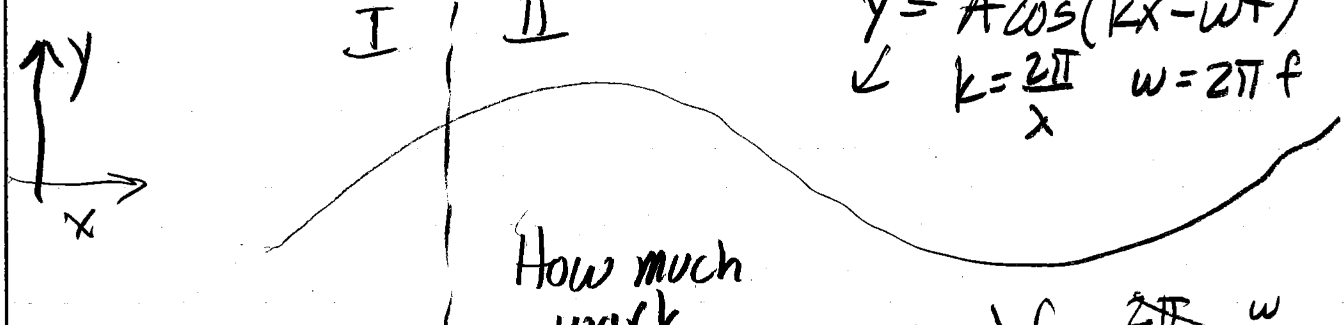
Power in Waves



$y = A\cos(kx - \omega t)$

$k = \frac{2\pi}{\lambda} \quad \omega = 2\pi f$

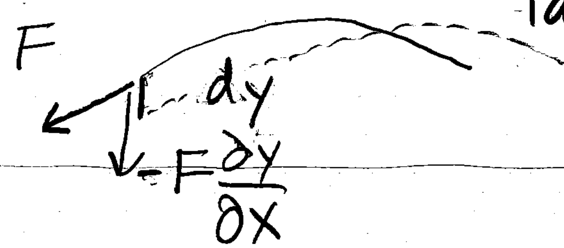
How much work done on region II by I?

$v = \lambda f = \frac{\cancel{2\pi}}{k} \cdot \frac{\omega}{\cancel{2\pi}}$

$= \frac{\omega}{k}$

How much <u>power</u>? (in dt

later, dt



$$dW = -F\frac{\partial y}{\partial x}\,dy$$

$$P = \frac{dW}{dt} = -F\frac{\partial y}{\partial x}\frac{\partial y}{\partial t}$$

$$y(x,t) = A\cos(kx - \omega t)$$

$$\frac{\partial y}{\partial x} = -kA\sin(kx - \omega t) \qquad \frac{\partial y}{\partial t} = +\omega A\sin(kx - \omega t)$$

y max, $\frac{\partial y}{\partial x} \cdot \frac{\partial y}{\partial t} \propto P$ <u>min</u>

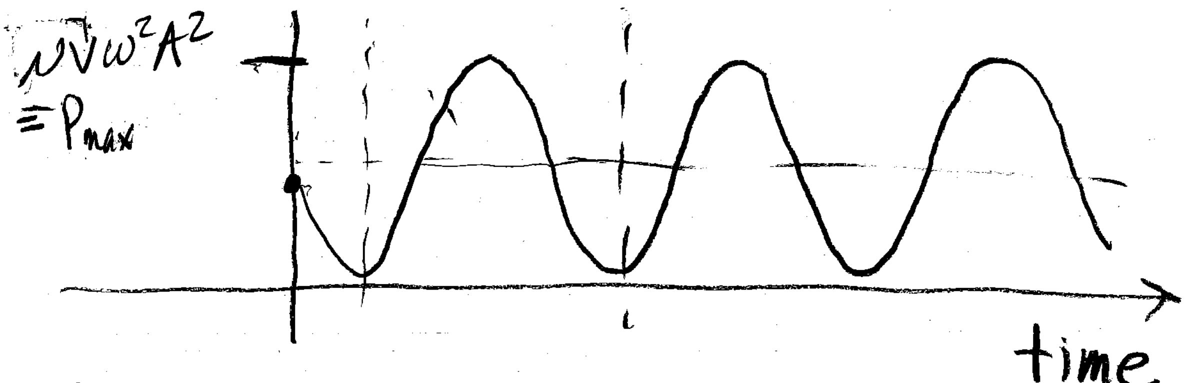
$$P = -F\left(-kA\sin(kx-\omega t)\right)\left(\omega A\sin(kx-\omega t)\right)$$

$$P = Fk\omega A^2 \sin^2(kx-\omega t)$$

$$k = \frac{\omega}{v} \qquad v = \sqrt{\frac{F}{\mu}} \quad F = \mu v^2$$

$$P = \mu v^2 \frac{\omega}{v} \omega A^2 \sin^2(kx-\omega t)$$

$$\boxed{P = \mu v \omega^2 A^2 \sin^2(kx-\omega t)}$$



$$P_{av} = \frac{1}{2} \cdot P_{max}$$

$$\boxed{P_{av} = \frac{1}{2}\mu v \omega^2 A^2}$$

never attenuates in <u>one</u>-dimension

2D/3D

"Intensity" = $\frac{\text{Power}}{(\text{unit boundary})}$

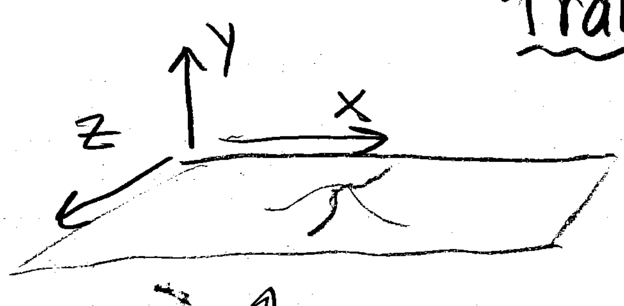
constant — grows w/ prop

More Dimensions

1-d: $F \frac{\partial^2 y}{\partial^2 x} = \mu \frac{\partial^2 y}{\partial t^2}$

2-d: y still displacement

transverse



membrane or surface

can have curvature $\frac{\partial^2 y}{\partial z^2}$ too!

m × acceleration = Σ Forces

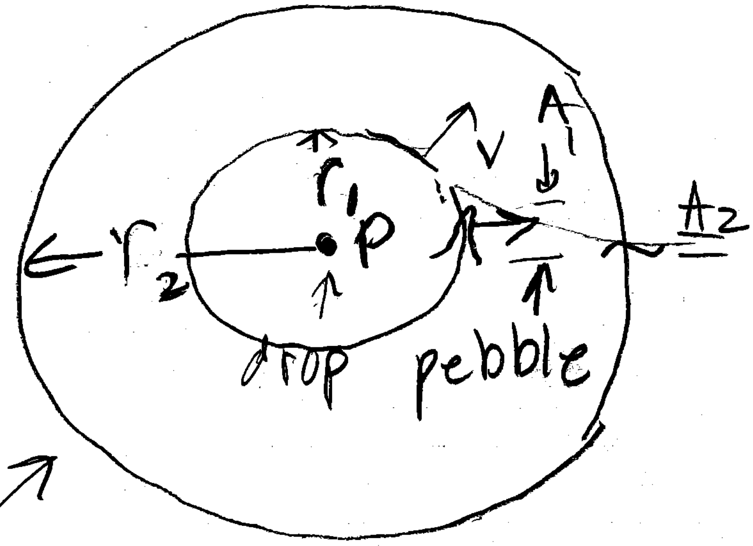
$$\boxed{\gamma \left(\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial z^2} \right) = \sigma \frac{\partial^2 y}{\partial t^2}}$$

mass/area!

2-d wave equation.

Consider energy again...

looking from above at a surface.



same P around circumference

$\left(\frac{\text{power}}{2\pi r}\right) \equiv$ intensity of power

goes down as $\frac{1}{r}$

why? 2 dimensions.

How: amplitude of wave decreases.

$$A \propto \frac{A_0}{\sqrt{r}}$$

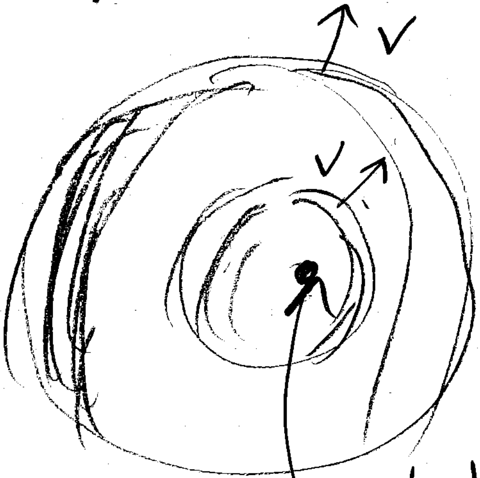
$$\text{Intensity} \propto \frac{A_0^2 \omega^2 v}{r}$$

doesn't change

3-d sound y (displacement) $\Rightarrow \xi$

now, can have longitudinal motion in addition to transverse.

$$P\left(\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} + \frac{\partial^2 \xi}{\partial z^2}\right) = \rho \frac{\partial^2 \xi}{\partial t^2}$$



initial disturbance.

$$\frac{\text{power}}{\text{surface area}} = \frac{\text{power}}{4\pi r^2} \equiv \text{intensity}$$

goes down as $\frac{1}{r^2}$

$$A \propto \frac{A_0}{r}$$

$$\text{Intensity} \propto \frac{A_0^2 \omega^2 v}{r^2}$$

Superposition

if $y_1(x,t)$ and $y_2(x,t)$ are both solutions to the wave equation,

$$a_1 y_1(x,t) + a_2 y_2(x,t)$$

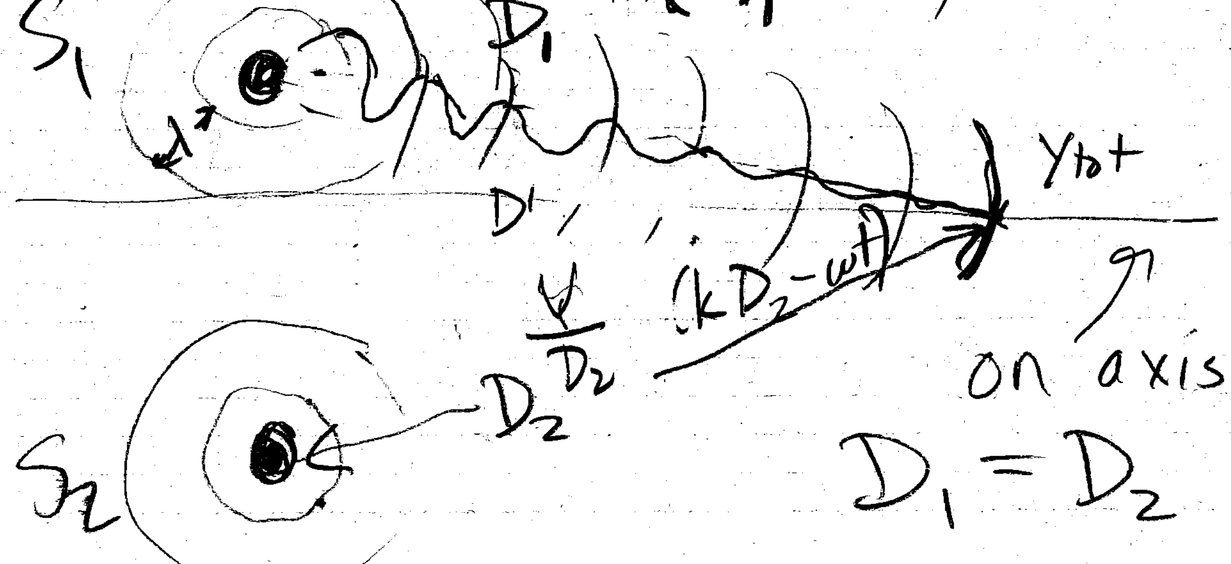
is also ... proof straightforward.

Consequences

(1) Interference ... bright spots, dead spots in concert halls.

$k = \frac{2\pi}{\lambda}$ $\omega = \frac{2\pi}{T} = 2\pi\nu$

$\frac{Y}{D_1}\sin(kD_1 - \omega t)$



on axis

$D_1 = D_2$

on axis... $D_1 = D_2 = D$

$$y_{tot} = \frac{Y}{D}\underbrace{\left(\sin(kD-\omega t)+\sin(kD-\omega t)\right)}_{2X \text{ amplitude}}$$

$\boxed{4X \text{ average power!}}$

when $D_1 \neq D_2$ $\quad \bar{D} \equiv \frac{D_1+D_2}{2}$

$$y_{tot} \simeq \frac{Y}{\bar{D}}\left(\sin(kD_1-\omega t)+\sin(kD_2-\omega t)\right)$$

"sum to product formulas"

$$\boxed{\sin B + \sin C = 2\sin\tfrac{1}{2}(B+C)\cos\tfrac{1}{2}(B-C)}$$

$B=C$? $\rightarrow 2\sin B$ ✓

$C=-B$ $\rightarrow 0$ ✓

$$y_{tot} = \underbrace{\frac{2Y}{\bar{D}}\cos\left(\tfrac{1}{2}k(D_1-D_2)\right)}_{\text{new amplitude}}\ \underbrace{\sin\left(\tfrac{1}{2}k(D_1+D_2)-\omega t\right)}_{\text{time variation}}$$

$\Rightarrow$ averages to $\frac{1}{2}$

$$\text{Power} \propto \frac{1}{2}\cdot\frac{4Y^2}{\bar{D}^2}\cos^2\left(\tfrac{1}{2}k(D_1-D_2)\right)$$

$$\frac{1}{2}k(D_1 - D_2) = 0, \pi, 2\pi, \ldots$$

maximum

$$\frac{\pi}{\lambda}(D_1 - D_2) = 0, \pi, 2\pi, \ldots$$

$$|D_1 - D_2| = 0, \lambda, 2\lambda$$

maximum

$$= \frac{\lambda}{2}, \frac{3}{2}\lambda, \frac{5}{2}\lambda \ldots$$

minimum!