Scattering and Structure ($\Rightarrow$ LHC)

Rutherford (~1910)

E ~ several MeV, $\beta \sim \frac{1}{20}$

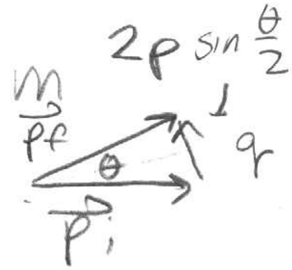
$\bigcirc \longrightarrow \vec{p}_i$

α, $Z_1 = 2$

$m \simeq 4$ nucleons

Z_2

$M \gg m$



$$\frac{d\sigma}{d\Omega} = \underbrace{\frac{Z_1^2 Z_2^2 e^4}{4\, p^2 V^2} \frac{1}{\sin^4 \theta/2}}_{\left.\begin{array}{c}\text{consequence of } 1/r \\ \text{potential}\end{array}\right\} \propto \frac{1}{q^4}}$$

Evidence for non-zero <u>size</u> of nuclei. You should just <u>know</u>

$$r_{\text{Nucleus}} \sim (1.2\ \text{fm})\, A^{1/3} \qquad \underline{\underline{1920\text{'s}}}$$

A: atomic weight

$\boxed{\text{How was this } \underline{\text{first}} \text{ deduced?}}$

"Chapter 2" of Rutherford Scattering . . .

Distance of "closest approach"
smallest is impact parameter = 0

$$\frac{1}{2}\mu V^2 = \frac{Z_1 Z_2 e^2}{r_{min}}$$

reduced mass

$$r_{min} = 2\,\frac{Z_1 Z_2 e^2}{\mu V^2}$$

$$= 2\, Z_1 Z_2 \frac{e^2}{\hbar c}\left(\frac{c}{V}\right)\frac{\hbar c}{\mu c^2 (V/c)}$$

$$r_{min} = 2 Z_1 Z_2\, \alpha\, \frac{1}{\beta}\, \lambda\!\!\!^{-}$$

de Broglie

comment
$r_{min} \gtrless \lambda\!\!\!^{-}$?

~ given by sources
until ~1930

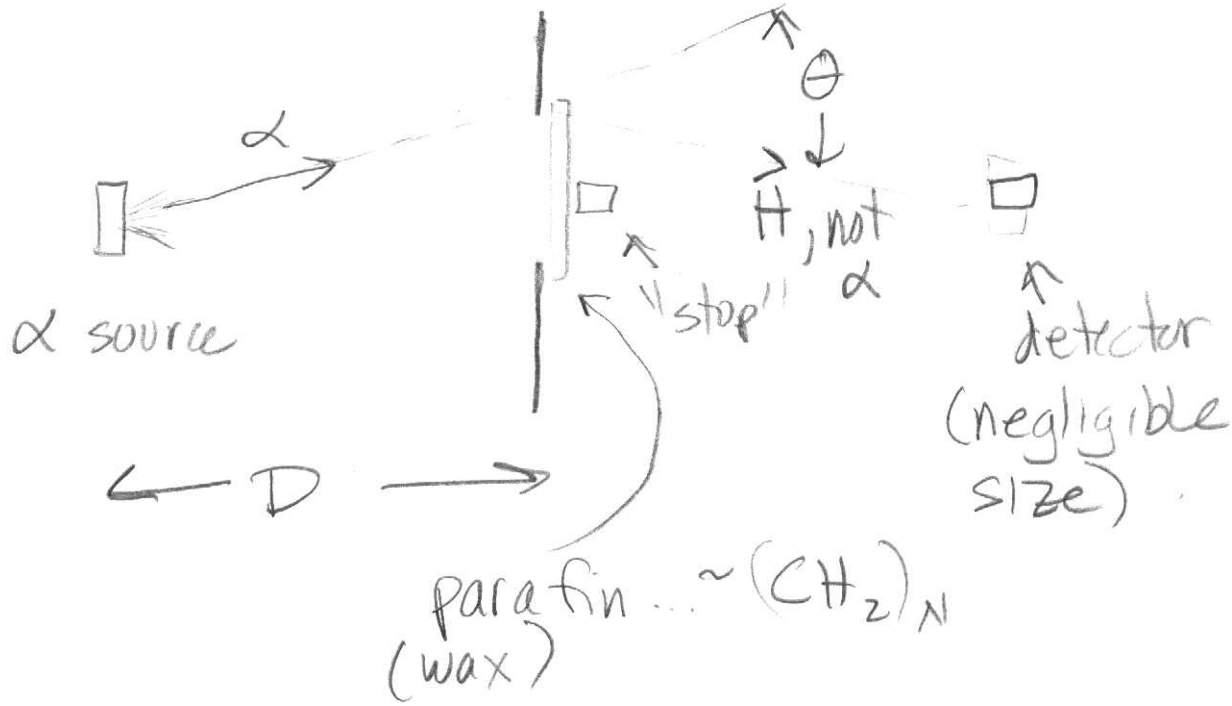
to make
r_{min} small, minimize Z_2

$\Rightarrow$ Just the opposite of "Classical" Rutherford ...

$Z_2 = 1$, hydrogen

Kinematics very different

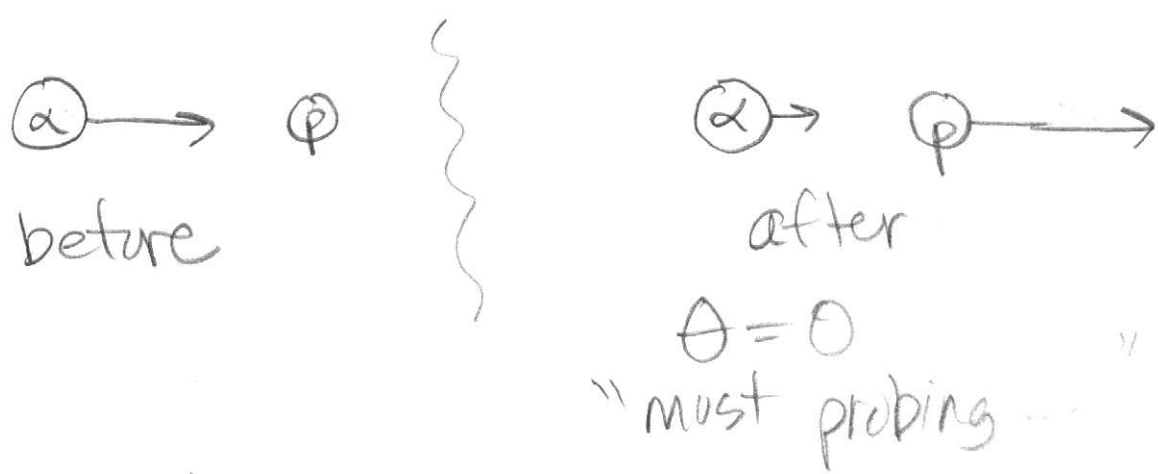
Experiment (Rutherford ~ 1919)



$\alpha + C \to C$ doesn't escape

$\alpha + H \to \alpha +$ <u>fast</u> H is point of experiment

"Best" case for probing nuclear structure ... θ <u>small</u>



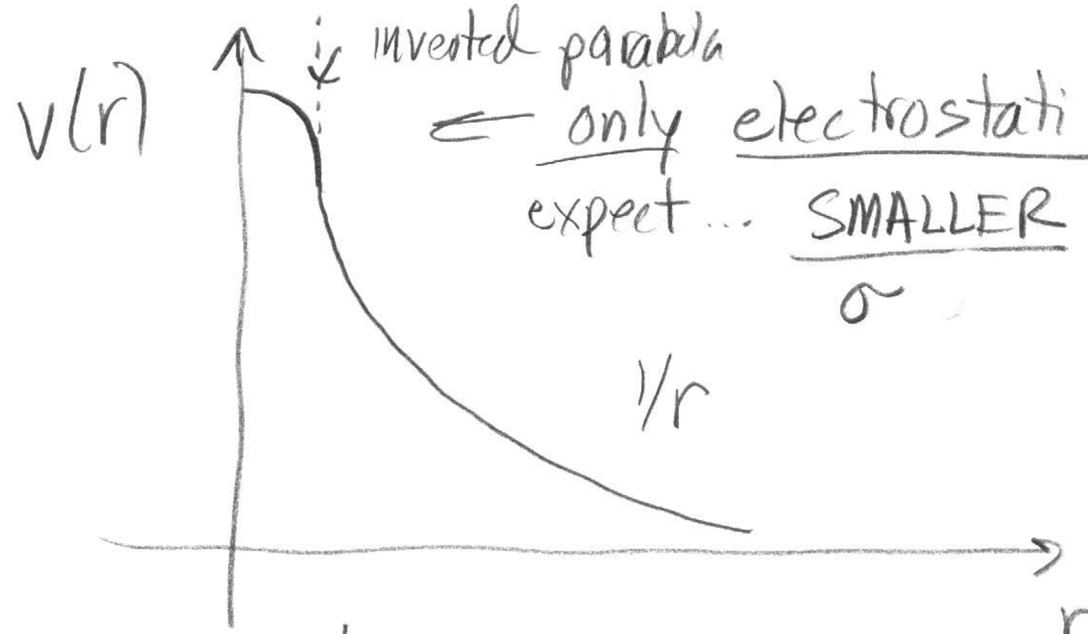
More kinematics → problems

View Pot

Comments: B' = pure electrostatic point charge

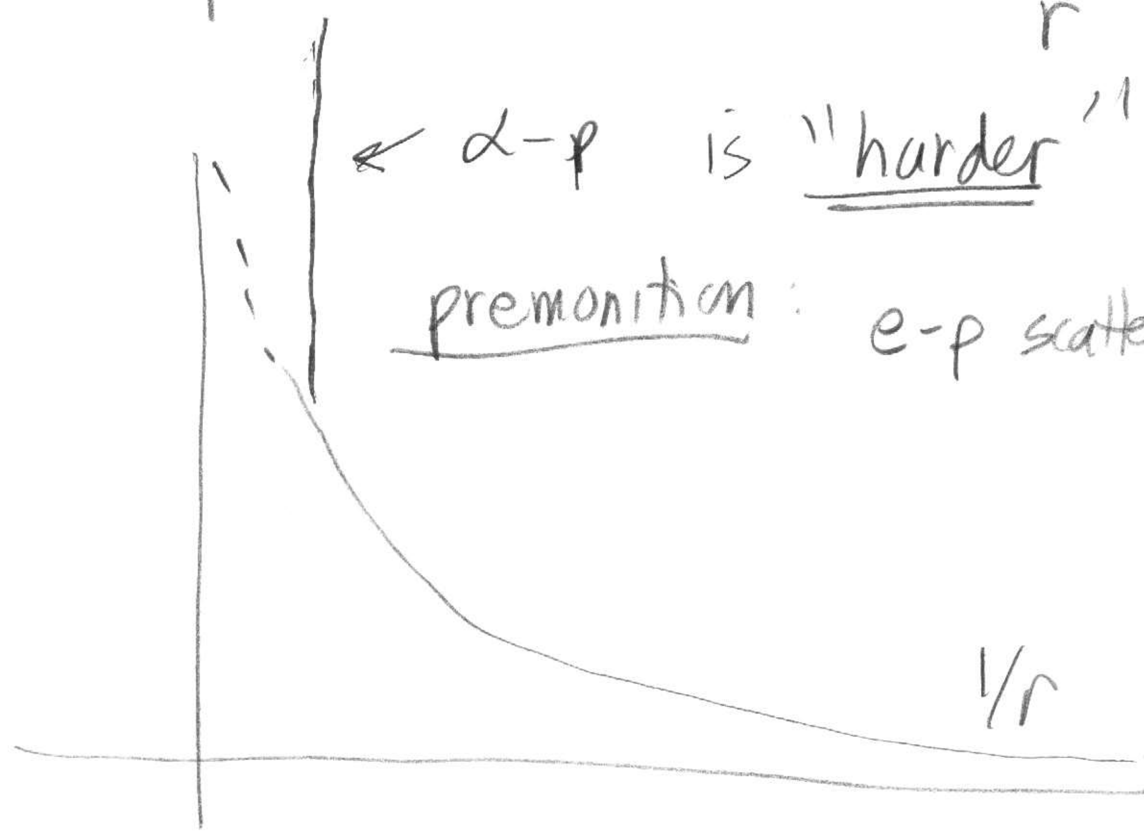
→ ball of charge (?)

inverted parabola



← only electrostatic

expect … SMALLER σ



← α-p is "harder"

premonition: e-p scattering

Confusion: α-p : harder potential

p.264 Fig.74 → α-He : "

268 Fig 76 → α-Al : softer!

α as probe: problem is, has
strong interactions
"closed shell"

Other probes: electron (Hofstadter
Hofstadter → (no strong) Nuclear struc;
$V(r) \propto 1/r$ F.K.T quarks).

complex
p.531 fig
11-16
221 6-7

neutron (make λ smaller)
(short-range).

neutrino ideal (low stats)

$$V(r) \underset{\sim}{\propto} \delta(r)$$

WIMPs (Dark Matter).

$$V(r) \propto \delta(r)$$

Neutron - Nucleus Scattering

Potential short range, ~ 1 fm
Like Light encountering a "cloudy sphere"... see figures...

"Diffraction" → note that central lobe gets narrower as A increases.
→ note ~ 2-3 orders of magnitude from central peak to first minimum

kind of fourier ... transform

Electron - Nucleus (1950's)

→ Experiment diagram. / → elastic/inelastic
→ Peak to valley ≃ 10 orders
→ $1/q^4$ nature of coulomb field.

→ idea of inelastic scattering
→ nucleus shape (electric charge) spherical

Electron - Proton

Proton : (a) spinless, point

(b) spin-$\frac{1}{2}$, Dirac ($g=2$)

(c) spin-$\frac{1}{2}$, anamolous ($g>2$) ~ 5.5

(d) charge spread out

proton F_1/F_2 : F_1 : dirac-like / F_2 : anamolous $\Big\}$ note, $1/q^4$ gone

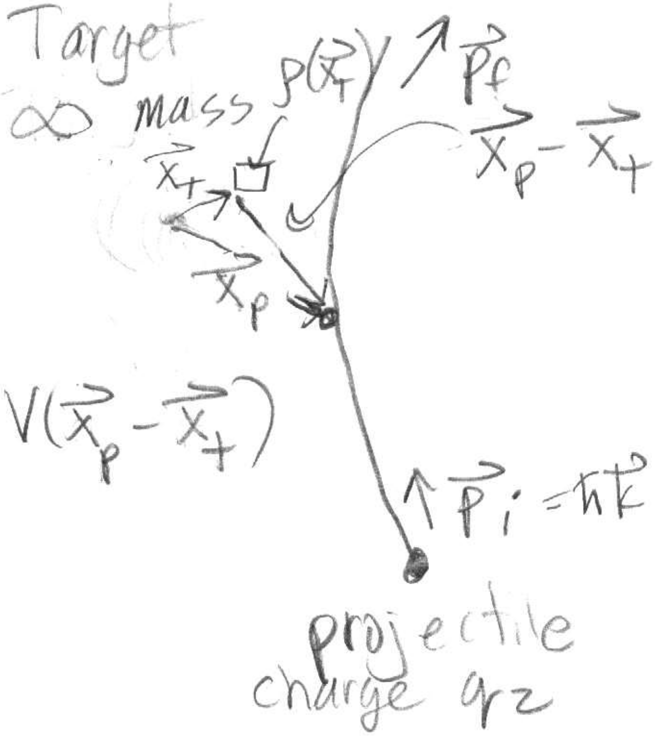
Deut : "Fermi" momentum

neutron : F_{2n} = moment

F_{1n} = dirac-like

"sandwich" + − +

Probing Nuclei/Particles/etc. Via Scattering (Elastic)



$$\vec{k}_f = \frac{1}{\hbar}\vec{p}_f$$

density of scatterers

$\rho(\vec{x}_t)$ ← "form factor"

$$\int \rho(\vec{x}_t)\, d^3x = 1$$

Fermi's Golden Rule ..

$$W = \frac{2\pi}{\hbar} |M_{if}|^2 \rho_f$$

W: transition Rate

M_{if}: transition matrix element

ρ_f: density of states

- experimentally always important

$\underline{M_{if}}$: $\langle f | \underset{\sim}{\mathcal{H}} | i \rangle$

$$|i\rangle \doteq \frac{1}{\sqrt{V}} e^{-i\vec{k}_i \cdot \vec{x}_p}, \quad |f\rangle = \frac{1}{\sqrt{V}} e^{-i\vec{k}_f \cdot \vec{x}_p}$$

V: quantization volume $\quad \vec{p}_i = \hbar k_i$, $\; p_f = \hbar k_f$

$$M_{if} = \frac{q^2}{V} \int d^3x_p \, d^3x_+ \, e^{-i\vec{k}_f \cdot \vec{x}_p} \rho(\vec{x}_+) V(\vec{x}_p - \vec{x}_+) \times e^{i\vec{k}_i \cdot \vec{x}_p}$$

$$= \frac{q^2}{V} \int d^3x_p \, e^{i(\vec{k}_i - \vec{k}_f) \cdot \vec{x}_p} \tilde{f}(\vec{x}_p)$$

$$\tilde{f}(\vec{x}_p) = \int d^3x_+ \, \rho(\vec{x}_+) \, V(\vec{x}_p - \vec{x}_+)$$

$$\rho(\vec{x}_+) = \frac{1}{(2\pi)^3} \int d^3\alpha \, \tilde{\rho}(\vec{\alpha}) e^{i\vec{\alpha} \cdot \vec{x}_+}$$

$$V(\vec{x}_p - \vec{x}_+) = \frac{1}{(2\pi)^3} \int d^3\beta \, \tilde{V}(\vec{\beta}) e^{i\vec{\beta} \cdot (\vec{x}_p - \vec{x}_+)}$$

[note $\int \rho(\vec{x}_+) d^3x_+ = 1 \Rightarrow \boxed{\tilde{\rho}(0) = 1}$

$$S(\vec{x}_p)$$

$$= \frac{1}{(2\pi)^6} \iiint d^3x_t \, d^3\alpha \, d^3\beta \, e^{\overbrace{i\vec{\alpha}\cdot\vec{x}_t + i\vec{\beta}\cdot(\vec{x}_p - x_t)}^{+i\vec{\beta}\cdot\vec{x}_p + i\vec{\alpha}\cdot\vec{x}_t - i\vec{\beta}\cdot\vec{x}_t}} \tilde{\rho}(\vec{\alpha})\, \tilde{V}(\vec{\beta})$$

$$\frac{1}{(2\pi)^3}\int d^3x_t \, e^{i(\vec{\alpha}-\vec{\beta})\cdot\vec{x}_t} = \delta(\vec{\alpha}-\vec{\beta})$$

can do the .. α integration ..

$$S(\vec{x}_p) = \frac{1}{(2\pi)^3}\int d^3\beta \, e^{i\vec{\beta}\cdot\vec{x}_p} \tilde{\rho}(\vec{\beta})\, \tilde{V}(\vec{\beta})$$

$$M_{if} = \frac{q_2}{V}\frac{1}{(2\pi)^3}\iint d^3x_p \, d^3\beta \, e^{i(\vec{k}_i - \vec{k}_f + \vec{\beta})\cdot\vec{x}_p} \tilde{\rho}(\vec{\beta})\, \tilde{V}(\vec{\beta})$$

can do the $\vec{x}_p$ integration

$\Rightarrow$ gives $\delta(\vec{k}_i - \vec{k}_f + \vec{\beta})$

$\Rightarrow$ can do the β integration

$$M_{if} = \frac{q_2}{V} \underbrace{\tilde{\rho}(\vec{k}_f - \vec{k}_i)}_{\text{distribution of scatterers in target}} \underbrace{\tilde{V}(\vec{k}_f - \vec{k}_i)}_{\text{interaction potential}}$$

$\tilde{\rho}(\vec{k}_f - \vec{k}_i)$: "form factor"

momentum transfer $\rightarrow \vec{q} = \hbar(\vec{k}_f - \vec{k}_i)$

$$\tilde{\rho}\left(\frac{\vec{q}}{\hbar}\right) = \int d^3x \, e^{-i\frac{\vec{q}\cdot\vec{x}}{\hbar}} \underbrace{\rho(\vec{x})}$$

spatial distribution of "charge" → e.m, strong, weak.

Simplest: $\rho(\vec{x}) = \delta(\vec{x})$

$$\tilde{\rho}\left(\frac{\vec{q}}{\hbar}\right) = \underline{1}$$

when $\rho(\vec{x})$ more spread out, $\tilde{\rho}\left(\frac{\vec{q}}{\hbar}\right)$ gets smaller

Elastic Form Factors Reduce Scattering Cross sections

$\tilde{V}(\vec{q})$: (setting $\hbar = 1$ ...)

$\Rightarrow$ about the <u>interaction</u>

<u>Most Famous:</u> $V(\vec{x}) \propto \frac{1}{r}$

(coulomb, fundamental above weak "threshold" color/strong)

$V(\vec{x}) \propto \delta(\vec{x})$ "effective" for weak interaction(s)

$\propto \frac{e^{-Mr}}{r}$ (Yukawa, effective strong)

motivation

coulomb potential

<u>Free space</u>: $\nabla^2 \phi = 0$

$\phi \propto 1/r \Rightarrow \frac{q}{r}$

$E^2 - c^2p^2 = m^2c^4$ (relativity)

(Klein Gordon) $\left(-\hbar^2\frac{\partial^2}{\partial t^2} + \hbar^2c^2\nabla^2\right)\varphi = \left(m^2c^4\right)\varphi$

Static $\frac{\partial^2 \psi}{\partial t^2} = 0$

$$\boxed{\nabla^2 \psi = \frac{m^2 c^2}{\hbar^2} \psi} \quad \text{(free space)}$$

$$\underbrace{\psi \to \phi, \ m = 0}_{\text{electrostatic potential}} , \ \frac{1}{r}$$

Spherical Symmetry

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) = \frac{m^2 c^2}{\hbar^2}\psi$$

<u>$m = 0$</u> : $\psi = \frac{g}{r}$ (q used!)

$$\frac{\partial \psi}{\partial r} \propto -\frac{g}{r^2}$$

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) = -\frac{\partial}{\partial r}(1) = 0$$

<u>$m \neq 0$</u> natural to try $\psi = g\frac{e^{ar}}{r}$

$$\frac{\partial \psi}{\partial r} = g\left[a - \frac{1}{r}\right]\frac{e^{ar}}{r}$$

$$r^2 \frac{\partial \psi}{\partial r} = g\left[ar - 1\right]e^{ar}$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right) = q\left[a + a^2 r - a\right]e^{ar}$$

$$= q a^2 r e^{ar}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right) = \frac{q a^2 e^{ar}}{r} \stackrel{?}{=} \frac{m^2c^2}{\hbar^2} q \frac{e^{ar}}{r}$$

$$a = -\frac{mc}{\hbar} \quad (+ \text{ diverges})$$

$$\boxed{\varphi(r) = \frac{q\, e^{-\frac{mc}{\hbar} r}}{r} \qquad c = \hbar = 1 \text{ could be used.}}$$

<u>massive</u> <u>field</u>

interpret as potential $V(\vec{x})$

<u>want</u> $$\tilde{V}(\vec{q}) = \int d^3x \, \frac{q\, e^{-\frac{mc}{\hbar} r}}{r} e^{-\frac{i\vec{q}\cdot\vec{x}}{\hbar}}$$

$\hbar = c = 1$

$$= \int d^3x \, \frac{q\, e^{-mr}}{r} e^{-\vec{q}\cdot\vec{x}}$$

$\vec{q}$: make it along z axis

$$\vec{q}\cdot\vec{x} = qr\cos\theta = qr\mu \quad \mu = \cos\theta$$

$$\tilde{V}(\vec{q}) = \int_0^\infty dr\, r^2 \int_{-1}^{1} d\nu \int_0^{2\pi} d\phi\, \frac{g\, e^{-mr}}{r} e^{-iqr\nu}$$

$$= 2\pi g \int_0^\infty dr\, \cancel{r}\, \frac{e^{-mr}\left(e^{-iqr} - e^{iqr}\right)}{-iq\cancel{r}}$$

$$= \frac{-2\pi g}{iq}\left(\frac{e^{-(m+iq)r}}{-m-iq} - \frac{e^{(-m+iq)r}}{-m+iq} \right)_0^\infty$$

$$= \frac{-2\pi g}{iq}\left(\frac{-1}{-m-iq} + \frac{1}{-m+iq} \right)$$

$$\left(\frac{\cancel{m} - iq - \cancel{m} - iq}{(-m-iq)(-m+iq)} \right)$$

$$\boxed{\tilde{V}(\vec{q}) = \frac{4\pi g}{q^2 + m^2}}$$

- $q^2 << m^2$? $=$ constant $\downarrow$ δ-function in space

- Troubles: $\tilde{V}(0) \neq 1$! Some people put the 4π in space

$$V(\vec{x}) = \frac{g}{4\pi r} e^{-mr} \qquad \tilde{V}(\vec{q}) = \frac{g}{q^2 + m^2}$$

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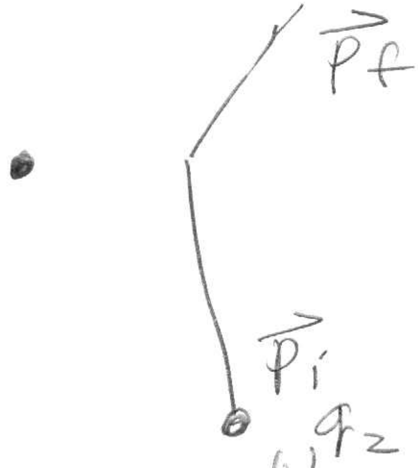
Connection to <u>EM</u>

Target q_1

<u>point</u>

$\tilde{\rho}(\vec{q}) = 1$



$$\underset{\sim}{H} \doteq \frac{q_1 q_2}{4\pi r}$$

look out!

$$= q_2 V(r)$$

$$M_{if} = \frac{1}{V} q_2 \tilde{V}(\vec{q}) \cdot 1$$

quant vol $\rightarrow V$

$$M_{if} = \frac{1}{V} \frac{q_1 q_2}{q^2}$$

<u>Interpretation</u>:

$$M_{if} = \frac{1}{(1)} \frac{q_1 q_2}{q^2}$$

quant vol $= 1$

- <u>Look out</u>: q here is a 3-vector. the "photon" in the diagram above transmits momentum BUT NOT ENERGY from the ∞ heavy target to the projectile.

Like $\underline{\underline{4}}$-momentum of that photon is: $p_{\gamma^*} = (0, \vec{q})$

$$p_{\gamma^*}^2 = 0 - \vec{q}^{\,2} = -q^2 \neq 0$$

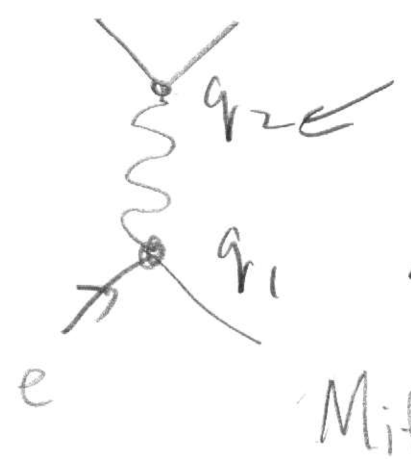
$$-q^2 < 0$$

"Spacelike" virtual photon

In other frames.. always spacelike.

Are there "timelike" virtual photons?

<u>YES</u>



· no net momentum transfer.

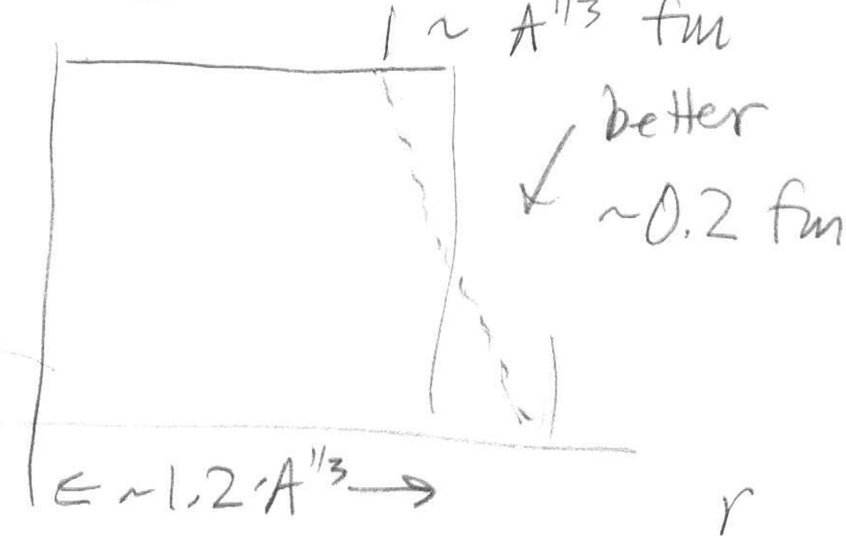
· energy transformed

$$M_{if} = -\frac{q_1 q_2}{Q^2}$$ Q^2 = 4-momentum squared

Other cases

"Structure" of nucleus

$\rho(\vec{x})$ of nucleus : (spherical)



Probes.

Strong : effectively $\boxed{\text{neutron}}$, $m \sim 140$ MeV

in $V(r) = \dfrac{g\, e^{-mr}}{4\pi r}$

$\Rightarrow$ $\boxed{\text{approximate}}$ as a δ-function

(actually true for WIMPs)

$\tilde{V}(\vec{q}) =$ constant

$\rightarrow$ $\tilde{\rho}(\vec{q}) \rightarrow$ "diffraction pattern"

(problem). PLOTS

E+M $\quad V(r) = \frac{q_1}{4\pi r}$

$$\hat{V}(\vec{q}) = \frac{q_1}{q^2}$$

Nucleus

$\hat{\rho}(q) \rightarrow$ diffraction pattern

$$M_{if} \propto (\text{diffraction}) \times \frac{1}{q^2}$$

Hofstadter 1950's

1961 Nobel Prize

$\Rightarrow$ PLOTS ... Carbon

Proton: No diffraction !

(no sharp edge)

$\rightarrow$ Rise of the magnetic moment!