_{if}$

$e^+ + e^- \rightarrow \tau^+ + \tau^-$

add antimatter to both sides!

(−4 momentum)

$\bar{b}$ $\bar{b}$

$a + (b\bar{b}) \rightarrow \bar{b} + c + d$

$a \rightarrow \bar{b} + c + d$ } same matrix element

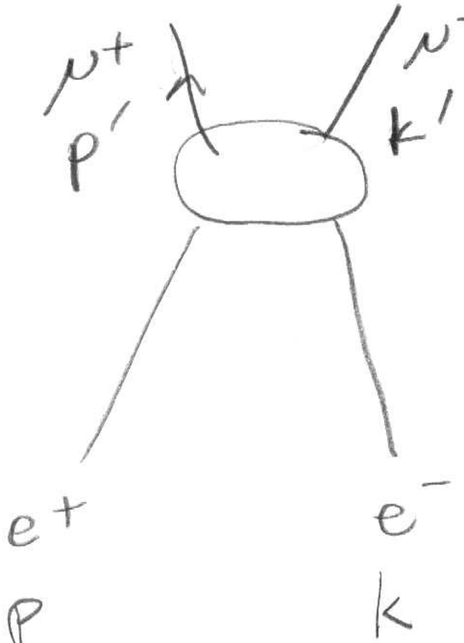
$a + \bar{c} \rightarrow \bar{b} + d$ same!

What is different?

$\Rightarrow$ Phase Space

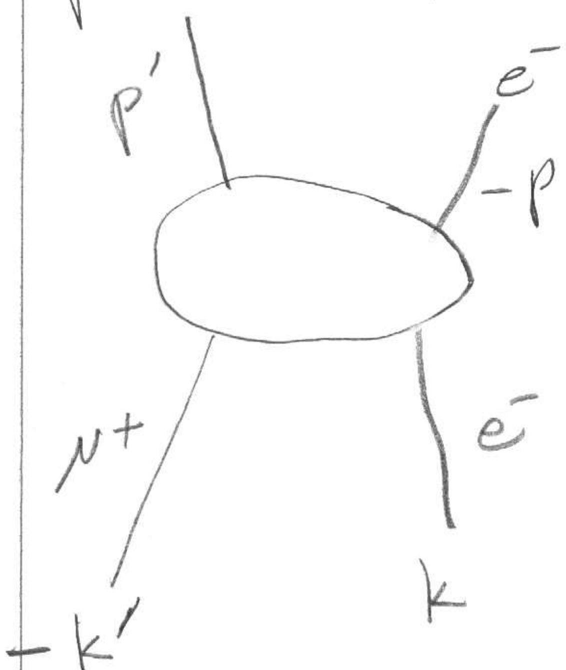
$\underbrace{\bar{c} + \bar{d}}_{\text{heavy}} \rightarrow \underbrace{\bar{a} + \bar{b}}_{\text{light}}$

$e^+e^- \to \mu^+\mu^-$



$$s = (p+k)^2$$
$$t = (k'-k)^2$$
$$u = (p'-k)^2$$

CROSS



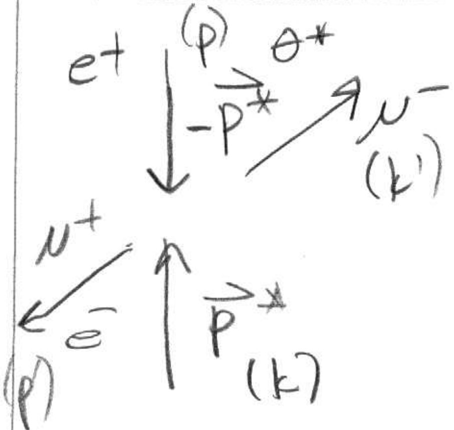
$$s_c = (-k'+k)^2 = t$$
$$t_c = (-p-k)^2$$
$$= (p+k)^2 = s$$
$$u_c = (p'-k)^2 = u$$

$e^-\mu^+ \to e^-\mu^+$

$$\mathcal{M}(e^+e^- \to \mu^+\mu^-) = f(s,t,u)$$

then $\mathcal{M}(e^-\mu^+ \to e^-\mu^+) = f(t,s,u)$

In center-of-mass frame



$$s = (p+k)^2$$
$$\simeq 2p\cdot k$$
$$\simeq 2(E^{*2} \underbrace{- \vec{p}\cdot\vec{k}}_{+|\vec{p}^*|^2}) \qquad |\vec{p}^*| = \frac{E}{2}$$

$$s \simeq E$$

$$t = (k'-k)^2 \simeq -2kk'$$
$$= -2(E^{*2} - p^{*2}\cos\theta^*)$$
$$= -E\tfrac{1}{2}(1-\cos\theta^*) = -E\sin^2\theta^*/2$$

$$u = (p'-k)^2 = -2p'k$$
$$= -E\tfrac{1}{2}(1+\cos\theta^*) = -E\cos^2\theta^*/2$$

<u>note</u>: $$s+t+u = E - E(\sin^2\theta^*/2 + \cos^2\theta^*/2)$$
$$= 0$$

$$t^2+u^2 = \frac{1}{4}E^2(1-2\cos\theta^* + \cos^2\theta^* + 1 + 2\cos\theta^* + \cos^2\theta^*)$$

$$t^2+u^2 = \tfrac{1}{2}E^2(1+\cos^2\theta^*)$$

$$\frac{t^2+u^2}{s^2} = \frac{1}{2}(1+\cos^2\theta^*)$$

$e^+e^- \to \mu^+\mu^-$: "Dimensionless Piece"

$$\frac{d\sigma}{d\Omega} = \underbrace{\frac{1}{2}\frac{\alpha^2}{E^2}}_{\text{dimensions}} \underbrace{\left(\frac{t^2+u^2}{s^2}\right)}$$

$e^-\mu^+ \to e^-\mu^+$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\frac{\alpha^2}{E^2}\left(\frac{t_c^2+u_c^2}{s_c^2}\right)$$

$$s_c = t \qquad t_c = s \qquad u_c = u$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\frac{\alpha^2}{E^2}\left(\frac{s^2+u^2}{t^2}\right)$$

$$= \frac{1}{2}\frac{\alpha^2}{E^2}\left(\frac{E^2 + E^2\cos^4\frac{\theta^*}{2}}{E^2\sin^4\theta^*/2}\right)$$

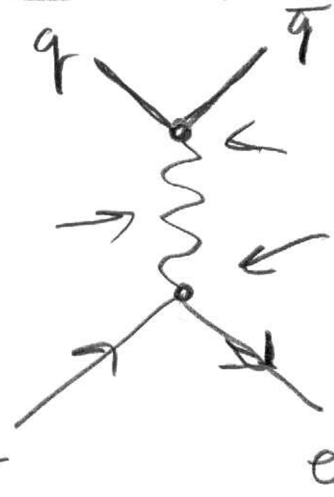
$$= \frac{1}{2}\frac{\alpha^2}{E^2}\left(\frac{1+\cos^4\theta^*/2}{\sin^4\theta^*/2}\right)$$

$$\propto \frac{(1+(1-y)^2)}{Q^4}$$

The Z^0

Nature of couplings (focos on annihillation)

Photon



$$\mathcal{M} \propto \frac{\sqrt{\alpha}\cdot\sqrt{\alpha}\cdot e_q}{Q^2}$$

Taking Spin into account:

same

$e^-_R e^+_L$ "Right Handed"

$e^-_L e^+_R$ "Left handed"

Z^0 not the same

$c_R \neq$

Z^0	ν_e, ν_μ, ν_τ	e, μ, τ	u, c, t	d, s, b
C_L	1	$-1+2\sin^2\theta_W$	$1-\frac{4}{3}\sin^2\theta_W$	$-1+\frac{2}{3}\sin^2\theta_W$
C_R	0	$2\sin^2\theta_W$	$-\frac{4}{3}\sin^2\theta_W$	$\frac{2}{3}\sin^2\theta_W$

For $\bar{e}, \bar{u}, \bar{d}$ etc: $\bar{C}_L = C_R$, $\bar{C}_R = C_L$

Like the W... Left handed ignored

θ_W is the Weak or Weinberg Mixing Angle

Expresses "Mixing" between

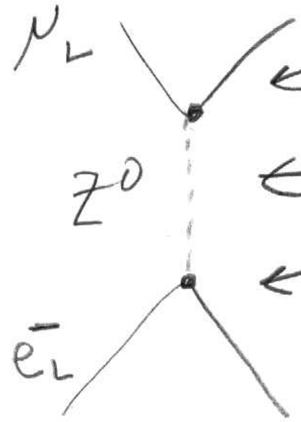
$\sin^2\theta_W \simeq 0.231$

Parity Violating (only C_L, $C_R = 0$)

Parity Conserving

historical

$G_F = 1.17 \cdot 10^{-5} \frac{1}{\text{GeV}^2}$



C_L: $-1+2\sin^2\theta_W$

propagator $\frac{\sqrt{2}\, G_F M_Z^2}{Q^2 - M_Z^2} \times 4\pi$

$-1+2\sin^2\theta_W$ C_L

as $Q^2 \ll M_Z^2$

$\simeq 4\pi \sqrt{2}\, G_F$

$$\mathcal{M} = C_L^2 \frac{\sqrt{2}\, G_F M_Z^2}{E^2 - M_Z^2} \times 4\pi$$

neglect Z^0 width

center of mass

Recalling

$$e^-_L e^+_R \to \gamma^* \to \mu^-_L \mu^+_R \qquad \mathcal{M} = \frac{4\pi\alpha}{E^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\frac{\alpha^2}{E^2}\left[1 + c_L^2 \frac{\sqrt{2}\, G_F E^2 M_Z^2}{\alpha(E^2 - M_Z^2)}\right]^2 (1+\cos\theta^*)^2$$

$$e^-_L e^+_R \to \mu^-_R \mu^+_L$$ ← final state "WRONG" Handedness

e.m doesn't care.

one $c_L \to c_R$

& $1+\cos\theta^* \to 1-\cos\theta^*$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\frac{\alpha^2}{E^2}\left[1 + \underbrace{c_R c_L \frac{\sqrt{2}\, G_F E^2 M_Z^2}{\alpha(E^2 - M_Z^2)}}_{\text{main Thing}}\right]^2 (1-\cos\theta^*)^2$$

$c_R \neq c_L$ messes up cancellation between $+2\cos\theta^*$ and $-2\cos$

must be careful..

$$e^-_R e^+_L \to \mu^-_L \mu^+_R \quad \left| 1 + c_R c_L(\quad) \right|^2 (1-\cos\theta^*)^2$$

$\Uparrow$ $\Downarrow$

$\uparrow$ $\downarrow$

$e^-_R e^+_L \to \mu^-_R \mu^+_L \qquad \left|1 + c_R^2(\ )\right|^2 (1+\cos\theta^*)^2$

coefficient of $\cos\theta^*$ term ..

$$\frac{1}{2}\frac{\alpha^2}{E^2}\left[\underline{-2(C_L^2 - 2C_R C_L + C_R^2)\frac{\sqrt{2}\,G_F E^2 M_Z^2}{E^2 - M_Z^2}}\right.$$

$$\left. + (C_L^4 - 2C_L^2 C_R^2 + C_R^4)\left(\quad '' \quad\right)^2\right]$$

Key point

$$\propto -\underbrace{(C_L - C_R)^2}\ \frac{G_F E^2}{\alpha}$$

$$-1\ \frac{G_F E^2}{\alpha} = -0.01\ ?$$

$$E^2 \simeq \frac{\alpha}{G_F}$$

$$E \simeq \sqrt{\frac{1}{137}}\ \frac{1}{\sqrt{10^{-5}}} \simeq 25\ \text{GeV}$$

Two accelerators devoted to this in the 1970's

$$A_{FB} \equiv \frac{F - B}{F + B}$$

where F : # between $0 < \cos\theta < 1$

B # between $-1 < \cos\theta < 0$

Plots : #1 from PETRA

then : #2 $e^+e^- \to q\bar{q}$

sum over all types.

as a function of beam energy.

#3 $A_{FB}(E)$