Physics 225b Problem Set 2

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due Monday, Jan. 26 in class

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1. Read Henry Kendall's Nobel lecture, available on the course website.

2. We've learned that the form factor of both the electric charge and magnetic moment distributions of the proton are governed by the form factor:

$$\tilde{\rho}(\vec{q}) = \frac{1}{(1 + b|\vec{q}|^2)^2}$$

where b=(1/0.71)(1/GeV^2), and $\vec{q}$ is the three-momentum transfer. Fourier transform this distribution (in three dimensions) and find the spatial 'wave function' of the proton. Use that wave function to evaluate, symbolically and numerically, the root-mean-squared radius of the proton.

3. In this problem, systematically work through the kinematics of the scattering of a very relativistic electron with a proton. Often, the kinematics discussions of this important reaction seem kind of spotty.

Take $c = 1$, and approximate the electron to be massless, $m_e = 0$. The energy and the magnitude of the three-momentum of the electron are initially equal to E, and after scattering the are equal finally to E'. The initial four-momentum of the electron is $k = (E, \vec{k})$, but $k^2 = E^2 - |\vec{k}|^2 = 0$, so $|\vec{k}| = E$. Often, people get sloppy about this, and just say $k = E$, confusing the four-momentum and the magnitude of the three-momentum, but get used to that. The proton is initially at rest, so its initial four-momentum is $p = (m_p, 0)$.

In the following, evaluate in terms of E and m_p (and, eventually, θ^*), and find the limits at $m_p \to 0$.

(a) The 'Mandelstam variable' $s = (k + p)^2$ (using four-vectors) is the square of the maximum mass attainable in the ep collision. Although in an elastic collistion a new particle with this maximum mass is never produced, s is still a very useful parameter.

(b) A particle with mass $\sqrt{s}$ made in the ep collision would be moving in the lab frame. Evaluate its β (speed relative to that of light) and Lorentz boost γ.

(c) The center of mass system is that where the particle with mass $\sqrt{s}$ is at rest. Find the energy of the electron, E^*, and that of the proton, E_p^*, in the center of mass frame. There are a bunch of ways to do this, but try this one: imagine $k(k + p)$ and $p(k + p)$ (four vector dot-products) in the center of mass frame, where $(k + p)$ (really, the 4-vector in the center of mass frame that corresponds to the lab-frame $(k + p)$, but that sloppiness is common) is super-easy, and then in the lab frame, where you know everything; the dot products evaluated in the different frames must be equal because they are relativistically invariant.

(d) In the center of mass system, evaluate the magnitude of the momentum of the electron, and that of the proton (individually). Of course, you should get the same expression, two different ways. Sometimes this expression is written:

$$|\vec{p}^*|^2 = \frac{\lambda(s, m_p^2, 0)}{4s}.$$

Determine the function $\lambda(a, b, c)$.

(e) The center of mass scattering angle θ^* is terrific way to explore the scattering parameter space *in the lab frame*; let's imagine θ^* as the principal independent variable to employ. In the center of mass frame, elastic scattering involves no energy transfer, but does involve momentum transfer, $|\vec{q}|$. Evaluate $|\vec{q}|$ (reminder: as a function of E, m_p, and θ^*). The four-vector q, in the center of mass frame, is $(0, \vec{q})$.

(f) Evaluate $\nu = E - E'$, which (on the right-hand side) are lab-frame quantities.

(g) Verify the expected relationship $2m_p\nu = -q^2$.

(h) The angle through which the electron is scattered in the lab is θ. Evaluate $\tan\theta$ in terms of γ, β (you need not put these in terms of E and m_p) and θ^*.

(i) The angle with which the proton is scattered in the lab with respect to the original electron direction is ϕ. Evaluate $\tan\phi$ in terms of γ, β, and θ^*.

(j) Imagine splitting the domain of θ^* from 0 to π into equal intervals; about 20 should do. For $E = 5\,\text{GeV}$ and $E = 20\,\text{GeV}$, make tables of E', θ, T_p', and ϕ as a function of θ^*. Here T_p' is the final kinetic energy of the proton. Make plots of E' versus θ and T_p' versus ϕ for the two energies (4 plots). Were you to design a spectrometer, these plots would be very useful.