∞ mass target

$\nearrow \vec{p}_f$

$\rho(\vec{x}_T)$ scatterer density

$$\int \rho(\vec{x}_T)\, d^3x = 1$$

Two typical

$$V(\vec{x}_p - \vec{x}_T) = \frac{g_T}{(4\pi)\,|\vec{x}_p - \vec{x}_T|}$$

"charge" on target

(?) norm convention

(e.m, etc)

projectile $\uparrow \vec{p}_i$

$$= g_T\, \delta^3(\vec{x}_p - \vec{x}_T)$$

$V(\vec{x}_p - \vec{x}_T)$ interaction potential

$$W = \frac{2\pi}{\hbar} |M_{if}|^2 \rho_f$$

projectile charge

$$M_{if} = \frac{g_p}{V} \tilde{\rho}(\vec{q})\, \tilde{V}(\vec{q})$$

norm volume $\rightarrow V$

$$\vec{q} = \hbar(\vec{k}_f - \vec{k}_i) = \vec{p}_f - \vec{p}_i$$ momentum transfer.

$$\tilde{\rho}(\vec{q}) = \int d^3x\, e^{-\frac{i\vec{q}\cdot\vec{x}}{\hbar}} \rho(\vec{x})$$

$$\tilde{V}(\vec{q}) = \int d^3x\, e^{i\vec{q}\cdot\vec{x}}\, V(\vec{x})$$

Cross Section

$$d\sigma = \frac{W}{\phi} \qquad \phi = \frac{v_i}{V}$$

$$\underbrace{\rho_f}_{\text{"one body"}} = \frac{p_f^2\, d\Omega_f\, V}{(2\pi\hbar)^3} \frac{dp_f}{dE}$$

remains of energy conserving δ-function

$$p_f = \frac{1}{c}\sqrt{E^2 - m^2c^4}$$

$$\frac{dp_f}{dE} = \frac{E}{c\sqrt{E^2-m^2c^4}} = \frac{E}{c^2 p_f} = \frac{mc^2\gamma}{mc^3\beta_f\gamma}$$

$$\frac{dp_f}{dE} = \frac{1}{c\beta_f} = \frac{1}{v_f}$$

$E = mc^2\gamma$
$p_f = mc\beta\gamma$

$$d\sigma = \frac{2\pi}{\hbar}\frac{g_p^2}{\cancel{V^2}}\left|\tilde{\rho}(\vec{q})\tilde{V}(\vec{q})\right|^2 \frac{\cancel{V}}{v_i} \cdot \frac{p_f^2\, d\Omega_f\, \cancel{V}}{(2\pi\hbar)^3}\frac{1}{v_i}$$

$$\frac{d\sigma}{d\Omega} = \frac{g_p^2}{4\pi^2\hbar^4}\left|\tilde{\rho}(\vec{q})\tilde{V}(\vec{q})\right|^2 \underbrace{\frac{p_f^2}{v_i v_f}}_{\text{in this case}}$$

$p_f = p = m\gamma\beta c$
$v_i = v_f = \beta c$

$$= \frac{g_p^2}{4\pi^2\hbar^4}\left|\tilde{\rho}(\vec{q})\tilde{V}(\vec{q})\right|^2 m^2\gamma^2$$

Look at "massive" propagator

$$\tilde{V}(\vec{q}) = \frac{4\pi g_T}{q^2 + m_v^2}\cdot(\hbar^2)$$

m_v: mass of "exchange" particle

$m=0$ photon

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{4\, g_P^2 g_T^2\, m^2\gamma^2}{(q^2 + m_V^2)^2} \underbrace{|\tilde{\rho}(\vec{q})|^2}}$$

function of q = magnitude of 3m. $\tilde{\rho}(0) = 1$

pre LEP
e^- $m_V = 0$
γ $m_V \sim m_Z$
CDF/LHC

Check: Coulomb $q = 2p\sin\theta/2$

$g_P = Z_2 e \quad g_T = Z_1 e$

$$\frac{d\sigma}{d\Omega} = \frac{Z_1^2 Z_2^2 e^4 m^2\gamma^2}{4\, p^4 \sin^4\theta/2} |\tilde{\rho}(\vec{q})|^2$$

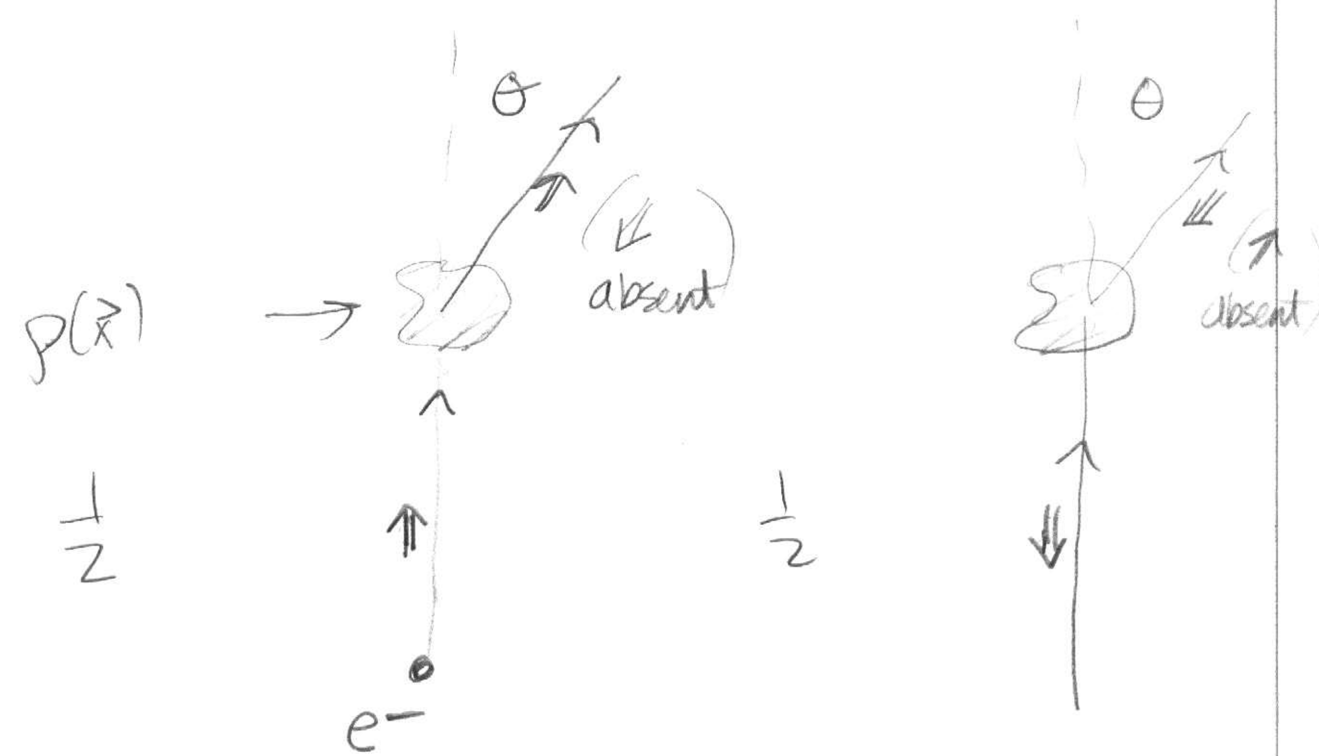
$p^2 \; m^2\gamma^2 v^2$

$$= \frac{Z_1^2 Z_2^2 e^4}{4\, p^2\, v^2 \sin^4\theta/2} |\tilde{\rho}(\vec{q})|^2$$

Mott Scattering (e off structureless target)

Key Insight: Helicity Conservation

In the "ultrarelativistic" limit, which is all we generally care about, electron's spin stays parallel or anti-parallel to $\vec{p}$. Must preserve total angular momentum.



Spin
Matrix Element

$$d^{\frac{1}{2}}_{\frac{1}{2}\,\frac{1}{2}}(\theta) = \cos\tfrac{1}{2}\theta$$

$$d^{1/2}_{-\frac{1}{2}\,-\frac{1}{2}}(\theta) = \cos\tfrac{1}{2}\theta$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{mott}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} \times \cos^2\left(\tfrac{1}{2}\theta\right)$$

spin matrix element

Key Point:

$\theta = \pi$, backscattering, suppressed

$$\left[1 - \beta^2 \sin^2\frac{\theta}{2}\right]$$

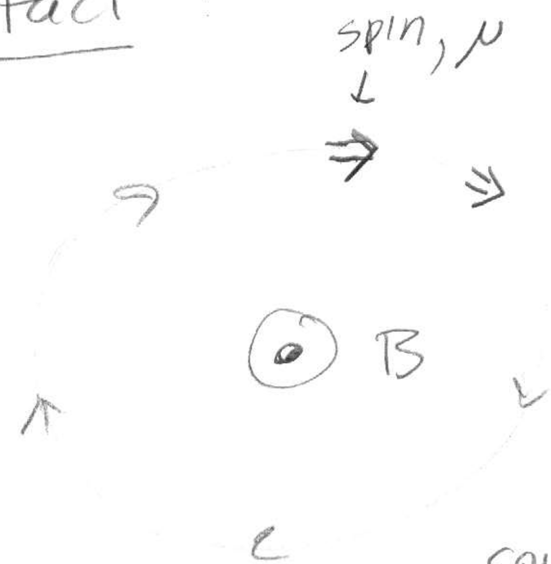
β of electron

e-p scattering underestimates

Back scattering becomes dominated by .... <u>magnetic moments</u>. (!)

<u>"dirac particle"</u> : $\mu \cong \frac{e\hbar}{2Mc}$ $\Big\}$ lots of factors of 2

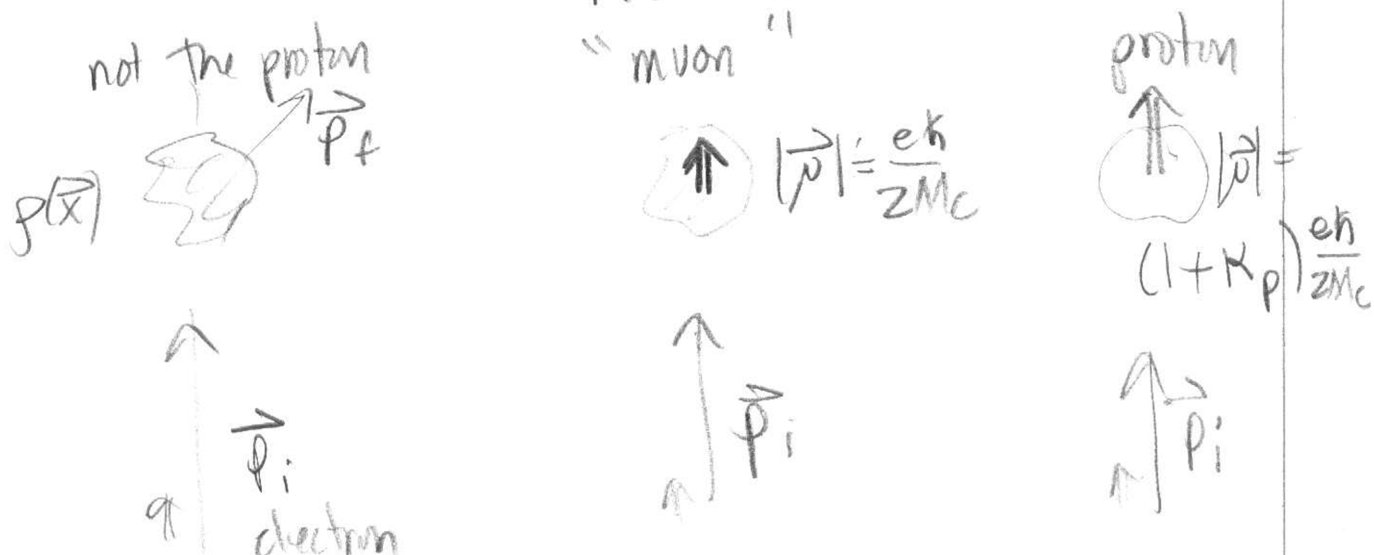
<u>Curious Fact</u> :

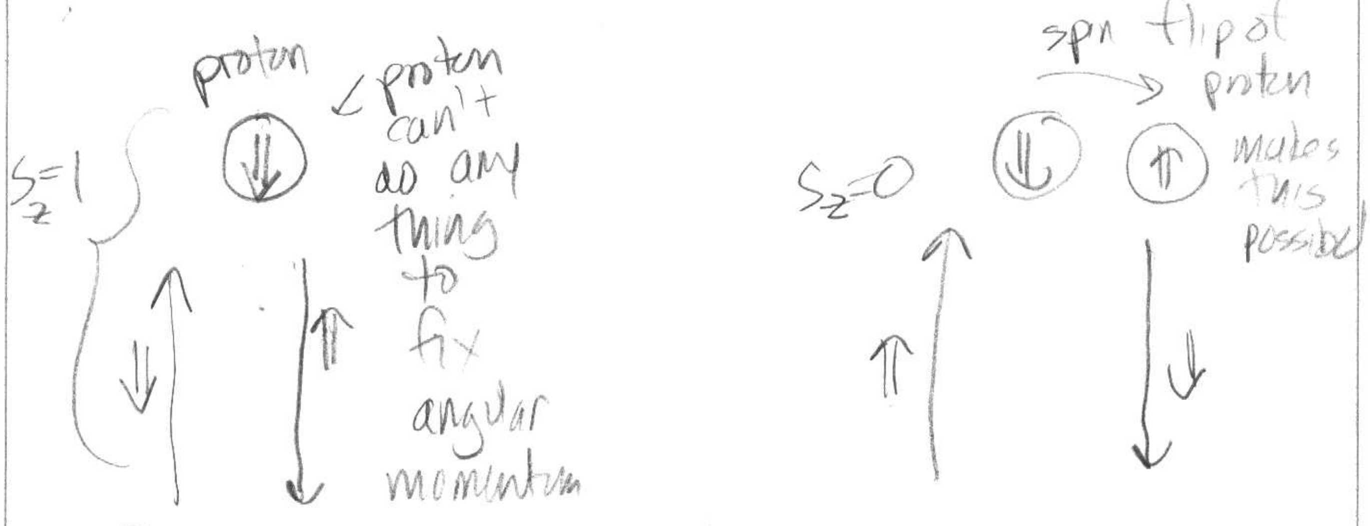


very relativistic electron, circular trajectory

$g=2$ keeps spin parallel to $\vec{p}$!

$g \neq 2$ due to higher order QFT.. experimentally, <u>lag</u> of $\Uparrow$ + $\vec{p}$ is the secret





$|Z_1| = |Z_2| = e$, be relativistic

⇒ First, imagine proton is Dirac point particle, mass M

⇒ electron ~ massless, initial energy E

$$\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \underbrace{\frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}}}_{\text{remnant of rutherford}} \cdot \frac{E'}{E} \, \underset{\text{point!}}{1} \left\{ \cos^2\frac{\theta}{2} + a(q^2)\sin^2\frac{\theta}{2} \right\}$$

no spin flip

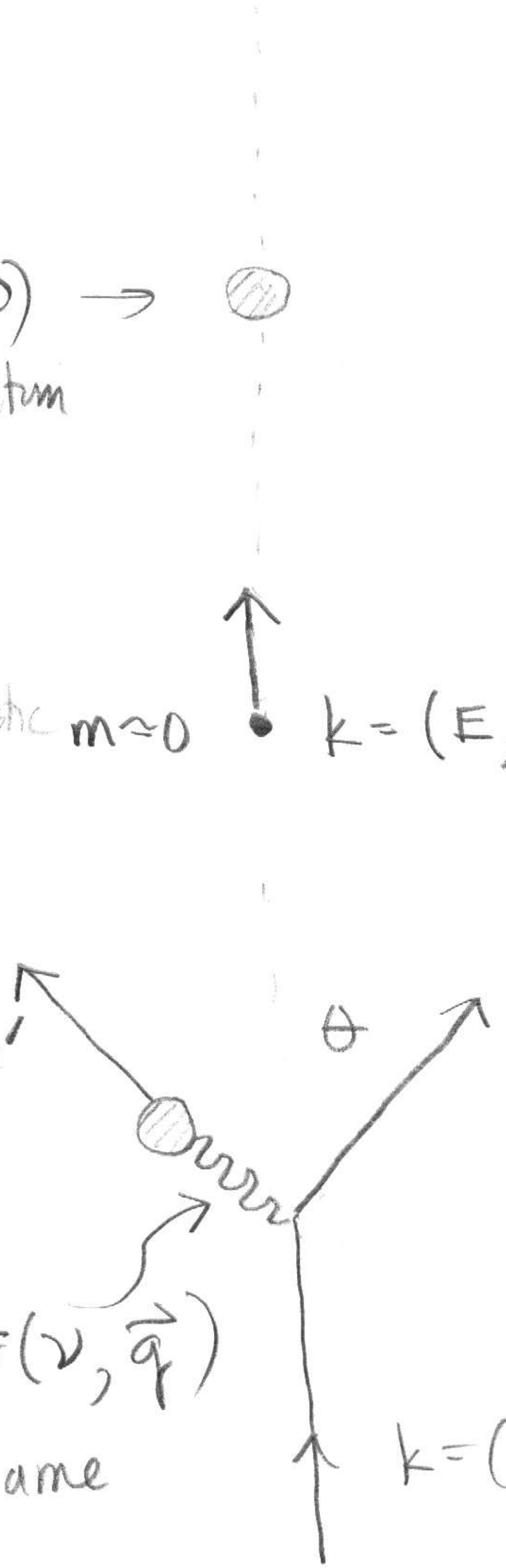
E' = final electron energy

E = initial

$E' \neq E \Rightarrow$ lab frame.

- q^2 is now 4-momentum transfer.
- $a(q^2) > 0$
- $a(0) = 0$ (helicity).

Scattering Kinematics (Lab Frame)



$p=(M,0)$ →
4 momentum

ultrarelativistic $m \simeq 0$ $k=(E,\vec{k})$

$E^2-|\vec{k}|^2=0$
$E=|\vec{k}|$
$c=1$

θ $k'=(E',\vec{k}')$

p'

4-momentum
$q \equiv (\nu, \vec{q})$

$k=(E,\vec{k})$

CM frame
$\nu=0$
$q^2=0-|\vec{q}|^2<0$
true in all frames!
$\nu=E-E'$ energy lost

$k=q+k'$ (4 mom)
$p+q=p'$

$$q^2 = (k-k')^2 = \underset{\substack{\| \\ 0 \\ \text{(neutrinos)}}}{k^2} - 2kk' + \underset{\substack{\| \\ 0}}{k'^2}$$

$$kk' = (E, \vec{k}) \cdot (E', \vec{k}')$$
$$= EE' - \vec{k} \cdot \vec{k}'$$
$$= EE' - EE' \cos\theta$$
$$= EE'(1-\cos\theta) = 2EE' \sin^2 \theta/2$$

$$\boxed{q^2 = -4EE' \sin^2(\theta/2) < 0}$$

$$p + q = p'$$

$$\underset{\uparrow \, M^2}{p^2} + \underset{\substack{\downarrow \\ 2(M,0)(\nu,\vec{q}) \\ \| \\ 2M\nu}}{2pq} + q^2 = \underset{\substack{\uparrow \\ M^2}}{p'^2}$$

$$q^2 = -2M\underset{\substack{E-E' \\ \downarrow}}{\nu} = -4EE'\sin^2\frac{\theta}{2}$$

$$\frac{-q^2}{2M} = \nu \quad \text{energy loss}$$

$$\frac{E'}{E} = \frac{1}{1 + \frac{2E\sin^2\frac{\theta}{2}}{M}} \to 1 \quad \text{as } M \to \infty$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{lab} = \frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}} \frac{1}{1+\frac{2E\sin^2\frac{\theta}{2}}{M}} \left\{\cos^2\frac{\theta}{2} + \left(\frac{\nu}{M}\sin^2\frac{\theta}{2}\right)\right\}$$

- just saying this here (no derivation)
- $\nu = 0$, nothing

$\boxed{\text{underestimates data}}$

$$E' = \frac{E}{1+\frac{2E\sin^2\theta/2}{M}}$$

$$E - E' = E\,\frac{2E\sin^2\theta/2\,/M}{\left(1+\frac{2E\sin^2\theta/2}{M}\right)}$$

max $\theta = \pi$

$$\frac{\nu}{M} = \frac{E-E'}{M} = \underbrace{\frac{E}{m}\,\frac{2E/M}{1+2E/M}}$$

need $E \sim 50\text{-}100$ MeV

$\boxed{\text{But More}}$

Proton Magnetic Moment

$$= (1+K_p)\frac{e\hbar}{2Mc}$$

"anamolous"

$\simeq 1.79$

hey, neutrons

$$+K_n\frac{e\hbar}{2Mc}$$

$K_n = -1.91$

If proton structureless

$$\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \underbrace{\frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}} \frac{E'}{E} \left\{ \left(1 + \frac{K_p^2 \nu}{2M}\right)\cos^2\frac{\theta}{2} + \frac{\nu}{M}(1+K_p)^2 \sin^2\frac{\theta}{2} \right\}}_{\text{shown on Hofstadter Plot}}$$

$\boxed{\text{Overestimates data}}$

$\Rightarrow$ $\boxed{\text{Form Factors}}$

"Dirac" $F_1(q^2)$ $\underbrace{F_1(0) = 1}_{\text{normalization}}$

"Anomalous" $F_2(q^2)$ $F_2(0) = 1$ (factor of K_p)

$$\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}} \frac{E'}{E} \left\{ \left(F_1^2 + \frac{K_p^2 \nu}{2M} F_2^2\right)\cos^2\frac{\theta}{2} + \frac{\nu}{M}\left(F_1 + K_p F_2\right)^2 \sin^2\frac{\theta}{2} \right\}$$

variable change

$$\underset{\text{electric}}{G_E(q^2)} \equiv F_1 + \frac{Kq^2}{4M^2} F_2$$

$$G_M(q^2) = F_1 + K F_2$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \underbrace{\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\cos^2\frac{\theta}{2}\,\frac{E'}{E}}_{\left(\frac{d\sigma}{d\Omega}\right)_{\text{No structure}}} \underbrace{\left[\frac{G_E^2+\tau G_M^2}{1+\tau}+2\tau G_M^2\tan^2\frac{\theta}{2}\right]}_{\text{"Form Factor Effect"}}$$

Both G_E + G_M

turn out to be...

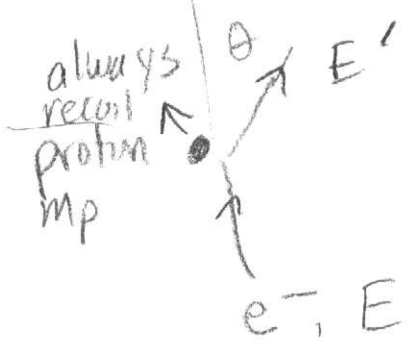
$$G_E = \frac{G_M}{\mu} = \frac{1}{\left(1+\frac{|\vec{q}|^2}{0.71\,(\mathrm{GeV})^2}\right)^2}$$

$$\mu = 1 + \kappa_p = 2.79$$

Plots (Taylor best)

On to inelastic scattering

elastic



E' is a completely predictable function of θ for elastic

$$E'_{elastic} = \frac{E}{1+\frac{2E\sin^2\theta/2}{M}}$$

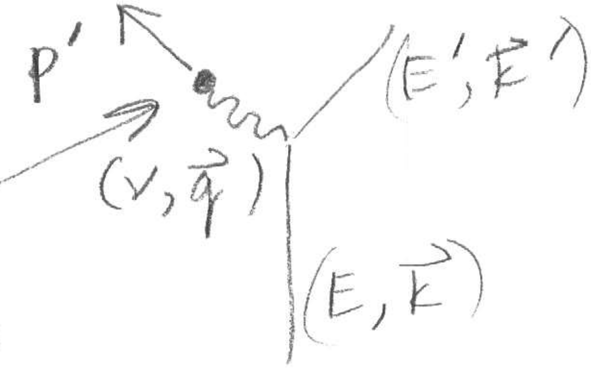
or $q^2 = -2m\nu$

Experimentally, $E'_{inelastic} < E'_{elastic}$

Cross section 'does not fall' Kendall?

Where does the energy go?

→ into excitation of the proton ...



4-vectors

initially, $m = m_p$

$p = (m_p, \vec{0})$ now, $p'^2 \neq m_p^2$

$$p'^2 > m_p^2$$

$$p'^2 \equiv W^2$$

$$p' = p + q \quad \text{4-vectors}$$

$$W^2 = p^2 + 2pq + q^2$$

$$W^2 = m_p^2 + 2m_p\nu + q^2$$

$$-q^2 = 2m_p\nu + \underbrace{(m_p^2 - W^2)}_{\text{inelastic, gets negative}}$$

$$x \equiv \frac{-q^2}{2m_p\nu} = 1 - \frac{(W^2 - m_p^2)}{2m_p\nu} \quad \begin{array}{l} = 1 \text{ elastic} \\ = 0 \text{ minimum, "small x"} \end{array}$$

Other variable $y = \frac{p \cdot q}{p \cdot k}$ } related to c.m. angle

Description of Cross Section

(1) think of $\frac{d\sigma}{dE' d\Omega}$

extra derivative.

for elastic scattering, this introduces a $\delta(E' - f(\theta))$ but not for inelastic!

(2)

$$\frac{d\sigma}{dE' d\Omega} = \frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}} \left\{ \overbrace{\underbrace{W_2(\nu, q^2)}_{\text{elastic, } q^2 = -2m_p\nu!}\cos^2\frac{\theta}{2}}^{\text{no flip}} + \overbrace{2W_1(\nu, q^2)\sin^2\frac{\theta}{2}}^{\text{helic flip}} \right\}$$

$1/E$ vanishes!

$\sim \delta(q^2 + 2m_p\nu)$

(3) First surprise (experimental)

VERY LITTLE q^2 dependence

$\Rightarrow$ whatever scattered not an extended object!

$\Rightarrow$ are they points?

Points of mass m ($\neq m_p$)

$$2W_1^{point}(\nu, \underset{-q^2}{Q^2}) \propto \underbrace{Q^2 \delta\left(1-\frac{Q^2}{2m\nu}\right)}_{\text{"magnetic"}}$$

$$W_2^{point}(\nu, Q^2) \propto \underbrace{\delta\left(1-\frac{Q^2}{2m\nu}\right)}_{\text{electrostatic}}$$

no anomalous moment.

When points

$$\frac{d\sigma}{dE' d\Omega} = \frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}} \left\{ \frac{1}{\nu}\delta\left(1-\frac{Q^2}{2m\nu}\right) + \underbrace{\frac{1}{m}\frac{Q^2}{2m\nu}\delta\left(1-\frac{Q^2}{2m\nu}\right)}_{\text{wins at high } Q^2} \right\}$$

function of $\frac{Q^2}{\nu}$, $\underbrace{\text{not } Q^2 \text{ alone}}$

would be evidence of quark form factor