Change of variables

$\boxed{x \equiv \frac{-q^2}{2M\nu}}$ $= 1$ when elastic collision.

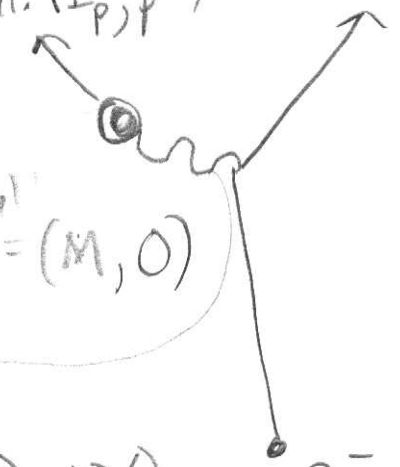
mass of proton

$\nu = E - E'$ … • E' much smaller when IN elastic.

• ν bigger

$x < 1$ when inelastic

$\boxed{y \equiv \frac{pq}{pk}}$ ⟸ 4-dot products



final: $(E'_p, \vec{p}\,')$

k' final $(E', \vec{k}\,')$

LAB FRAME

proton initial $p = (M, 0)$

e^- : k initial 4-momentum $(E, \vec{k})$

$q = (E - E', \vec{k} - \vec{k}\,')$

$pq = M(E - E') = M\nu$

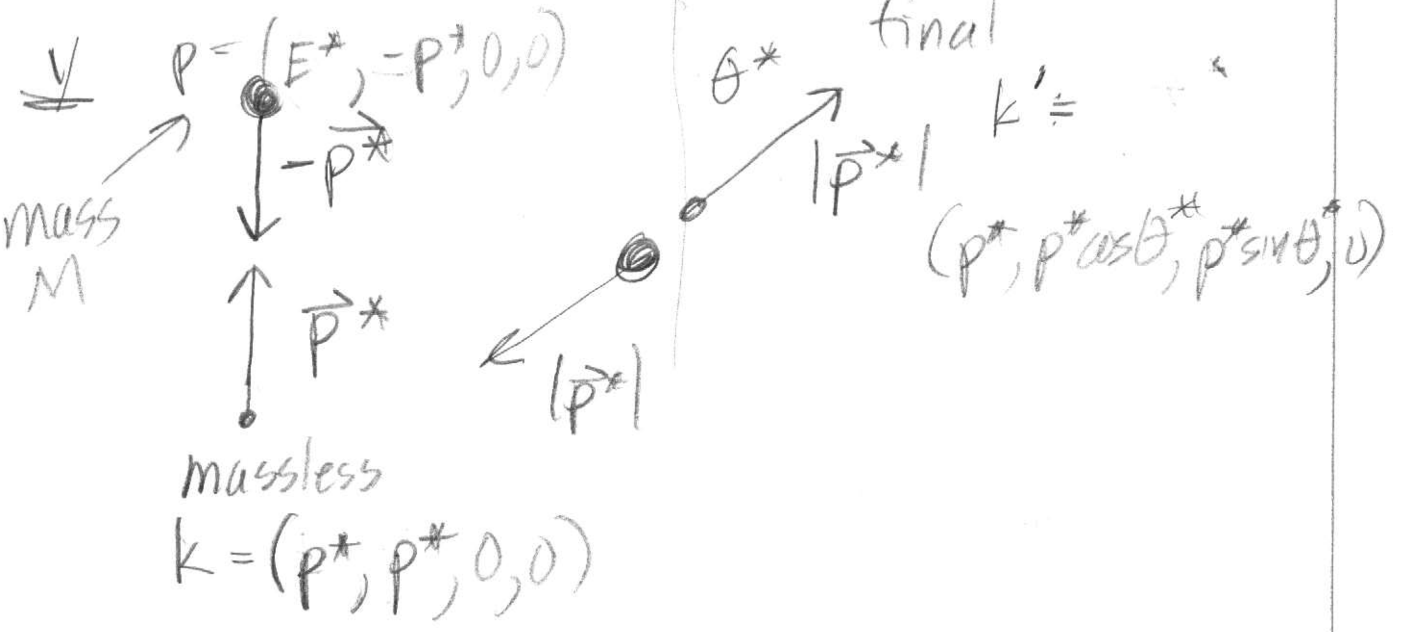
$pk = ME$

$y = \frac{M\nu}{ME} = \frac{\nu}{E} = \frac{E - E'}{E}$

$= 1 - \frac{E'}{E}$ } fractional energy loss

Relationship to Center of Mass-Frame Quantities

(Homework)



$$y = \frac{pq}{pk} = \frac{pk}{pk} - \frac{pk'}{pk} = 1 - \frac{\cancel{p^*}(E^* + p^*\cos\theta)}{\cancel{p^*}(E^* + p^*)}$$

$$= \frac{p^*}{E^* + p^*}(1 - \cos\theta^*)$$

$$\underset{\substack{\uparrow \\ \cos\theta^* = 1}}{0} < y < \frac{2p^*}{E^* + p^*} < 1, \ = 1 \text{ when } p^* \gg M$$

y is related to normalized energy loss and .. $\sin^2\theta^*/2$ or $1 - \cos\theta^*$.

<u>Angular</u>, <u>not collision energy</u>

Physically, what is x ?

Naive quark-parton model

Idea... super-simple carving up of the proton

"parton" → has mass $m = \xi M$

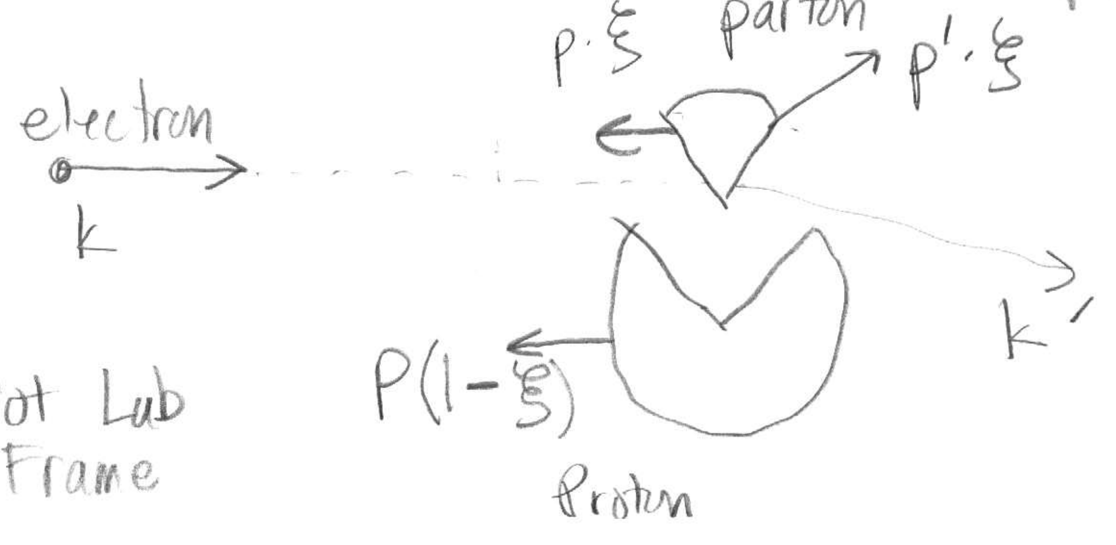
parton 0–1 proton

momentum $p = \xi P$

energy $E = \xi E_p$

What if electron elastically scatters off the parton...

→ looks like inelastic scattering off the proton... ~ independent of q^2 (point partons)



Not Lab Frame

What happens now?

$$\xi p + q = \xi p'$$

$$\underbrace{\xi^2 p^2} + 2\xi pq + q^2 = \underbrace{\xi^2 p'^2}$$

same.. elastic scattering off parton

$$2\xi M\nu + q^2 = 0$$

$$-\frac{q^2}{2\xi M\nu} = 1$$

$$\text{or} \quad \boxed{x = -\frac{q^2}{2M\nu} = \xi}$$

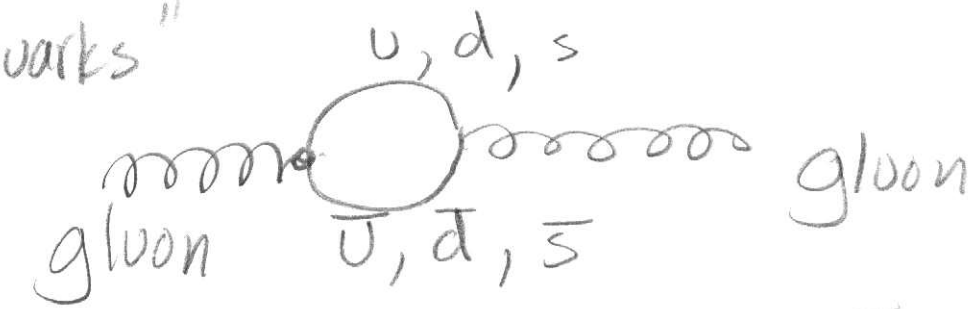
The kinematically invented variable x turns out to be the fraction ξ of the proton's property that the parton has! Just call the parton fraction... $\underline{\underline{x}}$.

At any moment, the proton is a superposition of lots of partons...

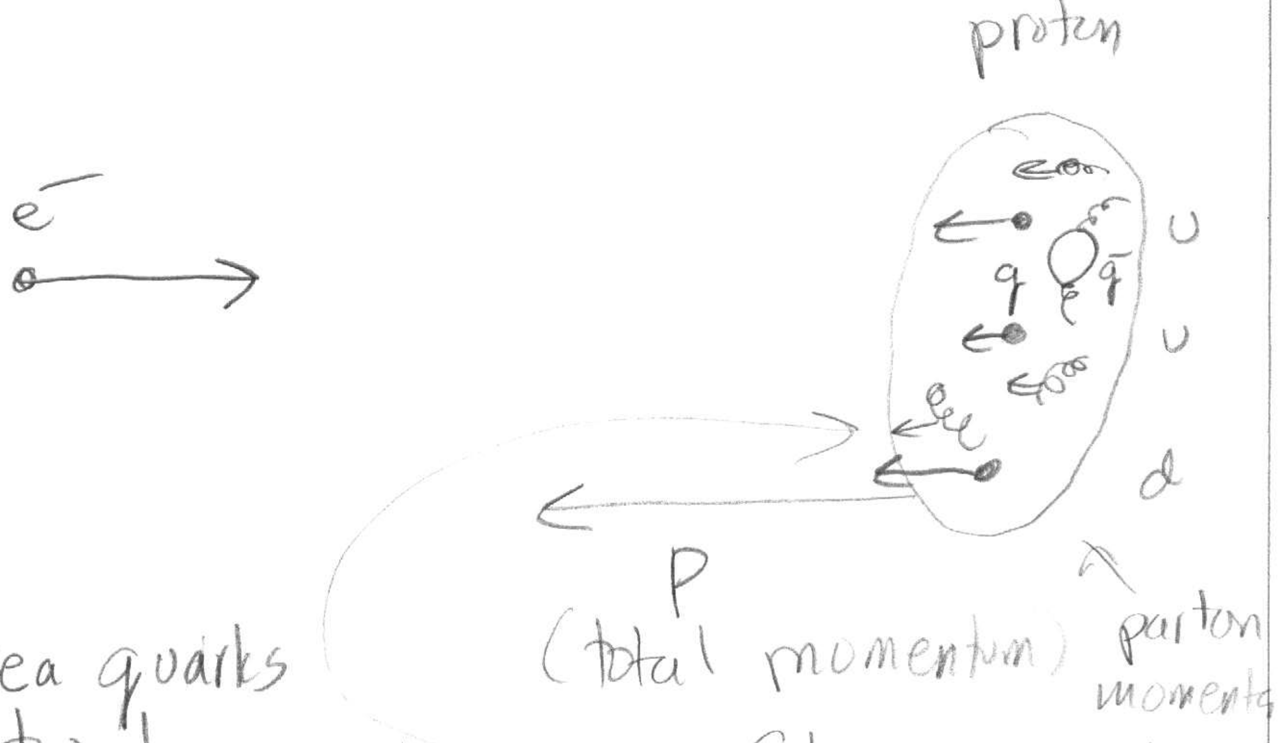
u quarks, d quarks, **gluons** (no electric charge)

"valence quarks"

"sea quarks"



sea quarks always in pairs!



sea quarks too!

Gluons, neutral

Probability of parton having momentum fraction x $\Rightarrow f_i(x)$ $\quad i = u, d, \bar{u}, \bar{d}, s, \bar{s}$, glue

$$\int_0^1 xP \sum_i f_i(x) \, dx = P$$

$$\int_0^1 x \sum f_i(x) dx = 1 \quad \text{(momentum)}.$$

$$\underset{\substack{\uparrow \\ \text{elastic} \\ \text{scattering}}}{\frac{d\sigma}{d\Omega}} \propto \sum_i e_i^2 \, x \sum f_i(x) dx$$

$e_u = +\frac{2}{3}e$ $\quad e_g = 0!$

$e_d = -\frac{1}{3}e = e_s$

Tricky: $dE' d\Omega \propto dx dy$

S: Mandelstam Variable

$s = \underbrace{(p+k)^2}_{\text{4-vector dot product}} \leftarrow$ p initial proton, k initial electron

$= m^2 \quad m =$ maximum mass attainable in collision.

$$\underset{ep\to eX}{\left(\frac{d\sigma}{dx\,dy}\right)} = \underset{\uparrow\ \text{photon propagator}}{\frac{2\pi\alpha^2}{Q^4}}\, s\, \underbrace{\left[1+(1-y)^2\right]}_{\text{angular dependence}} \sum e_i^2 x f_i(x)$$

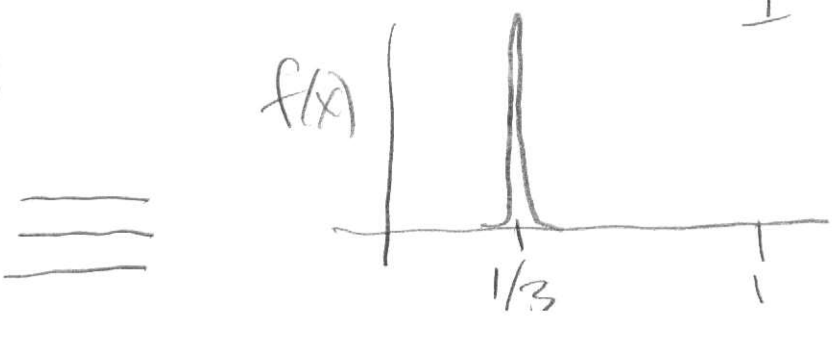
In this picture,
NO Q^2 dependence
⇒ no form factor
⇒ no new interaction

$f_u(x)$ → called $u(x)$
$f_d(x)$ → called $d(x)$
$f_{\bar{u}}(x)$ → called $\bar{u}(x)$
$f_{\bar{d}}(x)$ → called $\bar{d}(x)$ etc.

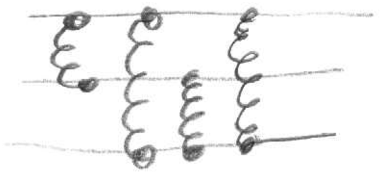
Qualitative Idea

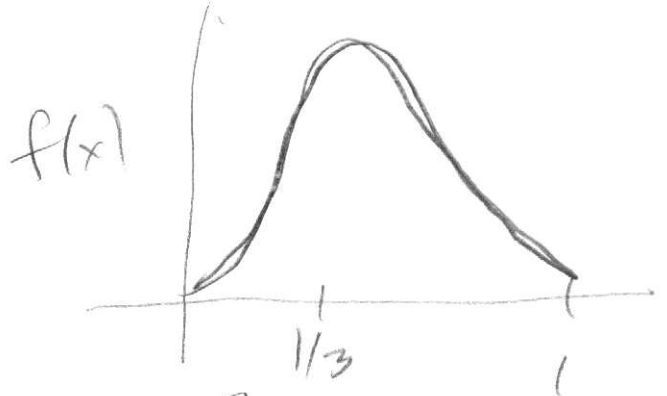
point proton

3 valence quarks (non interacting)

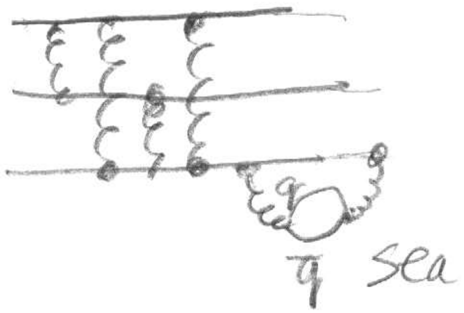


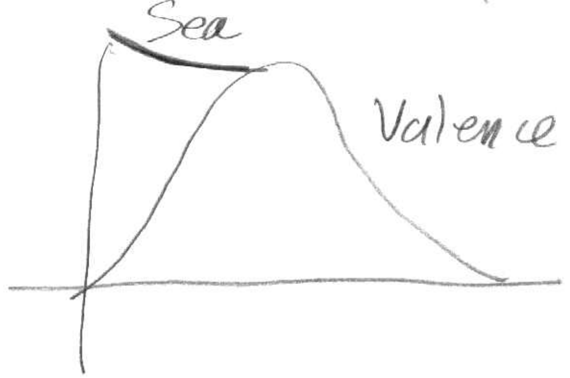
3 interacting valence quarks





QFT





Key point: compare results for proton + neutron .. all valence?

$$\left(\frac{d\sigma}{dx\,dy}\right)_{ep\to ex} \propto 2\times\left(\frac{2}{3}\right)^2 + 1\cdot\left(\frac{1}{3}\right)^2 = \frac{8}{9}+\frac{1}{9} = 1$$

$$en\to ex \propto \left(\frac{2}{3}\right)^2 + 2\cdot\left(\frac{1}{3}\right)^2 = \frac{4}{9}+\frac{2}{9} = \frac{2}{3}$$

expect $\frac{\text{neutron}}{\text{proton}} = \frac{2}{3}$.

Figure: true at $x \sim \frac{1}{3}$

But: at low x, $\frac{\text{neutron}}{\text{proton}} = 1$

WHY? → "sea" identical.

at high x, $\frac{\text{neutron}}{\text{proton}} \rightarrow \frac{1}{4}$!

Interpretation: high-x, one quark gets "antsy", steals the momentum.

p → an up quark

n → a down quark

$$\frac{\text{neutron}}{\text{proton}} \simeq \frac{(1/3)^2}{(2/3)^2} = 1/4$$

High x → short distance

Perhaps

Current Structure Functions

(figure)

Shock : $\int x(u+\bar{u}+d+\bar{d}+s+\bar{s})\,dx$

$\cong$ 54% of proton's momentum

$\simeq$ 46% carried by gluons

Soprisingly large fraction

$s+\bar{s}$! Important in dark matter