1. (a) $\frac{d\sigma}{dx\,dy} \propto e_q^2 q(x)$ or $e_q^2 \bar{q}(x)$

so for proton, $e_u = \frac{2}{3}$ $e_d = -\frac{1}{3}$

$$\frac{d\sigma_{ep}}{dx\,dy} \propto f_p(x) = \frac{4}{9}\left[u(x) + \bar{u}(x)\right] + \frac{1}{9}\left[d(x) + \bar{d}(x)\right]$$

for a neutron, $d_n(x) = u_p(x) = u(x)$
$u_n(x) = d_p(x) = d(x)$

$$\frac{d\sigma_{en}}{dx\,dy} \propto f_n(x) = \frac{1}{9}\left[d_n(x) + \bar{d}_n(x)\right] + \frac{4}{9}\left[u_n(x) + \bar{u}_n(x)\right]$$

$$= \frac{4}{9}\left[d(x) + \bar{d}(x)\right] + \frac{1}{9}\left[u(x) + \bar{u}(x)\right]$$

(b) $f_p(x) - f_n(x) =$

$$\frac{1}{3}\left[u(x) + \bar{u}(x)\right] - \frac{1}{3}\left[d(x) + \bar{d}(x)\right]$$

$$= \frac{1}{3}\left[u(x) - \bar{u}(x)\right] - \frac{1}{3}\left[d(x) - \bar{d}(x)\right]$$
$$+ \frac{2}{3}\left[\bar{u}(x) - \bar{d}(x)\right]$$

$$\int_0^1 (f_p(x) - f_n(x))\,dx = \frac{1}{3}\int_0^1 \left[u(x) - \bar{u}(x)\right]dx - \frac{1}{3}\int_0^1 \left[d(x) - \bar{d}(x)\right]dx$$

$$+\frac{2}{3}\int[\bar{u}(x)-\bar{d}(x)]\,dx$$

but $\int_0^1[u(x)-\bar{u}(x)]\,dx = 2$ $\quad \int_0^1[d(x)-\bar{d}(x)]\,dx = 1$

2 u quarks in proton $\qquad$ 1 d-quark in proton

$$\int_0^1(f_p(x)-f_n(x))\,dx = \frac{1}{3}\cdot 2-\frac{1}{3}\cdot 1 + \frac{2}{3}\int_0^1[\bar{u}(x)-\bar{d}(x)]\,dx$$

$$= \frac{1}{3}+\frac{2}{3}\int_0^1[\bar{u}(x)-\bar{d}(x)]\,dx$$

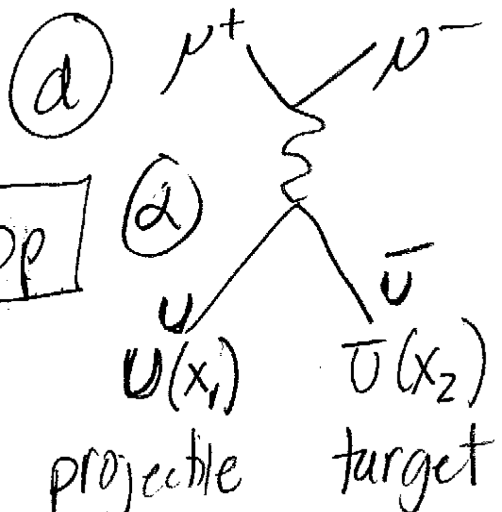
(c) $\frac{1}{4} \cong \frac{1}{3}+\frac{2}{3}\int_0^1[\bar{u}(x)-\bar{d}(x)]\,dx$

$$-\frac{1}{12} \cong \frac{2}{3}\int_0^1[\bar{u}(x)-\bar{d}(x)]\,dx$$

$$\int_0^1[\underset{\uparrow\text{ less}}{\bar{u}(x)}-\underset{\uparrow\text{ more}}{\bar{d}(x)}]\,dx \cong -\frac{1}{8}$$

Perhaps $u\bar{u}$ loops in the sea are suppressed by "Pauli Blocking" the **2** u quarks in the proton reduce available states for the u in the $u\bar{u}$ loop. Only **1** d quark, so "popping" a $d\bar{d}$ pair is perhaps <u>not</u> suppressed.

$\boxed{pp}$



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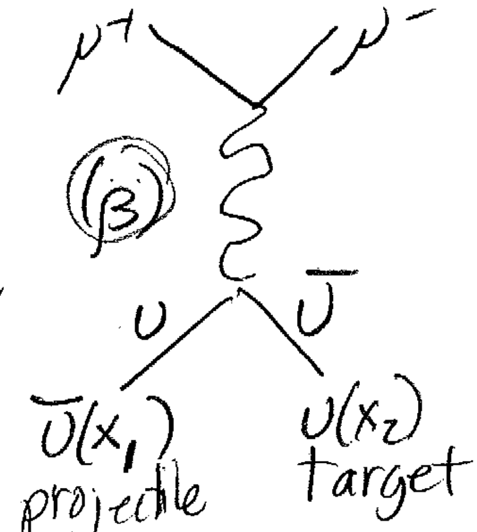
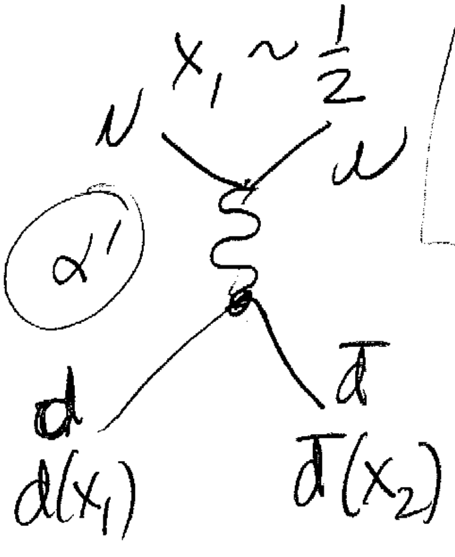
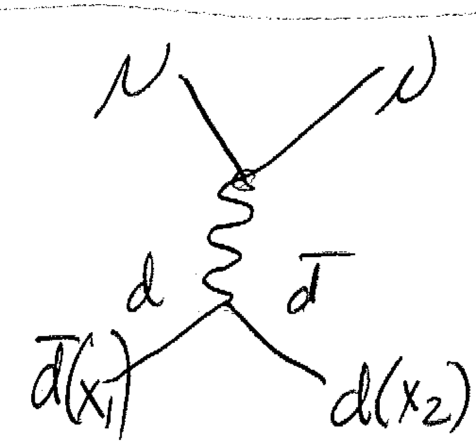


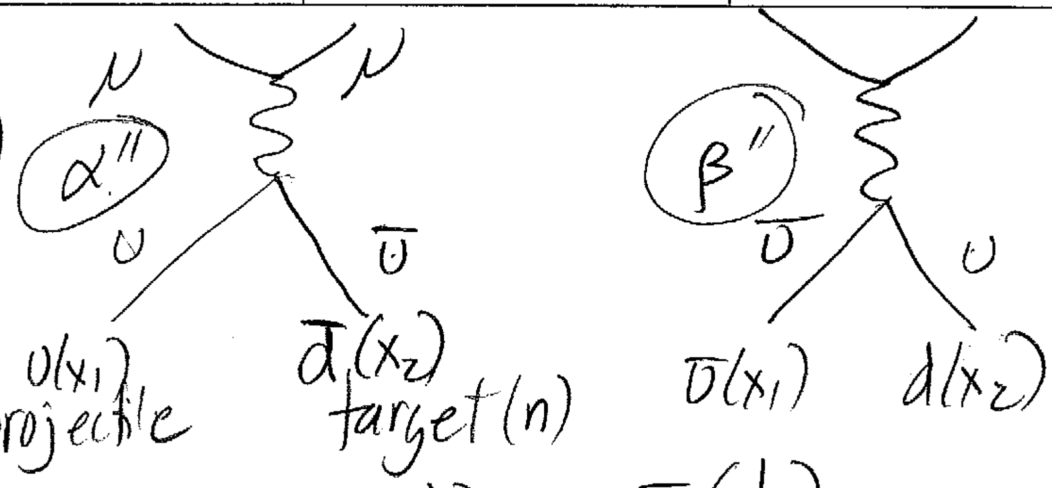
$\bar{u}(x_1) \ll u(x_1)$
can neglect (β)



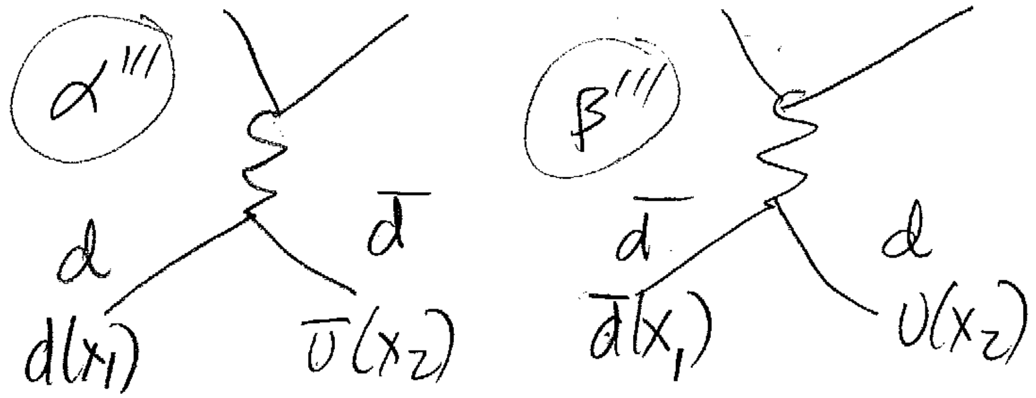


again,
$\bar{d}(x_1) \ll d(x_1)$

$\boxed{pn}$



$x_1 \ldots \quad u(\tfrac{1}{2}) >> \bar{u}(\tfrac{1}{2})$



neglecting <u>sea of projectile</u>

$$\frac{d\sigma_{pp}}{dx_1\,dx_2} \propto \left(\frac{2}{3}\right)^2 u(x_1)\bar{u}(x_2) + \left(\frac{1}{3}\right)^2 d(x_1)\bar{d}(x_2)$$

$$\frac{d\sigma_{pn}}{dx_1\,dx_2} \propto \left(\frac{2}{3}\right)^2 u(x_1)\bar{d}(x_2) + \left(\frac{1}{3}\right)^2 d(x_1)\bar{u}(x_2)$$

again, at $x_1 = \frac{1}{2}$, sea for x_1 is small.

so,

$$\frac{d\sigma_{pd}}{dx_1 dx_2} = \frac{4}{9}\left(u(x_1)\left(\bar{u}(x_2)+\bar{d}(x_2)\right)\right) + \frac{1}{9}\left(d(x_1)\left(\bar{u}(x_2)+\bar{d}(x_2)\right)\right)$$

$$2\frac{d\sigma_{pp}}{dx_1 dx_2} = \frac{8}{9}\, u(x_1)\bar{u}(x_2) + \underbrace{\frac{2}{9}\, d(x_1)\bar{d}(x_2)}_{\substack{\text{neglect} \\ \text{since } 4u(x_1) \gg d(x_1)}}$$

$$\frac{\dfrac{d\sigma_{pd}}{dx_1 dx_2}}{2\dfrac{d\sigma_{pp}}{dx_1 dx_2}} \simeq \frac{1}{2}\left[\frac{u(x_1)\left(\bar{u}(x_2)+\bar{d}(x_2)\right)}{u(x_1)\,\bar{u}(x_2)}\right]$$

$$\simeq \frac{1}{2}\left[1+\frac{\bar{d}(x_2)}{\bar{u}(x_2)}\right]$$

if $\bar{u}(x_2) = \bar{d}(x_2)$, $= 1$

since $\bar{d}(x_2) > \bar{u}(x_2)$, > 1

this is E866's result, shown in the figure ...

2 (a)(i)

$$\delta\left(\nu - \frac{Q^2}{2m}\right) : \quad \nu = E - E'$$

$$q = k - k'$$

$$q^2 = k^2 + k'^2 - 2kk'$$

$$= -2(EE' - EE'\cos\theta)$$

$$q^2 = -4EE'\sin^2\theta/2$$

$$Q^2 = -q^2 = 4EE'\sin^2\theta/2$$

$$\delta\left(\nu - \frac{Q^2}{2m}\right) = \delta\left(E - E' - \frac{2EE'}{m}\sin^2\theta/2\right)$$

$$= \delta\left(E'\left(1 + \frac{2E}{m}\sin^2\theta/2\right) - E\right)$$

(since δ function symmetric)

$$= \frac{1}{1 + \frac{2E}{m}\sin^2\theta/2}\,\delta\left(E' - \frac{E}{1 + \frac{2E}{m}\sin^2\theta/2}\right)$$

since $\delta(ax) = \frac{1}{a}\delta(x)$

(ii) $$\delta\left(\nu - \frac{Q^2}{2m}\right) = \frac{1}{\nu}\delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

$$= \frac{1}{\nu}\delta\left(\frac{M_p}{m}\left(\frac{m}{M_p} - \frac{Q^2}{2M_p\nu}\right)\right)$$

$$= \frac{m}{M_p\nu}\,\delta\left(x - \frac{m}{M_p}\right)$$

$$y = \frac{pq}{pk} = \frac{M_p(E - E')}{M_p E} = \frac{\nu}{E}$$

$$\nu = yE \qquad s = (p+k)^2 = M_p^2 + 2M_pE \sim 2M_pE$$

so

$$\delta\left(\nu - \frac{Q^2}{2m}\right) = \frac{m}{M_p E y}\,\delta\left(x - \frac{m}{M_p}\right)$$

(b) Already from above

$$y = 1 - \frac{E'}{E} \Rightarrow \underline{E' = E(1-y)}$$

so, $\dfrac{\partial E'}{\partial x} = 0 \Rightarrow$ will mean $\dfrac{\partial \Omega}{\partial y}$ irrelevant!

$$\frac{\partial E'}{\partial y} = -E$$

$$x = \frac{Q^2}{2M_p\nu} = \frac{4EE'\sin^2\theta/2}{2M_p(E-E')}$$

$$= \frac{EE'}{M_p(E-E')}\,(1-\cos\theta) \qquad \Omega = 2\pi(1-\cos\theta)$$

$$\Omega = 2\pi(1-\cos\theta) = 2\pi\,\frac{M_p(E-E')}{EE'}\,x$$

$$\frac{\partial \Omega}{\partial x} = \frac{2\pi M_p(E-E')}{EE'} = \frac{2\pi M_p y}{E'}$$

and so

$$\left|\frac{\partial(E',\Omega)}{\partial(x,y)}\right| = \begin{vmatrix} \frac{\partial E'}{\partial x} & \frac{\partial E'}{\partial y} \\ \frac{\partial \Omega}{\partial x} & \frac{\partial \Omega}{\partial y} \end{vmatrix} = \begin{vmatrix} 0 & -E \\ \frac{2\pi M_p y}{E'} & \boxed{} \end{vmatrix}$$

irrelevant

$$= \frac{2\pi M_p E y}{E'}$$

and so:

$$\frac{d\sigma}{dE'd\Omega} = \frac{4\alpha^2 e_q^2 E'^2}{Q^4}\left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2m^2}\sin^2\frac{\theta}{2}\right]\delta\left(\nu - \frac{Q^2}{2m}\right)$$

$$\frac{d\sigma}{dx\,dy} = \left|\frac{\partial(E',\Omega)}{\partial(x,y)}\right| \frac{d\sigma}{dE'd\Omega}$$

$$= \frac{2\pi M_p \cancel{Ey}}{E'} \cdot \underbrace{\frac{m}{\cancel{M_p Ey}}}_{\text{from } \delta \text{ function}} \frac{4\alpha^2 e_q^2 E'^2}{Q^4}\left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2m^2}\sin^2\frac{\theta}{2}\right]\delta\left(x - \frac{m}{M_p}\right)$$

$$= \frac{8\pi\alpha^2 e_q^2 m E'}{Q^4}\left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2m^2}\sin^2\frac{\theta}{2}\right]\delta\left(x - \frac{m}{M_p}\right)$$

(c) $\hat{s} \simeq 2mE$

to boost into the c.m. frame...

$$\gamma = \frac{E}{\sqrt{\hat{s}}} = \sqrt{\frac{E}{2m}} \qquad \beta = \sqrt{1 - \frac{1}{\gamma^2}} \simeq 1 - \frac{m}{E}$$

p^*: quark $\quad -|\vec{p}^*| = \gamma(0 - \beta m)$

$$= -\sqrt{\frac{mE}{2}} \quad \left(= -\frac{1}{2}\sqrt{\hat{s}}\right)$$

electron $\quad |\vec{p}^*| = \gamma(E - \beta E)$

$$= \sqrt{\frac{E}{2m}}\,E(1-\beta) = \sqrt{\frac{mE}{2}}$$

(just a check)

$$E' = \gamma(p^* + \beta p^* \cos\theta^*)$$

$$= \sqrt{\frac{E}{2m}}\sqrt{\frac{mE}{2}}(1+\cos\theta^*) = E\cdot\frac{1}{2}(1+\cos\theta^*)$$

$$E' = E\cos^2(\theta^*/2)$$

<u>Lab</u>

$$p_{\parallel e} = \gamma(p^*\cos\theta^* + p^*)$$

$$= \sqrt{\frac{E}{2m}}\sqrt{\frac{mE}{2}}(1+\cos\theta^*)$$

$$= E\cdot\frac{1}{2}(1+\cos\theta^*) = E\cos^2\theta^*/2$$

$$p_{\perp e} = p^*\sin\theta^*$$

$$= \sqrt{\frac{mE}{2}}\sin\theta^* = 2\sqrt{\frac{mE}{2}}\sin\frac{\theta^*}{2}\cos\frac{\theta^*}{2}$$

$$\tan\theta \simeq \theta = \frac{p_{\perp e}}{p_{\parallel e}} = \frac{2\sqrt{\frac{mE}{2}}\sin\frac{\theta^*}{2}\cos\frac{\theta^*}{2}}{E\cos^2\theta^*/2}$$

$$\simeq \sqrt{\frac{2m}{E}}\tan\theta^*/2$$

<u>and so</u> $$\cos^2\frac{\theta}{2} \simeq 1 - \underbrace{\frac{2m}{E}\tan^2\theta^*/2}_{\text{negligible}}$$

<u>because</u> $$\frac{Q^2}{2m^2}\sin^2\frac{\theta}{2} \Rightarrow$$

$$Q^2 = 4|p^*|^2\sin^2\frac{\theta^*}{2} = 4\frac{mE}{2}\sin^2\frac{\theta^*}{2}$$

$$\sin^2\frac{\theta}{2} = \frac{1}{4}\cdot\frac{2m}{E}\tan^2\theta^*/2$$

7/5/6

$$\frac{Q^2}{2m^2}\sin^2\frac{\theta}{2} = \frac{2\cancel{m}\cancel{E}}{2m^{\cancel{2}}}\sin^2\frac{\theta^*}{2}\cdot\frac{m}{2\cancel{E}}\tan^2\theta^*/2$$

$$= \frac{1}{2}\,\frac{\sin^4\theta^*/2}{\cos^2\theta^*/2}$$

so, $2mE = 2M_p E\left(\frac{m}{M_p}\right) = s\left(\frac{m}{M_p}\right)$

(d)

$$\frac{d\sigma}{dx\,dy} = \frac{8\pi\alpha^2 e_q^2\, m \overbrace{E'}^{E\cos^2\theta^*/2}}{Q^4}\left[\underbrace{\cos^2\frac{\theta}{2}}_{1} + \underbrace{\frac{Q^2}{2m^2}\sin^2\frac{\theta}{2}}_{\frac{1}{2}\frac{\sin^4\theta^*/2}{\cos^2\theta^*/2}}\right]\delta\left(x-\frac{m}{M_p}\right)$$

$$= \frac{4\pi\alpha^2 e_q^2\, s\cdot\left(\frac{m}{M_p}\right)}{Q^4}\cdot\frac{1}{2}\left[2\cos^2\frac{\theta^*}{2} + \sin^4\theta^*/2\right]\delta\left(x-\frac{m}{M_p}\right)$$

$$= \frac{2\pi\alpha^2 e_q^2 s}{Q^4}\left[2\cos^2\frac{\theta^*}{2}+\sin^4\frac{\theta^*}{2}\right]\left(\frac{m}{M_p}\right)\delta\left(x-\frac{m}{M_p}\right)$$

because of the δ-function, this can be changed to x.

$$2\cos^2\frac{\theta^*}{2}+\sin^4\frac{\theta^*}{2} = 2\cos^2\frac{\theta^*}{2} + \left(1-\cos^2\frac{\theta^*}{2}\right)^2$$

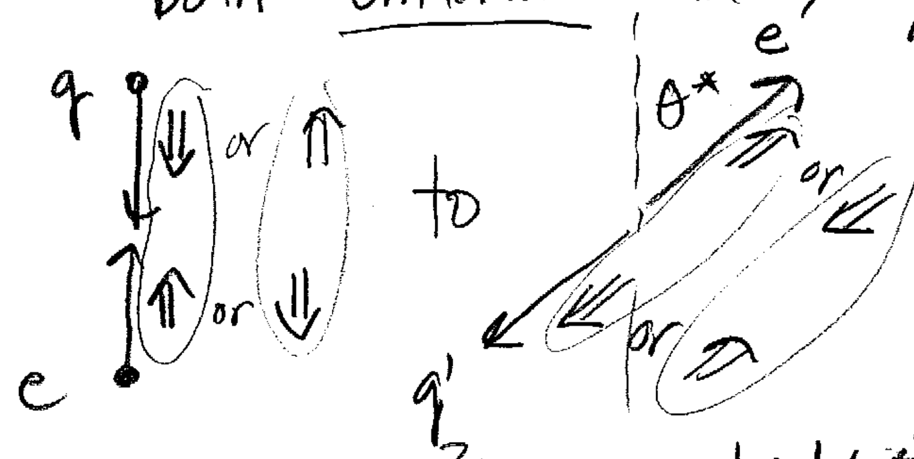
$$= 1+\cos^4\frac{\theta^*}{2}$$

but $\frac{E'}{E} = \cos^2\frac{\theta^*}{2} = 1-y$

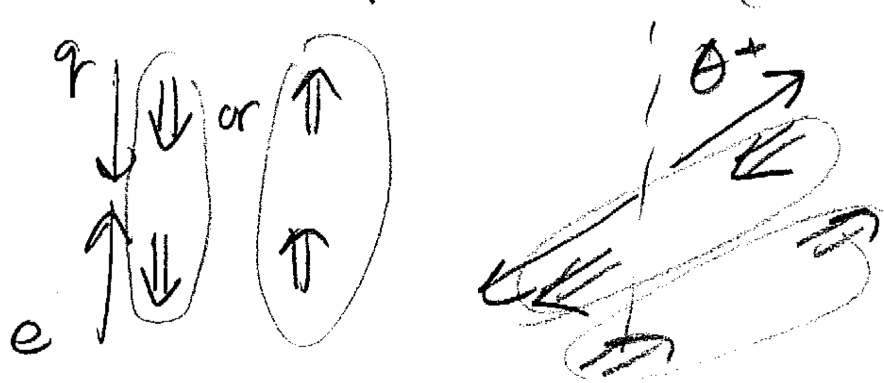
so, finally,

$$\boxed{\frac{d\sigma}{dx\,dy} = \frac{2\pi\alpha^2 e_q^2 s}{Q^4}\underbrace{\left[1+(1-y)^2\right]}_{\text{with electromagnetic interactions..}}\times\delta\left(x-\frac{m}{M_p}\right)}$$

both <u>uniform..</u> in , y



and $(1-y)^2$ ... $\propto \left|d^1_{11}(\theta^*)\right|^2$



Compare with νq (uniform)

$\nu \bar{q}$ $(1-y)^2$

$\bar{\nu}\bar{q}$ (uniform)

$\bar{\nu} q$ $(1-y)^2$