Probability of parton having momentum fraction x $\Rightarrow f_i(x)$ $\quad i = u, d, \bar{u}, \bar{d}, s, \bar{s}$, glue

$$\int_0^1 x P \sum_i f_i(x)\, dx = P$$

$$\int_0^1 x \sum f_i(x)\, dx = 1 \qquad \text{(momentum).}$$

$$\frac{d\sigma}{d\Omega} \propto \sum_i e_i^2\, x \sum f_i(x)\, dx$$

$\uparrow$ elastic scattering

$e_u = +\frac{2}{3} e \qquad e_g = 0!$

$e_d = -\frac{1}{3} e = e_s$

<u>Tricky</u>: $dE' d\Omega \propto dx\,dy$

s: <u>Mandelstam Variable</u>

$$s = \underbrace{(p+k)^2}_{\text{4-vector dot product}} \Leftarrow \begin{array}{l} p \text{ initial proton} \\ k \text{ initial electron} \end{array}$$

$= \mathcal{M}^2 \qquad \mathcal{M}$ = maximum mass attainable in collision

$2\cos^2\theta^*/2 + \sin^4\theta^*/2$

$1 + \cos^4\theta^*/2$

$$\left(\frac{d\sigma}{dx\,dy}\right)_{ep\to eX} = \frac{2\pi\alpha^2}{Q^4}\, s \underbrace{\left[1+(1-y)^2\right]}_{\text{angular dependence}} \sum e_i^2 x f_i(x)$$

photon propagator

$y \sim \frac{1}{2}(1-\cos\theta^*)$

$(1-y)^2 \sim \frac{1}{4}(1+\cos\theta^*)^2$

$\sim \cos^4\theta^*/2$

$$\frac{2\pi\alpha^2\, 4p^{*2}\left[2\cos^2\frac{\theta^*}{2}+\sin^4\frac{\theta^*}{2}\right]}{16\, p^{*4}\sin^4\frac{\theta^*}{2}} \times \sum e_i^2 x f_i(x)$$

$$\left(\frac{d\sigma}{dx\,dy}\right)_{ep\to eX} = \frac{\pi}{2}\frac{\alpha^2}{p^{*4}\sin^4\frac{\theta^*}{2}}\left[2\cos^2\frac{\theta^*}{2}+\sin^4\frac{\theta^*}{2}\right]\sum e_i^2 x f_i(x)$$

In this picture, NO Q^2 dependence

$\Rightarrow$ no form factor

$\Rightarrow$ no new interaction

$f_u(x) \to$ called $u(x)$

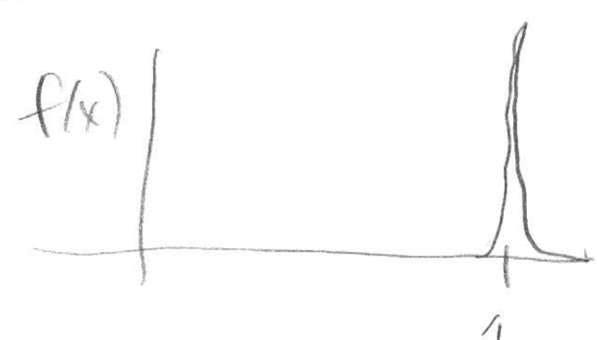
$f_d(x) \to$ called $d(x)$

$f_{\bar{u}}(x) \to$ called $\bar{u}(x)$

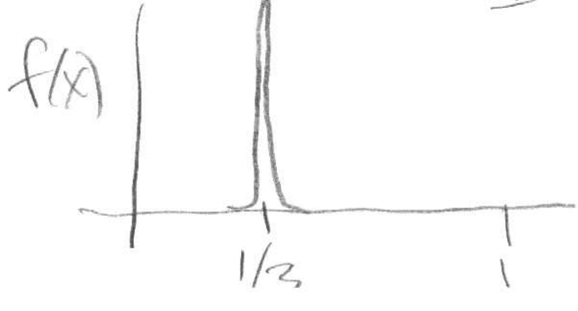
$f_{\bar{d}}(x) \to$ called $\bar{d}(x)$ etc.

Qualitative Idea

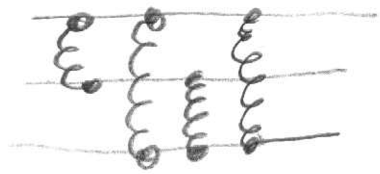
point proton

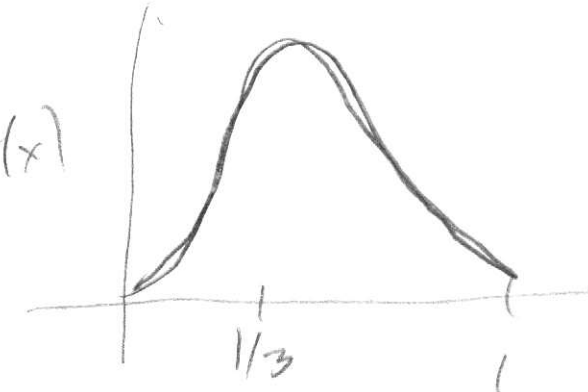


3 valence quarks (non interacting) $\equiv$

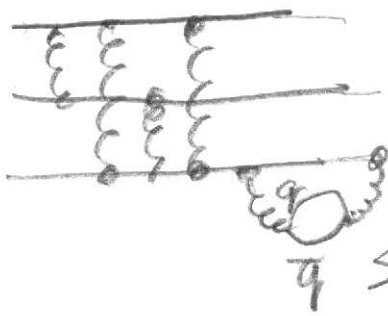


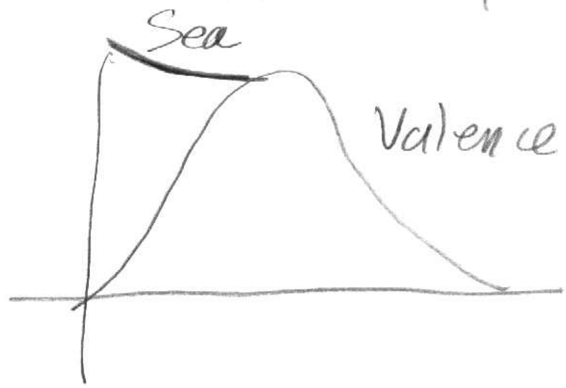
3 interacting valence quarks





QFT





Key point: compare results for proton + neutron .. all valence?

$$\left(\frac{d\sigma}{dx\,dy}\right)_{ep\to eX} \propto 2\times\left(\frac{2}{3}\right)^2 + 1\cdot\left(\frac{1}{3}\right)^2 = \frac{8}{9} + \frac{1}{9}$$
$$= 1$$

$$en\to eX \propto \left(\frac{2}{3}\right)^2 + 2\cdot\left(\frac{1}{3}\right)^2 = \frac{4}{9} + \frac{2}{9}$$
$$= \frac{2}{3}$$

expect $\frac{\text{neutron}}{\text{proton}} = \frac{2}{3}$.

Figure: true at $x \sim \frac{1}{3}$

But: at low x, $\frac{\text{neutron}}{\text{proton}} = 1$

WHY? → "sea" identical.

at high x, $\frac{\text{neutron}}{\text{proton}} \to \frac{1}{4}$!

Interpretation: high-x, one quark gets "antsy," steals the momentum

p → an up quark

n → a down quark

$$\frac{\text{neutron}}{\text{proton}} \simeq \frac{(1/3)^2}{(2/3)^2} = 1/4$$

High x → short distance

Perhaps

neutron proton

Current Structure Functions

(figure)

<u>Shock</u>: $\int x(u+\bar{u}+d+\bar{d}+s+\bar{s})\,dx$

$\cong 54\%$ of proton's momentum

$\simeq 46\%$ carried by gluons

Suprisingly large fraction

$s+\bar{s}$! Important in dark matter

<u>Cross-Check</u>: $e^- N$ scattering and $\nu/\bar{\nu}$ N scatterings

N: "isoscalar" target (^{56}Fe: $\begin{matrix}26\ p \\ 30\ n\end{matrix}$)

= equal parts p + n

$e^- p$: $\frac{d\sigma_p}{dx\,dy} \propto \left(\frac{2}{3}\right)^2\left[u(x)+\bar{u}(x)\right] + \left(\frac{1}{3}\right)^2\left(d(x)+\bar{d}(x) + s(x)+\bar{s}(x)\right)$

<u>note</u> $\int(u(x)-\bar{u}(x))\,dx = 2$

$\int(d(x)-\bar{d}(x))\,dx = 1$

$c, \bar{c}, b, \bar{b}, t, \bar{t} \rightarrow$ neglect

$\underline{e^- n}$: $\quad u_n(x) = d_p(x) \qquad d_n(x) = u_p(x)$
$\qquad\qquad = d(x) \qquad\qquad\quad = u(x)$

$$e^- n \Rightarrow \frac{d\sigma_n}{dx\,dy} = \left(\frac{2}{3}\right)^2 \left[u_n(x) + \bar{u}_n(x)\right] + \left(\frac{1}{3}\right)^2 \left[d_n(x) + \bar{d}_n(x) + s_n(x) + \bar{s}_n(x)\right]$$

$$= \left(\frac{2}{3}\right)^2 \left[d(x) + \bar{d}(x)\right] + \left(\frac{1}{3}\right)^2 \left[u(x) + \bar{u}(x) + s(x) + \bar{s}(x)\right]$$

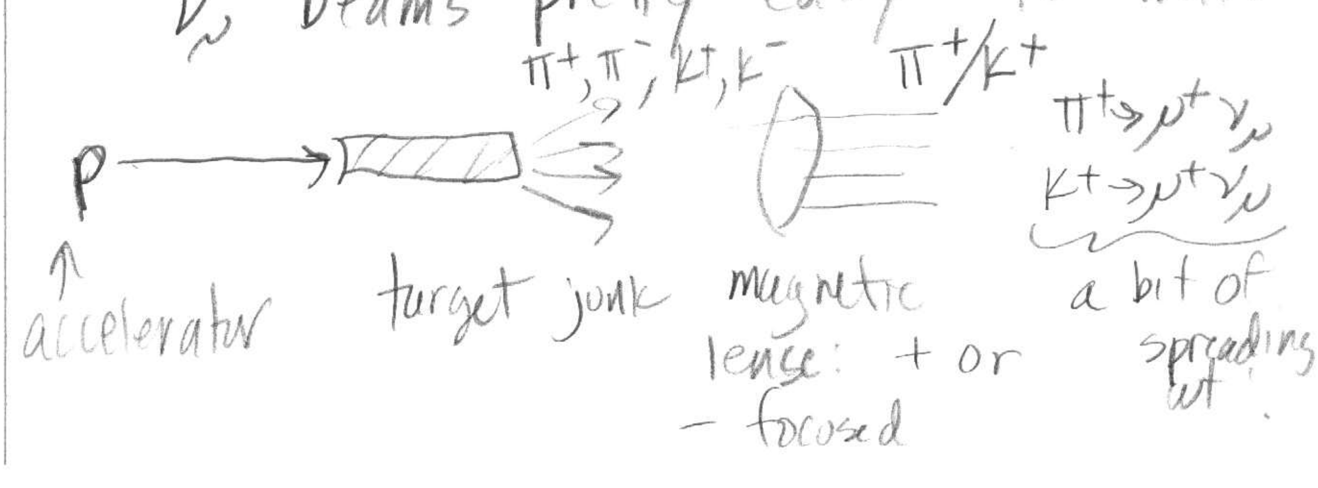
$\underline{e^- N}$:
$$\frac{d\sigma_N}{dx\,dy} = \frac{1}{2}\left(\frac{d\sigma_p}{dx\,dy} + \frac{d^2\sigma_n}{dx\,dy}\right)$$

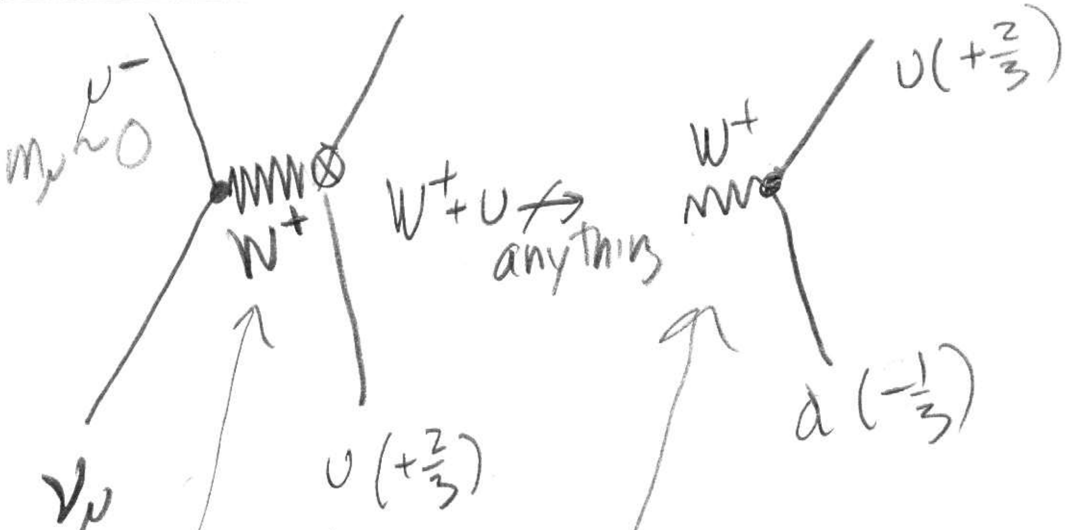
$$\frac{1}{2}\left(\frac{2}{3}\right)^2 + \frac{1}{2}\left(\frac{1}{3}\right)^2 = \frac{1}{18}\left[2^2+1\right] = \frac{5}{18}$$

$$\frac{d\sigma_N}{dx\,dy} \propto \frac{5}{18}\left(u(x) + \bar{u}(x) + d(x) + \bar{d}(x)\right) + \underbrace{\frac{1}{9}\left(s(x) + \bar{s}(x)\right)}_{\text{small}}$$

$\underline{(\nu_\mu / \bar{\nu}_\mu)\, N}$:

ν_μ beams pretty easy to make:



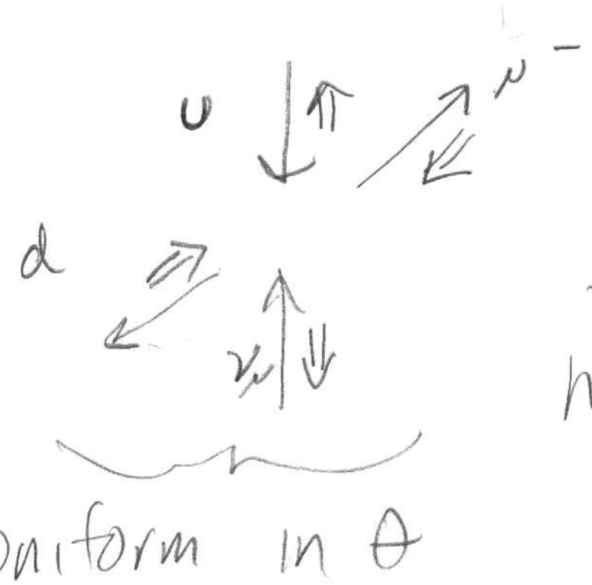


photon $\frac{e^4}{Q^4}$

$\underline{W}$: $\frac{f^4}{M_W^4}$ $\Leftarrow$ no $\frac{1}{\sin^4(\theta/2)}$!

f = fudge factor, related to G_F, some other $\underline{\text{stuff}}$.

<u>another comment</u>



ν_μ left handed.

$\nu_\mu + \bar{u} \rightarrow \mu^- \bar{d}$ goes

$\nu_\mu + \bar{d} \not\rightarrow$ no way

but:

right handed ANTIMATTER

left handed MATTER

$$\propto \left| d^{1}_{-1\,-1}(\theta) \right|^2 = \frac{1}{4}(1+\cos\theta^*)^2$$

$$y = \frac{1}{2}(1-\cos\theta^*)$$

$$1-y = \frac{1}{2}(1+\cos\theta^*)$$

$$\frac{d\sigma_{\nu p}}{dx\,dy} \propto d(x) + (1-y)^2\,\bar{u}(x)$$

$$\frac{d\sigma_{\nu p}}{dx} \propto d(x) + \underbrace{\int_0^1 dy\,(1-y)^2}_{\int_0^1 d\xi\,\xi^2 = \frac{1}{3}}\,\bar{u}(x)$$

$$\frac{d\sigma_{\nu p}}{dx} \propto d(x) + \frac{1}{3}\,\bar{u}(x)$$

$$\frac{d\sigma_{\nu N}}{dx} \propto \underset{Q(x)}{u(x)+d(x)} + \frac{1}{3}\underset{\overline{Q}(x)}{\left(\overline{u}(x)+\overline{d}(x)\right)}$$

$$\frac{d\sigma_{\overline{\nu} N}}{dx} \propto \underset{\overline{Q}(x)}{\overline{u}(x)+\overline{d}(x)} + \frac{1}{3}\underbrace{\left(u(x)+d(x)\right)}_{Q(x)}$$

$$\frac{\sigma_{\overline{\nu}N}}{\sigma_{\nu N}} = \frac{\overline{Q} + 1/3\, Q}{Q + \frac{1}{3}\overline{Q}} = \frac{1 + 3\,\overline{Q}/Q}{3 + \overline{Q}/Q}$$

= "R" in old ν physics.

Figure

$$\cong 0.45 \Rightarrow \frac{\overline{Q}}{Q} \simeq 0.15$$

a great result.

More, though!

can extract

$$u(x) + d(x) + \overline{u}(x) + \overline{d}(x)$$

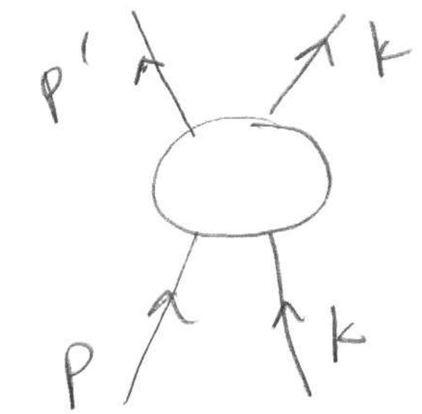
from $\overline{\nu}N$, νN cross sections

[also, $u(x) - \overline{u}(x) + d(x) - \overline{d}(x)$]

$eN \Rightarrow \frac{5}{18} \times$ that, $\boxed{\text{Figure}}$ (amazing too).

Mandelstam Variables

(4-momentum)



$s \equiv (p+k)^2$ square of c.m. energy

$t \equiv (k'-k)^2 = -Q^2$

$= (p-p')^2$

$(p+k = p'+k')^2$

$u \equiv (p'-k)^2 = (p-k')^2$

$m=0$ approximation ..

$s = 2pk \quad t = -2kk' \quad u = -2p'k$

$s+t+u = 2\left[pk - kk' - p'k\right]$

$p+k-k'$

$p'k = pk - kk'$

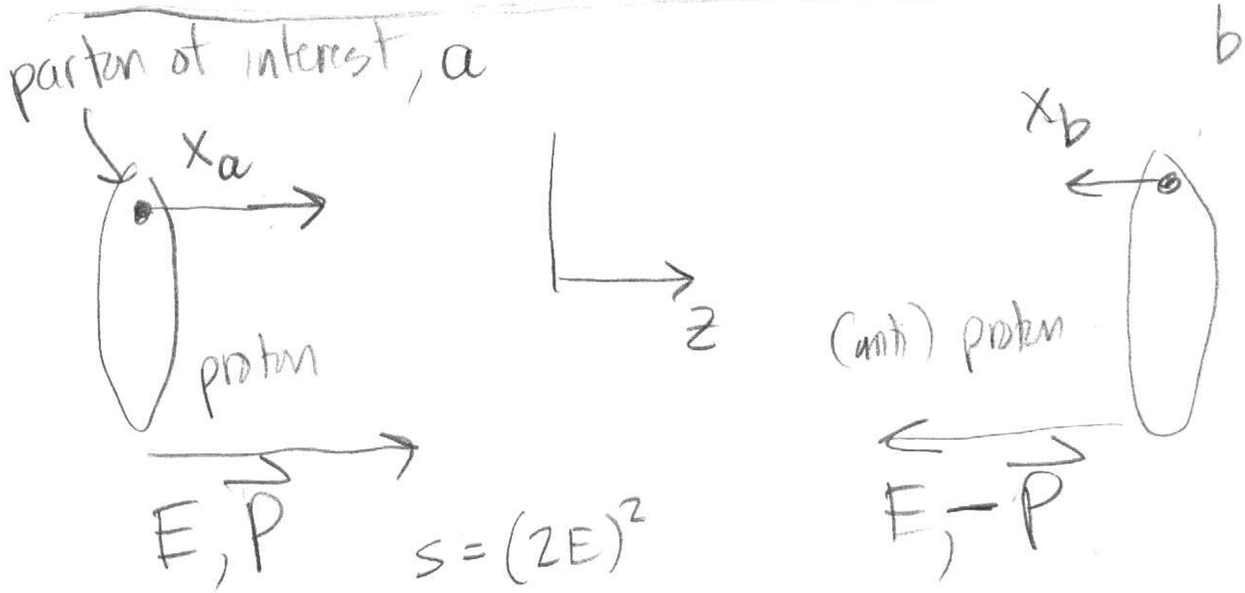
$= 2\left[pk - kk' - pk + kk'\right] = 0$

$s+t+u = 0$ ($m=0$ approx)

$= \sum_{i=1}^{4} m_i^2$ ($m \neq 0$)

→ only 2 independent variables

Proton - (anti proton) Proton Collisions



$s = (2E)^2$

a, b : $g, u, d, \bar{u}, \bar{d}, s, \bar{s}, \ldots$

$|\vec{p}_1| = |\vec{p}_2|$ but not for

$x_1\vec{p}_1$ + $x_2\vec{p}$

4-vectors

$\boxed{\underline{\text{parton}}\ a}$ $x_a E(1,0,0,1)$ b : $x_b E(1,0,0,-1)$

$$p_a + p_b = \left((x_a + x_b)E, 0, 0, (x_a - x_b)E\right)$$

$$\hat{s} = (p_a + p_b)^2 = \left[(x_a + x_b)^2 - (x_a - x_b)^2\right]E^2$$

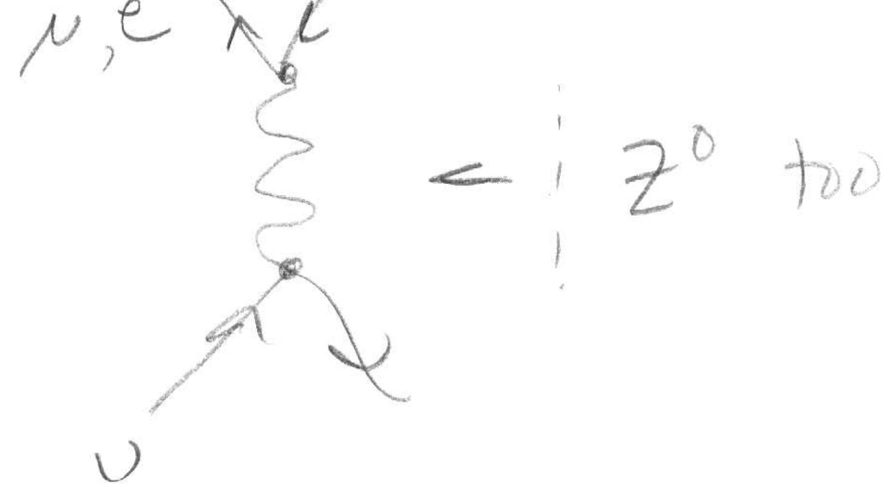
$$= 4x_a x_b E^2$$

$$\boxed{\hat{s} = x_a x_b\, s \quad \sqrt{\hat{s}} = \sqrt{x_a x_b}\,\sqrt{s}}$$

$\sigma(ab \to X)$ examples:

$d\bar{d},\ u\bar{u} \to \mu^+\mu^-$, e^+e^-

"Drell-Yan"



$gg \to \tilde{u}\,\tilde{u}$ (up-squarks)

$gg \to H^0$ (Higgs)

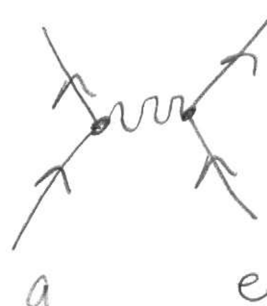
$$\sigma(p\overset{(-)}{p} \to X) \cong \iint dx_a\, dx_b\, \underbrace{f(x_a)\, f(x_b)}_{\text{"scaling approx"}}\, \sigma(ab \to X)$$

↑ for study, $C_X\, \delta(\sqrt{x_a x_b}\, s - \hat{s})$

(Figure). Q^2 dependence neglected

<u>To go further</u>, must understand <u>parton - parton cross sections</u>

We've talked about....



or W, Z^0

μ^-, q $\quad$ $e^-, (\nu)$

easiest to always work in center of mass frame.

$$a + b \rightarrow c + d$$

$$m_a \quad m_b \rightarrow \underbrace{m_c + m_d}$$

when $m_c \neq m_a$ or $m_b = m_d$

<u>Inelastic</u>

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi^2(\hbar^4)} |\mathcal{M}_{if}|^2 \frac{p_f^2}{v_i} \frac{dp_f}{dE_0}$$

<u>$\mathcal{M}_{if}$</u> : evaluate from

- Feynman Rules (simplified)
- Helicity conservation
- simplest angular momentum $\Rightarrow d'(\theta)$
- CROSSING

CROSSING

$$a + b \rightarrow c + d$$

Mif

$\bar{b}$ add antimatter to both sides! $\bar{b}$

(−4 momentum)

$$a + (b\bar{b}) \rightarrow \bar{b} + c + d$$

$$\left. a \rightarrow \bar{b} + c + d \right\}$$ same matrix element

$$a + \bar{c} \rightarrow \bar{b} + d$$ same!

What is different?

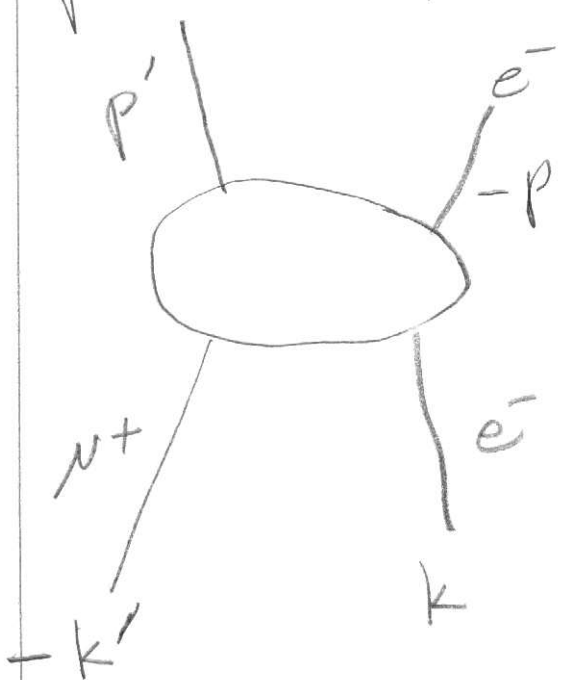
$\Rightarrow$ Phase Space

$$\underbrace{\bar{c} + \bar{d}}_{\text{heavy}} \rightarrow \underbrace{\bar{a} + \bar{b}}_{\text{light}}$$

$e^+e^- \to \mu^+\mu^-$

$$s = (p+k)^2$$
$$t = (k'-k)^2$$
$$u = (p'-k)^2$$

<u>CROSS</u>



$$s_c = (-k'+k)^2 = t$$
$$t_c = (-p-k)^2$$
$$= (p+k)^2 = s$$
$$u_c = (p'-k)^2 = u$$

$e^-\mu^+ \to e^-\mu^+$

$$\mathcal{M}(e^+e^- \to \mu^+\mu^-) = f(s,t,u)$$

then $\mathcal{M}(e^-\mu^+ \to e^-\mu^+) = f(t,s,u)$