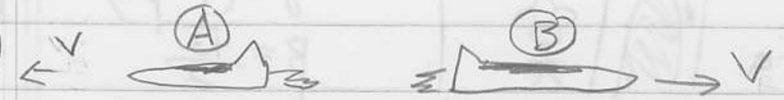
PHYS 22 HW 9

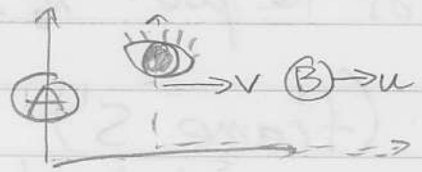
(12.5)



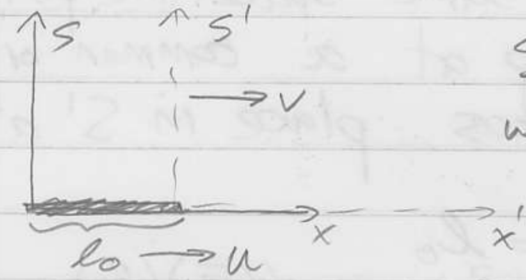
In the lab frame, we see these 2 ships flying away from one another w/ $v = 0.99c$.

So using the velocity addition formulas, to spaceship (A), it is like we are moving w/ velocity $v = 0.99c$, and spaceship (B) is moving in our frame w/ velocity $u = 0.99c$

So $\boxed{u' = \frac{u+v}{1+uv/c^2} = \frac{2(.99c)}{1+(.99)^2 c^2/c^2} = \frac{1.98}{1.9801}\,c = .99995c}$



(12.6)



So we have a rod moving w/ speed u in x in frame S. + want to know l'. Since we know for stationary objects $(u=0)$ $l' = \frac{l_0}{\gamma}$

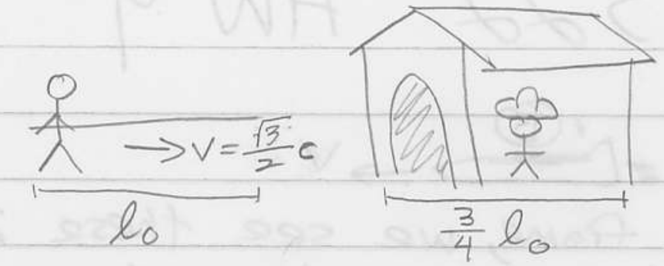
So in S', we have $u' = \frac{u-v}{1-uv/c^2}$ as the speed.

Now our $\gamma = \frac{1}{\sqrt{1-(u'/c)^2}} = \frac{1}{\sqrt{1-\frac{c^2}{c^4}\frac{(u-v)^2}{(c^2-uv)^2}}}$

$$\gamma = \frac{c^2-uv}{\sqrt{(c^2-uv)^2 - c^2(u-v)^2}} = \frac{c^2-uv}{\sqrt{c^4 - c^2(u^2+v^2) + u^2v^2}}$$

So back in $l' = \frac{l_0}{\gamma}$ form, $\boxed{l' = l_0\,\frac{\sqrt{(c^2-u^2)(c^2-v^2)}}{c^2-uv}}$

(12.10)



$\gamma = (1-\beta^2)^{-1/2} = \left(\frac{1}{4}\right)^{-1/2} = 2$

$\beta = \frac{\sqrt{3}}{2}$

So we have a discrepancy between events (i.e. pts. in spacetime) for observers in 2 reference frames, the farmer & pole vaulter.

<u>What the farmer sees:</u> (frame S)

Event A: slamming the barn door $\quad x_A = 0,\ t_A = 0$

Event B: (simultaneous w/ the slamming): $t_B = 0$

the measurement of the pole: $x_B = l = \frac{l_0}{\gamma} = \frac{l_0}{2}$

<u>What the pole vaulter sees:</u> (frame S')

Event A occurs at the same spacetime pt. (this is where the frames overlap at a common origin)

Event B: this now takes place in S' at

$$x'_B = \gamma(x_B - v\cancel{t_B}) = \gamma x_B = l_0$$

$$t'_B = \gamma\left(\cancel{t_B} - \frac{x_B v}{c^2}\right) = \gamma\left(-x_B \frac{\beta}{c}\right) = -2\left(\frac{\sqrt{3}}{\cancel{2}c}\right)\left(\frac{l_0}{\cancel{2}}\right) = -\frac{\sqrt{3}}{2c} l_0$$

So we see t'_B is negative, meaning event B in S' occured before event B in S, <u>and</u> before event A, so the pole crashed before the door was even shut... identifying this as event C

$$x_C = \frac{3}{4} l_0, \quad t_C = \frac{x_C - x_B}{v} = \frac{l_0}{4v}$$

$$t'_C = \gamma\left(t_C - \frac{\beta}{c} x_C\right) = 2\left(\frac{l_0}{4v} - \frac{1}{c}\left(\frac{\sqrt{3}}{2}\right)\left(\frac{3l_0}{4}\right)\right)$$

$$t'_C = 2\frac{l_0}{c}\left[\frac{2}{4\sqrt{3}} - \frac{3\sqrt{3}}{8}\right] = -\frac{5\sqrt{3}}{12}\frac{l_0}{c}$$

So this is still negative & less than t_A, so he crashes!

PHYS 22 HW 9

(1) a) $L = x_B - x_A = 30\text{ cm} - 0\text{ cm} = 30\text{ cm}$

$T = t_B - t_A = 0.75\text{ ns}$

$\frac{L}{T} = \frac{30\text{ cm}}{0.75\text{ ns}} = 40\frac{\text{cm}}{\text{ns}} > c = 30\frac{\text{cm}}{\text{ns}} \Rightarrow$ <u>spacelike separation</u>

We know there exists some frame where the events appear simultaneous ($T' = 0$, $L' > 0$)

$\beta = \frac{c}{L/T} = \frac{v}{c} \Rightarrow \boxed{v = \left(\frac{30}{40}\right)c = 0.75c}$

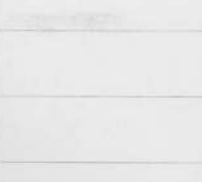
Check: $\boxed{\begin{array}{l} T' = \gamma\left(T - \beta\frac{L}{c}\right) = \gamma T\left(1 - \frac{\beta(L/T)}{c}\right) = \gamma T(1-1) = 0 \\ L' = \gamma(L - \cancel{\beta c T}) = \gamma L = (1.511)L = 45.33\text{ cm} \end{array}}$

b) $\frac{L}{T} = \frac{-30\text{ cm}}{1\text{ ns}} = \left|-30\frac{\text{cm}}{\text{ns}}\right| = c \Rightarrow$ <u>on the light cone</u>

c) $\frac{L}{T} = \frac{-60\text{ cm}}{1.5\text{ ns}} = \left|-40\frac{\text{cm}}{\text{ns}}\right| > c \Rightarrow$ <u>space like separation</u>

$\beta = \frac{c}{L/T} = \frac{v}{c} \Rightarrow \boxed{v = \left(\frac{c}{L/T}\right)c = \left(\frac{30}{-40}\right)c = -0.75c}$

✓ $\boxed{\begin{array}{l} T' = \gamma\left(T - \beta\frac{L}{c}\right) = \gamma T\left(1 - \frac{\beta(L/T)}{c}\right) = \gamma T\left(\cancel{1 - \left(\frac{-3}{4}\right)\left(-\frac{4}{3}\right)}\right) = 0 \\ L' = \gamma(L - \cancel{\beta c T}) = \gamma L = (1.511)L = -45.33\text{ cm} \end{array}}$



d) $L = \sqrt{L_x^2 + L_y^2} = \sqrt{40^2 + 30^2} = 50\text{ cm}$, $T = 4 - 5 = -1\text{ ns}$

$\frac{L}{T} = \frac{50\text{ cm}}{-1\text{ ns}} = \left|-50\frac{\text{cm}}{\text{ns}}\right| > c \Rightarrow$ <u>spacelike separation</u>

$\beta = \frac{c}{L/T} = \frac{v}{c} \Rightarrow \boxed{v = \left(\frac{c}{L/T}\right)c = \left(\frac{30}{-50}\right)c = -0.6c}$

$\boxed{L' = \gamma(L - \cancel{\beta c T}) = \gamma L = (1.25)L = 62.5\text{ cm}}$