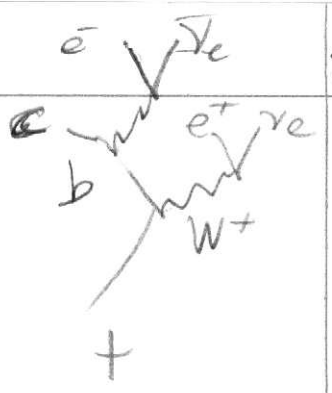



(4) Semileptonic Decays of
b-quark
t-quark.

(5) $Z^0 \to e^+e^-$ + $W^- \to e^-\bar{\nu}_e$

LHC : $H^0 \to Z^0 Z^0 \to e^+e^-$ $\left(\frac{1}{30}\right)^2$
$\to e^+e^-$

(CMS) problem BR $Z^0 \to e^+e^-$ ] $\simeq 3.4\%$

## Experimental Detection of Electrons

common to all charged particles

(1) "$\frac{dE}{dx}$" → how much energy lost per distance

$\Rightarrow dx \to \rho dx$ ] $\frac{gm}{cm^2}$]

$\propto \frac{\#\ nucleons}{cm^2}$

"nuclear"

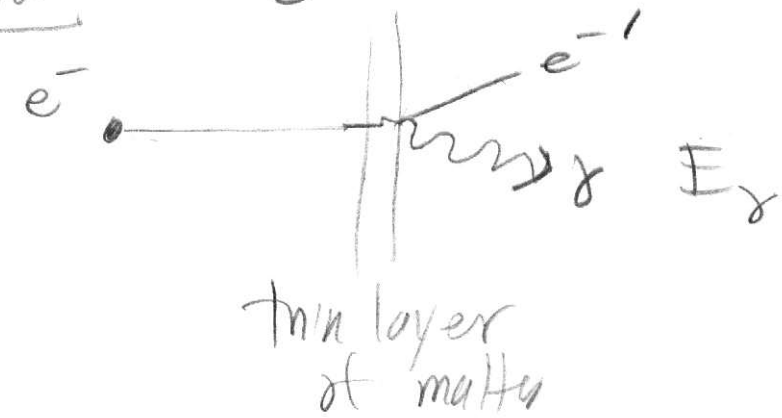
guess: 2 $\frac{MeV}{gm/cm^2}$

(2) Scattering ] why vacuum important.

→ Pic, Low energy

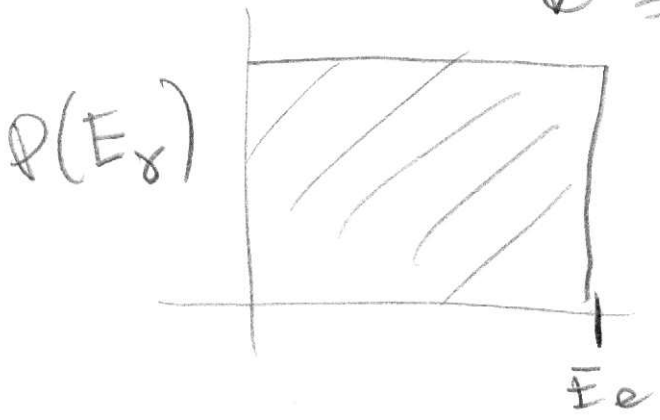
(3) Bremsstrahlung.

Key point: $E_e$



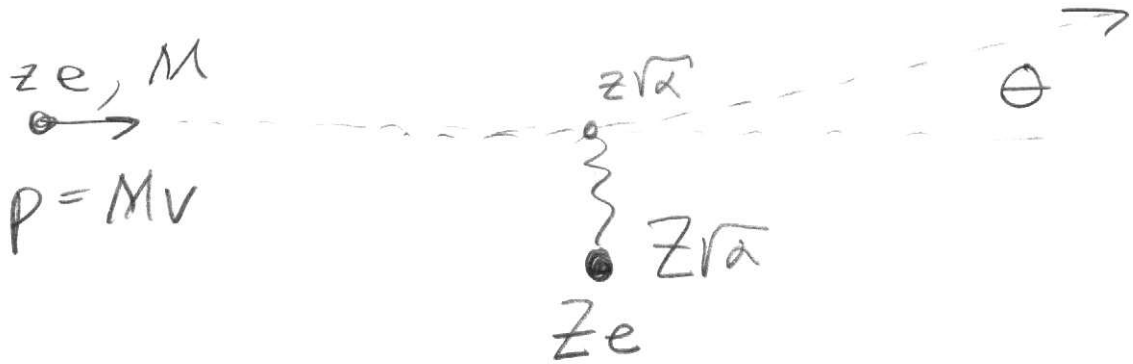

For sufficient $E_e$,

↙ flat



# Multiple Coulomb Scattering

You know... Rutherford Scattering.. (Jackson 13.7)




(N.R.) $\frac{d\sigma}{d\Omega} = \underbrace{\frac{z^2 Z^2 e^4}{4 p^2 v^2}}_{\text{trick: relativistically correct}} \frac{1}{\sin^4 \frac{\theta}{2}}$ ← dimension $(E^2 \ell^2)$

$$= \frac{1}{4} \underbrace{\frac{z^2 Z^2 e^4}{(\hbar c)^2 (v/c)^2}}_{\text{dimensionless}} \underbrace{\frac{(\hbar c)^2}{(cp)^2 \sin^4 \theta/2}}_{\text{descended from } 1/q^4 \text{ propagator } (\gamma)}$$

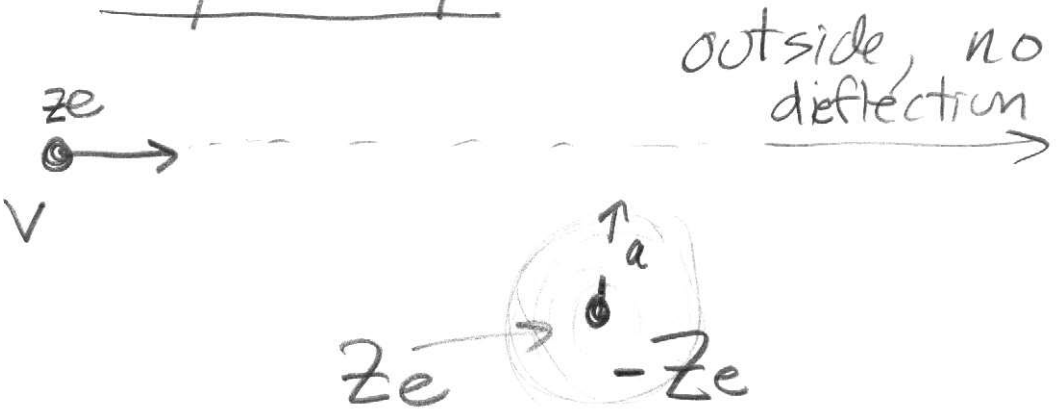
$(z Z \sqrt{\alpha}\, r\sqrt{\alpha})^2$

Problem

$$\int_0^{\pi/2} \frac{d\sigma}{d\Omega} d\Omega = \infty$$

important

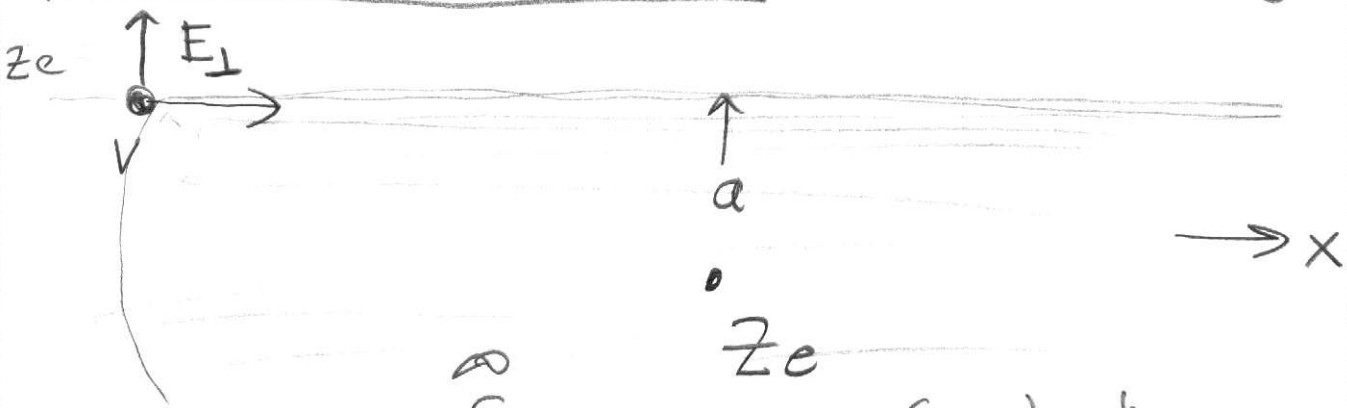
# Physically




Thomas Fermi:

$$a \simeq \frac{a_0}{Z^{1/3}}$$ (for all Z electrons)

What Momentum Transfer $q$ Corresponds to $a$? (no screening)




$$q \simeq \int_{-\infty}^{\infty} dt\, ze E_\perp$$ (neglecting deflection)

$$\Phi_E = \int dx\, (2\pi a) E_\perp = 4\pi Ze$$

$$dx = v\,dt$$

$$q_r \simeq \int dt\; ze\underline{E}_\perp = \frac{4\pi z Z e^2}{2\pi a v}$$

"classical" → $$\boxed{q_c \simeq \frac{2zZe^2}{va}}$$

Look out: uncertainty principle.

quantum → $$q_q a \gtrsim \hbar$$

$$q_q \gtrsim \frac{\hbar}{a}$$

want $\dfrac{q_c}{q_q} > 1$ (avoid U.P. issues)

$$\frac{q_c}{q_q} \cdot \frac{\dfrac{2zZe^2}{va}}{\dfrac{\hbar}{a}} = 2\frac{zZe^2}{\hbar v}$$

Particle Physics: $z = 1$

$Z \simeq 14$ (silicon)

$$\frac{q_c}{q_q} = \frac{2 \cdot 14 \cdot e^2}{\hbar c} \cdot \frac{1}{\beta}$$

$$\simeq \frac{28}{137} \cdot \frac{1}{\beta}\ \Big]\ \begin{matrix}\text{quantum} \\ \beta \to 1 \\ \text{classical } e^-\end{matrix}$$

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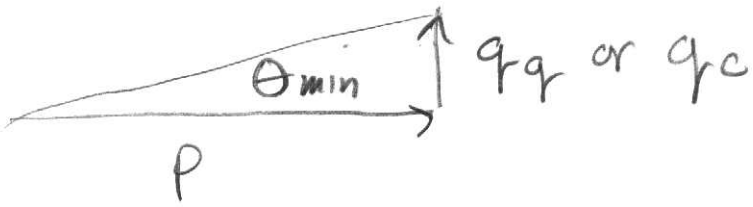
Estimate $\theta_{min}$




1 keV electron

$$\theta_{min} \simeq \frac{2zZe^2}{pVa} \sim \frac{zZ}{T}\frac{e^2}{\hbar c}\frac{\hbar c}{a}$$

Silicon: $Z = 14$

$$a \simeq \frac{a_0}{14^{1/3}} \sim \frac{\frac{1}{2}\cdot 10^{-8}\,cm}{2.4} = 2\cdot 10^{-9}\,cm$$

$$\simeq 0.02\,nm$$

$$\hbar c \simeq 200\ MeV\cdot fm \simeq 200\ eV\cdot nm$$

$$\theta_{min} \simeq \frac{14}{10^3}\cdot\frac{1}{137}\cdot\frac{200}{0.02}$$

$\theta_{min} \simeq 1$ radian — 1 keV electron "lots of scattering"

1 GeV charged, relativistic particle

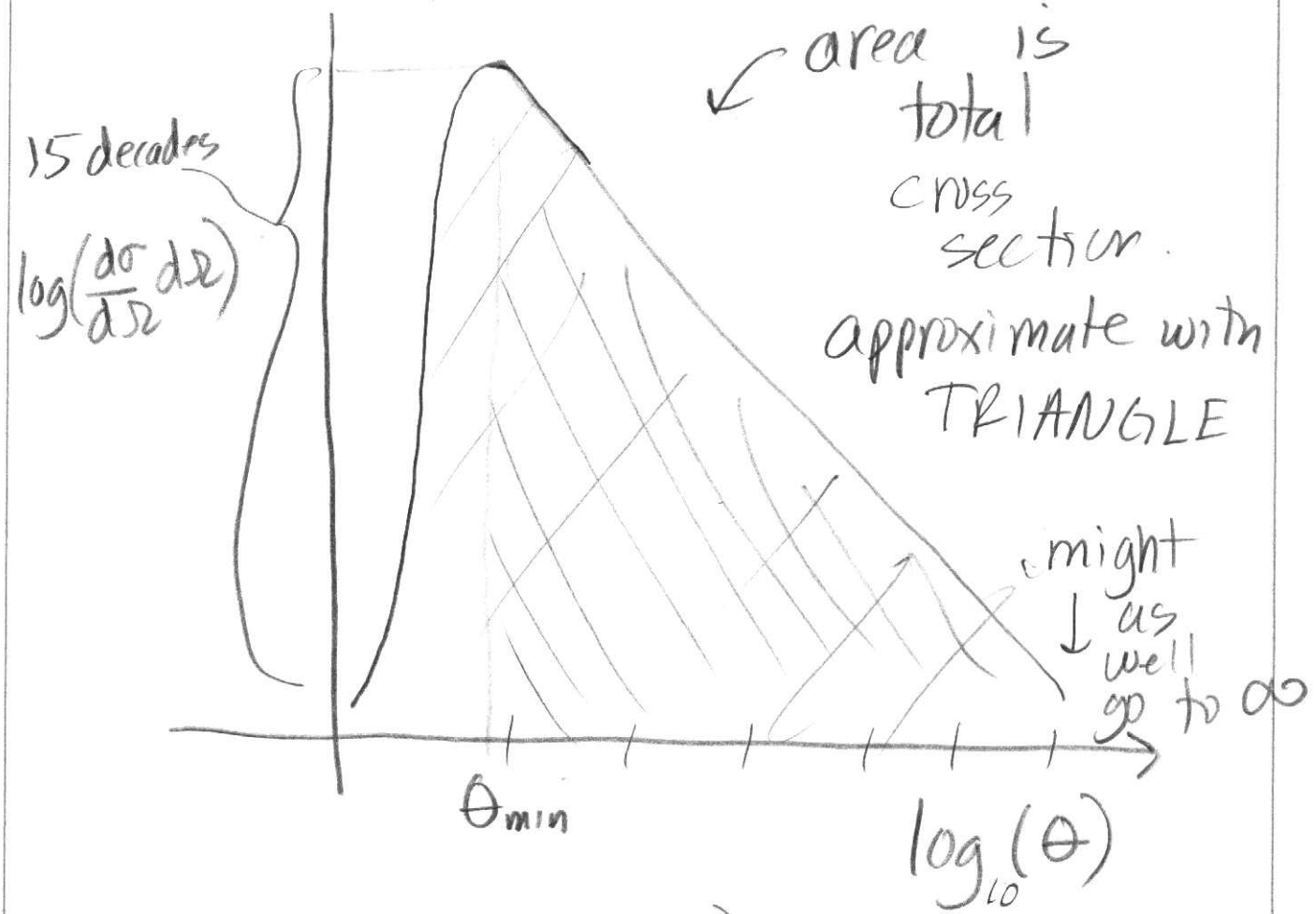
$$\theta_{min} \simeq \frac{\hbar}{pa} \simeq \frac{\hbar c}{cp\cdot a} \simeq \frac{200\cdot eV\cdot nm}{10^9\cdot 0.02}$$

$\theta_{min} \simeq 10^{-5}$ Radians

$$\frac{d\sigma}{d\Omega} \propto \frac{1}{\theta^4}, \quad \left(\frac{d\sigma}{d\Omega}\right) d\Omega \propto \frac{2\pi\theta}{\theta^4}$$

$$\propto \frac{2\pi}{\theta^3}$$

in $\int \frac{d\sigma}{d\Omega} d\Omega$, region near $\theta \sim 0$ dominates




(1 GeV)

5 decades

$$\sigma_{tot} \simeq \int_{\theta_{min}}^{\infty} \frac{d\sigma}{d\Omega} d\Omega$$

$$\simeq \frac{4 z^2 Z^2 \alpha^2 (\hbar c)^2}{\beta^2 (cp)^2} \int_{\theta_{min}}^{\infty} \frac{2\pi\theta}{\theta^4}$$

$$\simeq \frac{4\pi z^2 Z^2 \alpha^2 (\hbar c)^2}{\beta^2 c^2 p^2} \quad \left( \frac{1}{\theta_{min}^2} = \frac{p^2 a^2}{\hbar^2} \right)$$

$$\sigma_{tot} \simeq 4 \frac{z^2 Z^2 \alpha^2}{\beta^2} (\pi a^2 \to \text{size})$$

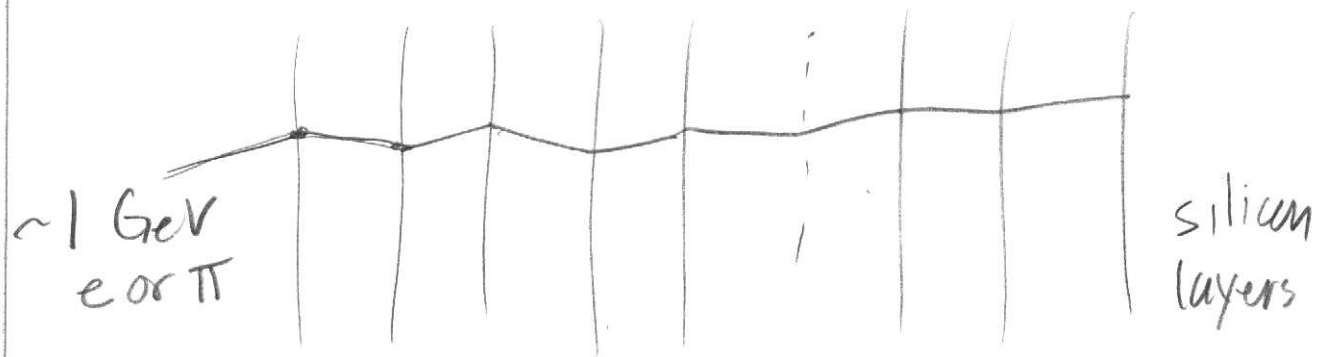
we are relativistic : $\beta \simeq 1$

$$\text{Silicon} \simeq 4 \cdot 14^2 \cdot \left(\frac{1}{137}\right)^2 \cdot \pi \cdot \left(\frac{1}{50}\right)^2 \cdot 10^{-18} \ m^2$$

$$\simeq 5 \cdot 10^{-19} \ cm^2$$

## Multiple Scattering

CMS: >12 layers silicon

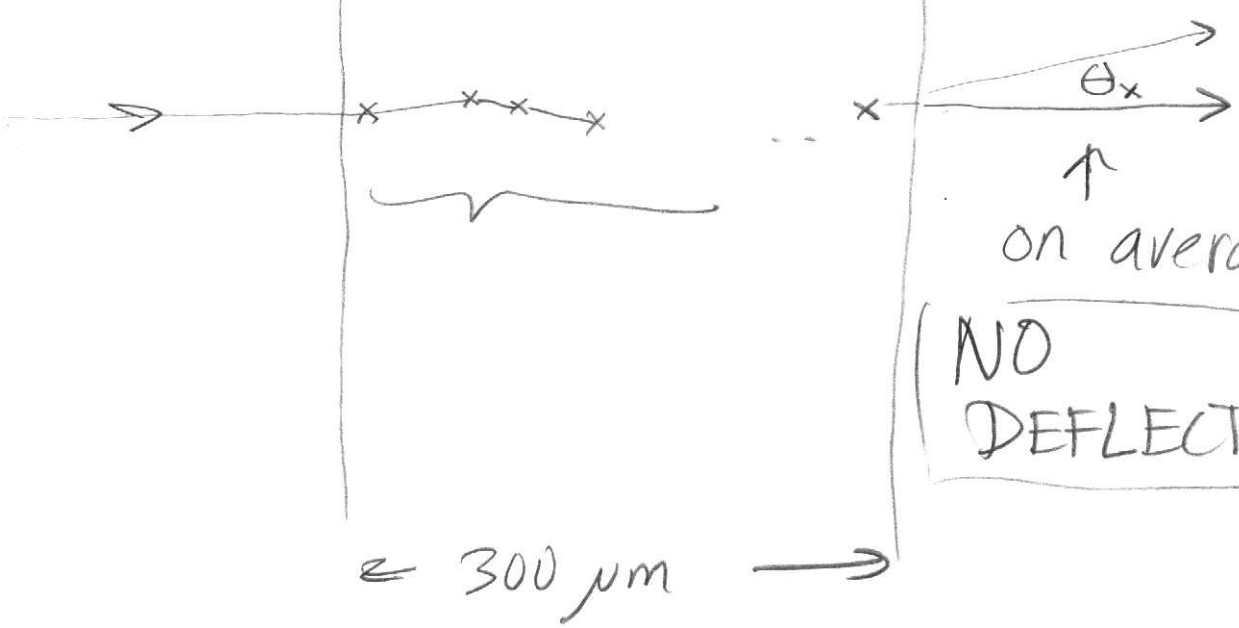



Worse: $\theta_{min}$ so small, many scatters

Like: Stat Mech
or Random Walk.

Project into Plane of Page

x = plane of page
y = out of plane




Of interest: width of angular distribution

"Probability Distribution Function" of $\theta_x$.

→ mean value is 0

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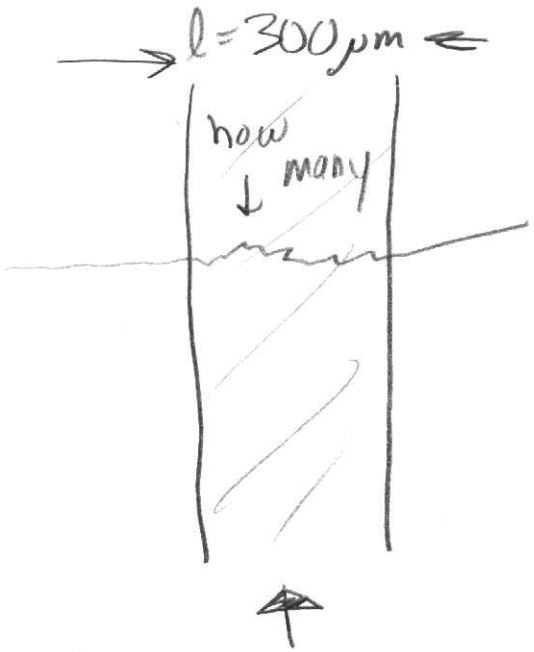
It is worse: Silicon




density $\simeq 2.3\ gm/cm^3$

$A \simeq 28\ gm/mole.$

Number Density: $\frac{2.3}{28} \cdot 6 \cdot 10^{22}\ \frac{1}{cm^3}$

$n \simeq 5 \cdot 10^{22}/cm^3$

# interactions:

$N \simeq n \cdot \sigma \cdot l$

$\simeq 5 \cdot 10^{22} \cdot 5 \cdot 10^{-19} \cdot 3 \cdot 10^{-2}$

$N \simeq 750$ interactions

LOTS! Good News } each only $\simeq 10^{-5}$ radians!

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At each scatter: $P(\theta) \propto \frac{1}{\theta^4}$

Many Scatters: $P(\theta_x) \propto e^{-\theta_x^2 / 2\langle\theta_x^2\rangle}$

Your Background:
- Random Walk
- Stat Mech: BE, FD into Maxwellian
- Central Limit Theorem.

(Figure)

- Pythagorean: $\theta^2 = \theta_x^2 + \theta_y^2$

full spatial angle — assume azimuthal symmetry.

$$\langle\theta^2\rangle = 2\langle\theta_x^2\rangle$$

or $$\langle\theta_x^2\rangle = \frac{1}{2}\langle\theta^2\rangle$$

- Many interactions ... independent

$$\theta = \theta_1 + \theta_2 + \theta_3 + \cdots + \theta_N$$

$$\langle\theta\rangle = \langle\theta_1\rangle + \langle\theta_2\rangle + \cdots + \langle\theta_N\rangle$$

$$= 0$$

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$$\theta^2 = \theta_1^2 + 2\theta_1\theta_2 + \theta_2^2 + \theta_2\theta_3 + \theta_1\theta_3 + \theta_3^2 + \ldots$$

$$\langle\theta^2\rangle = \langle\theta_1^2\rangle + 2\underbrace{\langle\theta_1\theta_2\rangle}_{\text{all cross terms average to zero}} + \ldots$$

statistical independence.

$$\langle\theta_1^2\rangle = \langle\theta_2^2\rangle = \langle\theta_3^2\rangle = \ldots$$

$$\equiv \langle (H)^2 \rangle \quad \text{(variance of single scatter)}$$

$$\langle\theta_x^2\rangle = \frac{1}{2}\langle\theta^2\rangle = \frac{1}{2} N \langle (H)^2 \rangle$$

$N$ = # interactions $= n\sigma_{tot}\ell$

$$\langle (H)^2 \rangle \cong \frac{\int_{\theta_{min}}^{\infty} \theta^2 \frac{d\sigma}{d\Omega} d\Omega}{\int_{\theta_{min}}^{\infty} \frac{d\sigma}{d\Omega} d\Omega}$$

$$\simeq \frac{\int_{\theta_{min}}^{\infty} \frac{\theta^3}{\theta^4} d\theta}{\int_{\theta_{min}}^{\infty} \frac{\theta}{\theta^4} d\theta} = \theta_{min}^2 \ln\left(\frac{\infty}{\theta_{min}}\right)$$

ugg!

A bit of a problem ... $\infty \to 1$ or $\pi/2$

$$\langle \theta_x^2 \rangle = \frac{1}{2} n \sigma_{tot} \ell \, \theta_{min}^2 \cdot \ln\left(\frac{pa}{\hbar}\right)$$

$$= \frac{1}{2} n \sigma_{tot} \ell \left(\frac{\hbar^2}{p^2 a^2}\right) \ln\left(\frac{pa}{\hbar}\right)$$

$$\theta_0 \equiv \langle \theta_x^2 \rangle^{1/2}$$

$$= \frac{\sqrt{\ell}}{p} \times \underbrace{\left[\frac{1}{2} n \sigma_{tot} \left(\frac{\hbar}{a}\right)^2 \ln\left(\frac{pa}{\hbar}\right)\right]^{1/2}}$$

similar problems afflict Bremmstrahlung!

Usual to measure $\ell$ in Radiation Lengths

(Silicon ... $X_0 \simeq 22 \frac{gm}{cm^2}$

$$Z_0 = \frac{X_0}{\rho} = \frac{22}{2.3}$$

$$\simeq 9.5 \text{ cm}$$

then $\theta_0 \simeq \frac{14 \text{ MeV}}{\beta c p} z \sqrt{\frac{\ell}{Z_0}} \times (1 + \text{small})$.

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